

# Computer algebra independent integration tests

2-Exponentials/2.2-c+d-x<sup>m</sup>-F<sup>g</sup>-e+f-x<sup>n</sup>-a+b-F<sup>g</sup>-e+f-x<sup>n</sup><sup>p</sup>

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 93 ]. This is test number [ 54 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 93 )	% 0.00 ( 0 )
Mathematica	% 90.32 ( 84 )	% 9.68 ( 9 )
Maple	% 80.65 ( 75 )	% 19.35 ( 18 )
Maxima	% 77.42 ( 72 )	% 22.58 ( 21 )
Fricas	% 100.00 ( 93 )	% 0.00 ( 0 )
Sympy	% 53.76 ( 50 )	% 46.24 ( 43 )
Giac	% 55.91 ( 52 )	% 44.09 ( 41 )
Mupad	% 56.99 ( 53 )	% 43.01 ( 40 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.



grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

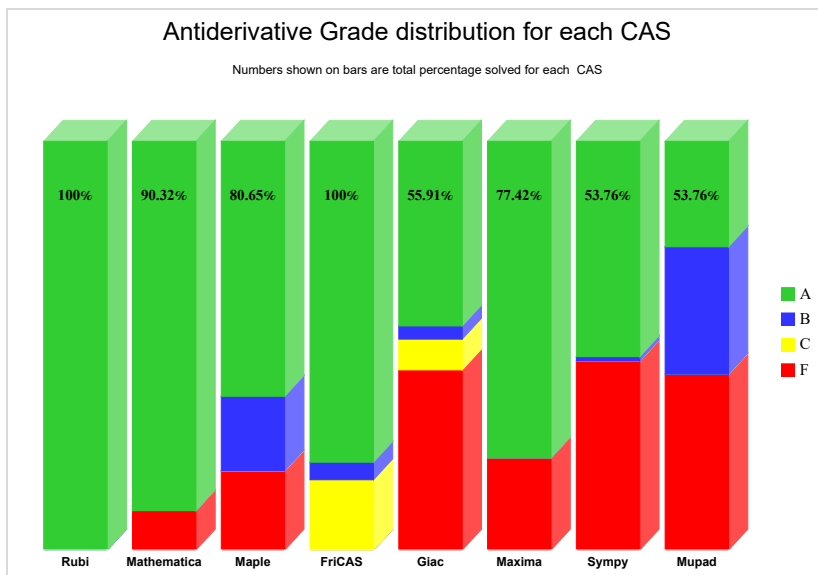
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

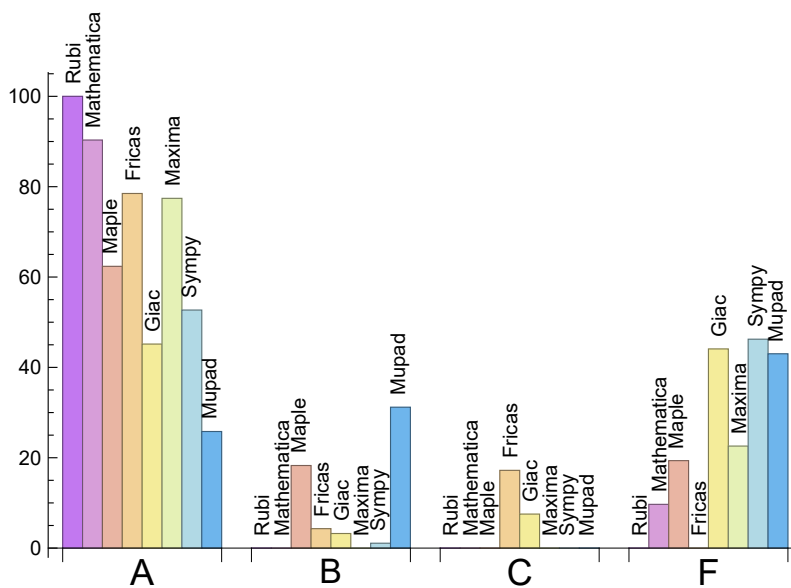
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	90.32	0.00	0.00	9.68
Maple	62.37	18.28	0.00	19.35
Maxima	77.42	0.00	0.00	22.58
Fricas	78.49	4.30	17.20	0.00
Sympy	52.69	1.08	0.00	46.24
Giac	45.16	3.23	7.53	44.09
Mupad	25.81	31.18	0.00	43.01

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	9	100.00 %	0.00 %	0.00 %
Maple	18	100.00 %	0.00 %	0.00 %
Maxima	21	100.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Sympy	43	79.07 %	20.93 %	0.00 %
Giac	41	95.12 %	4.88 %	0.00 %
Mupad	40	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

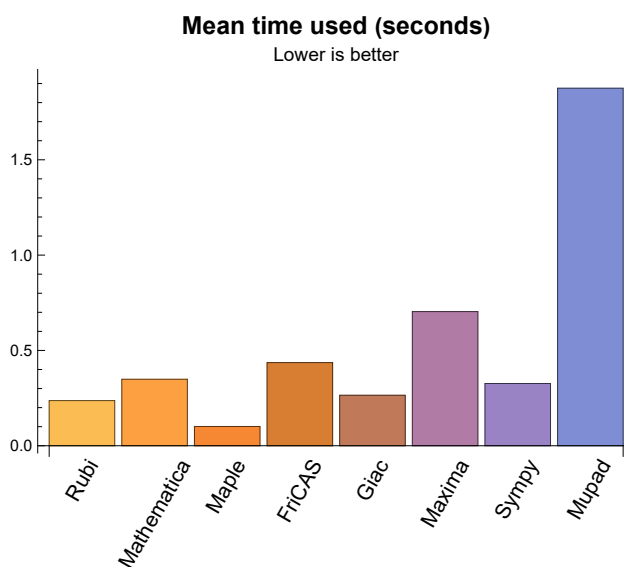
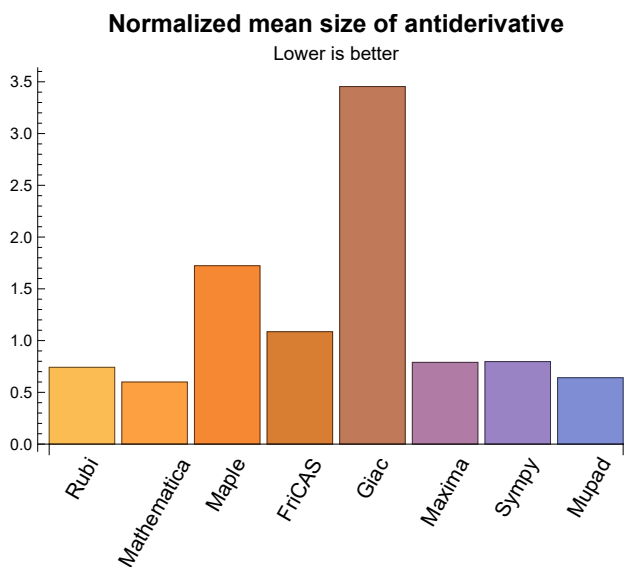
## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.24	121.83	0.74	84.00	1.00
Mathematica	0.35	80.51	0.60	62.00	0.76
Maple	0.10	326.47	1.72	77.00	1.20
Maxima	0.70	106.72	0.79	69.00	1.00
Fricas	0.44	211.45	1.09	100.00	1.07
Sympy	0.33	116.32	0.80	35.50	0.83
Giac	0.26	612.33	3.45	26.00	0.96
Mupad	1.88	75.11	0.64	24.00	0.91

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



## 1.4 list of integrals that has no closed form antiderivative

{5, 6, 13, 14, 21, 22, 50, 51, 56, 57, 62, 63, 67, 68, 69, 73, 74, 75, 80, 81, 86, 87, 92, 93}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>



[ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](http://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

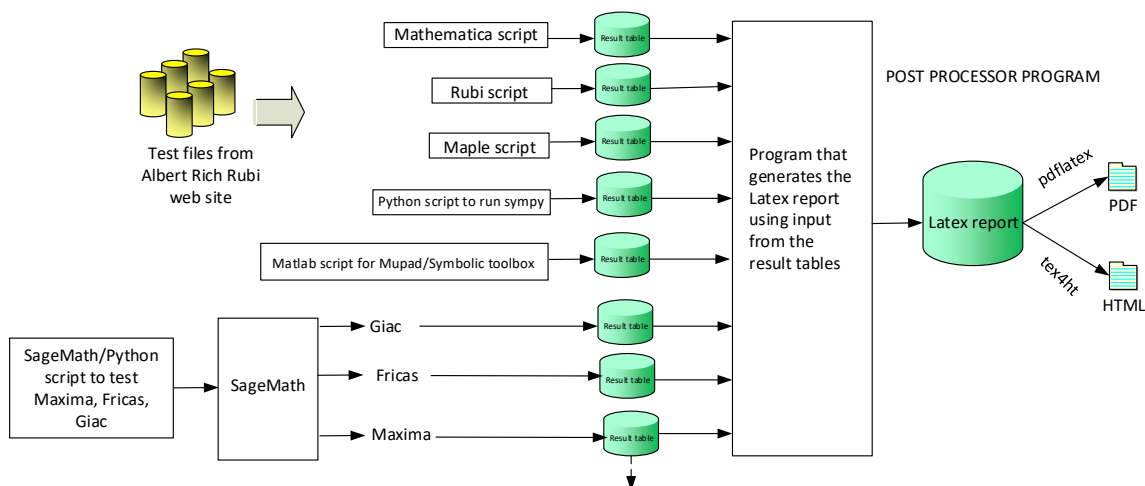
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
  2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
  3. integer. Leaf size of result.
  4. integer. Leaf size of the optimal antiderivative.
  5. number. CPU time used to solve this integral. 0 if failed.
  6. string. The integral in Latex format
  7. string. The input used in CAS own syntax.
  8. string. The result (antiderivative) produced by CAS in Latex format
  9. string. The optimal antiderivative in Latex format.
  10. integer. 0 or 1. Indicates if problem has known antiderivative or not
  11. String. The result (antiderivative) in CAS own syntax.
  12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
  15. integer. Integrand leaf size.
  16. real number. Ratio of field 14 over field 15
  17. integer. 1 if result was verified or 0 if not verified.
  18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { }

C grade: { }

F grade: { 52, 53, 54, 58, 59, 60, 70, 71, 72 }

### 2.1.3 Maple

A grade: { 1, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 26, 27, 28, 34, 35, 42, 49, 50, 51, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 2, 3, 10, 11, 19, 46, 47, 48, 52, 53, 54, 58, 59, 60, 77, 78, 83 }

C grade: { }

F grade: { 25, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 43, 44, 45, 70, 71, 72 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 32, 33, 34, 35, 39, 40, 41, 42, 49, 50, 51, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { }

C grade: { }

F grade: { 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 52, 53, 54, 58, 59, 60, 70, 71, 72 }

### 2.1.5 FriCAS

A grade: { 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 83, 84, 85, 86, 87, 90, 91, 92, 93 }

B grade: { 19, 54, 60, 89 }

C grade: { 1, 2, 9, 10, 17, 18, 46, 47, 52, 53, 58, 59, 76, 77, 82, 88 }

F grade: { }

### 2.1.6 Sympy

A grade: { 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 27, 28, 32, 33, 34, 35, 39, 40, 41, 42, 49, 50, 55, 56, 57, 61, 64, 65, 66, 67, 68, 69, 79, 80, 81, 84, 85, 86, 87, 90, 92, 93 }

B grade: { 91 }

C grade: { }

F grade: { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 58, 59, 60, 62, 63, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 88, 89 }

## 2.1.7 Giac

A grade: { 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 28, 35, 42, 49, 50, 51, 55, 56, 57, 61, 62, 63, 67, 68, 69, 73, 74, 75, 79, 80, 81, 85, 86, 87, 91, 92, 93 }

B grade: { 64, 65, 66 }

C grade: { 25, 26, 27, 33, 34, 40, 41 }

F grade: { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 32, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 52, 53, 54, 58, 59, 60, 70, 71, 72, 76, 77, 78, 82, 83, 84, 88, 89, 90 }

## 2.1.8 Mupad

A grade: { 5, 6, 13, 14, 21, 22, 50, 51, 56, 57, 62, 63, 67, 68, 69, 73, 74, 75, 80, 81, 86, 87, 92, 93 }

B grade: { 4, 7, 8, 12, 15, 16, 20, 23, 24, 25, 26, 27, 28, 32, 33, 34, 35, 39, 40, 41, 42, 49, 55, 61, 79, 84, 85, 90, 91 }

C grade: { }

F grade: { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 52, 53, 54, 58, 59, 60, 64, 65, 66, 70, 71, 72, 76, 77, 78, 82, 83, 88, 89 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	112	191	94	120	0	0	-1
normalized size	1	1.00	1.02	1.74	0.85	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.012	0.021	0.978	0.435	0.000	0.000	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	166	72	100	0	0	-1
normalized size	1	1.00	0.99	1.98	0.86	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.007	0.019	0.968	0.420	0.000	0.000	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	133	48	72	0	0	-1
normalized size	1	1.00	0.91	2.29	0.83	1.24	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.003	0.018	0.969	0.455	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	35	32	24	17	31	24
normalized size	1	1.00	1.00	1.35	1.23	0.92	0.65	1.19	0.92
time (sec)	N/A	0.018	0.004	0.010	0.903	0.446	0.123	0.330	3.497

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.048	0.045	0.053	0.000	0.442	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.045	0.131	0.047	0.000	0.458	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	21	37	34	23	17	33	23
normalized size	1	1.00	0.81	1.42	1.31	0.88	0.65	1.27	0.88
time (sec)	N/A	0.021	0.017	0.010	0.884	0.431	0.127	0.337	3.474

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	19	41	34	25	19	33	25
normalized size	1	1.00	0.68	1.46	1.21	0.89	0.68	1.18	0.89
time (sec)	N/A	0.023	0.016	0.010	0.881	0.428	0.132	0.378	0.048

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	158	382	195	330	0	0	-1
normalized size	1	1.00	0.73	1.76	0.90	1.52	0.00	0.00	-0.00
time (sec)	N/A	0.507	0.243	0.141	1.030	0.462	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	113	324	149	263	0	0	-1
normalized size	1	1.00	0.68	1.96	0.90	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.399	0.244	0.114	1.022	0.478	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	85	231	95	176	0	0	-1
normalized size	1	1.00	0.79	2.16	0.89	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.118	0.030	1.008	0.429	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	54	51	60	39	51	66
normalized size	1	1.00	0.87	1.17	1.11	1.30	0.85	1.11	1.43
time (sec)	N/A	0.033	0.039	0.012	1.006	0.442	0.149	0.440	3.559

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.047	0.816	0.158	0.000	0.417	0.000	0.000	0.000



Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.044	0.636	0.174	0.000	0.442	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	58	55	65	39	54	68
normalized size	1	1.00	0.88	1.21	1.15	1.35	0.81	1.12	1.42
time (sec)	N/A	0.034	0.055	0.013	1.093	0.426	0.158	0.239	0.163

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	64	57	73	42	58	76
normalized size	1	1.00	0.67	1.23	1.10	1.40	0.81	1.12	1.46
time (sec)	N/A	0.035	0.058	0.011	0.953	0.486	0.168	0.225	3.499

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	241	548	303	702	0	0	-1
normalized size	1	1.00	0.72	1.65	0.91	2.11	0.00	0.00	-0.00
time (sec)	N/A	1.078	0.254	0.167	1.084	0.447	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	203	385	234	521	0	0	-1
normalized size	1	1.00	0.84	1.58	0.96	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.764	0.190	0.139	1.064	0.412	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	120	393	149	338	0	0	-1
normalized size	1	1.00	0.75	2.47	0.94	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.332	0.148	0.038	1.055	0.468	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	62	74	84	127	76	65	121
normalized size	1	1.00	0.90	1.07	1.22	1.84	1.10	0.94	1.75
time (sec)	N/A	0.042	0.062	0.012	0.913	0.432	0.188	0.217	3.745

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.045	1.004	0.187	0.000	0.427	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.044	0.777	0.201	0.000	0.458	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	79	88	132	78	71	124
normalized size	1	1.00	0.86	1.10	1.22	1.83	1.08	0.99	1.72
time (sec)	N/A	0.044	0.093	0.014	0.900	0.435	0.199	0.207	3.651

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	54	87	92	140	85	75	132
normalized size	1	1.00	0.69	1.12	1.18	1.79	1.09	0.96	1.69
time (sec)	N/A	0.047	0.096	0.013	0.875	0.420	0.205	0.185	3.700

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	130	0	294	267	332	5732	225
normalized size	1	1.00	0.85	0.00	1.92	1.75	2.17	37.46	1.47
time (sec)	N/A	0.238	0.278	0.079	1.351	0.452	0.278	0.576	3.815

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	91	199	178	166	196	2726	135
normalized size	1	1.00	0.79	1.73	1.55	1.44	1.70	23.70	1.17
time (sec)	N/A	0.154	0.169	0.050	1.256	0.457	0.220	0.442	3.672

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	73	105	88	87	94	1117	72
normalized size	1	1.00	0.95	1.36	1.14	1.13	1.22	14.51	0.94
time (sec)	N/A	0.076	0.167	0.038	1.078	0.495	0.170	0.374	3.595

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	31	37	32	32	31
normalized size	1	1.00	1.00	1.03	1.03	1.23	1.07	1.07	1.03
time (sec)	N/A	0.016	0.026	0.004	0.927	0.469	0.125	0.201	3.604

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	0	0	50	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.150	0.079	0.000	0.445	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	0	0	87	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.278	0.077	0.000	0.427	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	111	0	0	160	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.333	0.075	0.000	0.430	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	239	0	604	482	709	0	438
normalized size	1	1.00	0.74	0.00	1.88	1.50	2.20	0.00	1.36
time (sec)	N/A	0.486	0.558	0.095	1.885	0.429	0.476	0.000	3.970

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	171	0	368	287	439	5699	267
normalized size	1	1.00	0.72	0.00	1.54	1.20	1.84	23.85	1.12
time (sec)	N/A	0.320	0.370	0.083	1.612	0.430	0.368	1.232	3.816

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	117	220	187	139	233	2312	146
normalized size	1	1.00	0.75	1.41	1.20	0.89	1.49	14.82	0.94
time (sec)	N/A	0.157	0.263	0.056	1.245	0.436	0.272	0.981	3.669

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	52	90	75	61	94	77	75
normalized size	1	1.00	0.78	1.34	1.12	0.91	1.40	1.15	1.12
time (sec)	N/A	0.037	0.038	0.022	1.194	0.429	0.177	0.489	3.719

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	108	0	0	95	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.315	0.088	0.000	0.424	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	136	0	0	171	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.85	0.00	0.00	-0.00
time (sec)	N/A	0.341	0.685	0.077	0.000	0.418	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	217	0	0	314	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.730	0.076	0.000	0.463	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	341	0	914	708	1074	0	652
normalized size	1	1.00	0.69	0.00	1.84	1.43	2.17	0.00	1.31
time (sec)	N/A	0.705	0.743	0.093	2.369	0.432	0.653	0.000	4.372

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	248	0	558	414	653	8854	399
normalized size	1	1.00	0.68	0.00	1.52	1.13	1.78	24.19	1.09
time (sec)	N/A	0.464	0.530	0.088	1.814	0.441	0.479	1.662	3.993

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	161	0	284	192	350	3572	219
normalized size	1	1.00	0.68	0.00	1.20	0.81	1.48	15.14	0.93
time (sec)	N/A	0.239	0.375	0.033	1.421	0.433	0.340	1.330	3.765

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	74	124	115	84	153	102	124
normalized size	1	1.00	0.72	1.20	1.12	0.82	1.49	0.99	1.20
time (sec)	N/A	0.049	0.050	0.014	1.562	0.433	0.209	0.435	3.775

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	160	0	0	140	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.354	0.498	0.080	0.000	0.443	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	250	0	0	259	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.85	0.00	0.00	-0.00
time (sec)	N/A	0.491	1.325	0.077	0.000	0.435	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	325	0	0	475	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	1.06	0.00	0.00	-0.00
time (sec)	N/A	0.695	1.059	0.078	0.000	0.435	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	166	3075	0	413	0	0	-1
normalized size	1	1.00	0.86	16.02	0.00	2.15	0.00	0.00	-0.01
time (sec)	N/A	0.326	0.112	0.091	0.000	0.437	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	121	1677	0	271	0	0	-1
normalized size	1	1.00	0.83	11.57	0.00	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.035	0.114	0.000	0.414	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	74	679	0	147	0	0	-1
normalized size	1	1.00	0.76	6.93	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.012	0.092	0.000	0.421	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	65	66	44	27	74	44
normalized size	1	1.00	1.00	1.62	1.65	1.10	0.68	1.85	1.10
time (sec)	N/A	0.030	0.008	0.020	1.144	0.433	0.155	0.349	3.449

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.137	0.134	0.132	0.000	0.412	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.128	0.385	0.155	0.000	0.427	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	0	3319	0	1390	0	0	-1
normalized size	1	1.00	0.00	8.55	0.00	3.58	0.00	0.00	-0.00
time (sec)	N/A	0.867	1.814	0.085	0.000	0.462	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	0	1650	0	834	0	0	-1
normalized size	1	1.00	0.00	5.61	0.00	2.84	0.00	0.00	-0.00
time (sec)	N/A	0.671	1.183	0.077	0.000	0.440	0.000	0.000	0.000



Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	0	591	0	400	0	0	-1
normalized size	1	1.00	0.00	3.09	0.00	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.340	1.131	0.112	0.000	0.432	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	99	100	100	66	111	80
normalized size	1	1.00	0.84	1.34	1.35	1.35	0.89	1.50	1.08
time (sec)	N/A	0.049	0.061	0.022	1.190	0.423	0.190	0.415	3.445

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.127	1.015	0.589	0.000	0.423	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.120	1.052	0.446	0.000	0.426	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	0	4005	0	2704	0	0	-1
normalized size	1	1.00	0.00	6.74	0.00	4.55	0.00	0.00	-0.00
time (sec)	N/A	1.901	1.814	0.095	0.000	0.496	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	0	1887	0	1518	0	0	-1
normalized size	1	1.00	0.00	4.30	0.00	3.46	0.00	0.00	-0.00
time (sec)	N/A	1.319	1.371	0.081	0.000	0.478	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	0	668	0	696	0	0	-1
normalized size	1	1.00	0.00	2.42	0.00	2.52	0.00	0.00	-0.00
time (sec)	N/A	0.564	0.977	0.069	0.000	0.430	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	84	134	145	190	116	136	142
normalized size	1	1.00	0.76	1.21	1.31	1.71	1.05	1.23	1.28
time (sec)	N/A	0.063	0.107	0.021	1.216	0.424	0.241	0.382	3.502

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.128	1.867	0.439	0.000	0.430	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.121	1.490	0.712	0.000	0.433	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	77	82	70	85	132	-1
normalized size	1	1.00	1.00	1.08	1.15	0.99	1.20	1.86	-0.01
time (sec)	N/A	0.088	0.102	0.018	1.173	0.434	2.135	0.337	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	134	144	160	133	184	250	-1
normalized size	1	1.00	0.92	0.99	1.10	0.92	1.27	1.72	-0.01
time (sec)	N/A	0.185	0.324	0.017	2.485	0.425	3.213	0.418	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	196	211	238	196	291	368	-1
normalized size	1	1.00	0.88	0.94	1.06	0.88	1.30	1.64	-0.00
time (sec)	N/A	0.264	0.797	0.018	2.415	0.419	4.360	0.527	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.040	0.683	0.088	0.000	0.420	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.040	1.265	0.088	0.000	0.420	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.039	2.365	0.099	0.000	0.442	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	270	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.467	0.336	0.080	0.000	0.448	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	0	0	0	191	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.215	0.081	0.000	0.450	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	111	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.123	0.079	0.000	0.442	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.123	0.108	0.098	0.000	0.425	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.120	0.137	0.463	0.000	0.421	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.119	0.287	0.209	0.000	0.441	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	225	133	134	0	0	-1
normalized size	1	1.00	1.00	1.96	1.16	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.013	0.072	0.482	0.441	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	194	107	108	0	0	-1
normalized size	1	1.00	1.00	2.28	1.26	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.009	0.064	0.485	0.434	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	154	79	75	0	0	-1
normalized size	1	1.00	1.00	2.85	1.46	1.39	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.006	0.061	0.481	0.430	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	17	24	23
normalized size	1	1.00	1.00	1.04	1.00	1.00	0.74	1.04	1.00
time (sec)	N/A	0.037	0.007	0.004	0.429	0.418	0.138	0.421	3.429

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.070	0.086	0.083	0.000	0.426	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	0.068	0.080	0.000	0.428	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	137	274	134	246	0	0	-1
normalized size	1	1.00	0.98	1.96	0.96	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.161	0.053	0.524	0.423	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	103	231	106	186	0	0	-1
normalized size	1	1.00	0.96	2.16	0.99	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.093	0.049	0.498	0.417	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	54	67	72	74	58	0	63
normalized size	1	1.00	0.78	0.97	1.04	1.07	0.84	0.00	0.91
time (sec)	N/A	0.074	0.071	0.025	0.470	0.416	0.183	0.000	3.623

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	25	26	26	31
normalized size	1	1.00	1.00	1.04	1.00	1.00	1.04	1.04	1.24
time (sec)	N/A	0.035	0.021	0.002	0.429	0.428	0.123	0.316	3.463

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.118	0.131	0.086	0.000	0.434	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.118	0.161	0.088	0.000	0.432	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	220	501	263	577	0	0	-1
normalized size	1	1.00	0.84	1.92	1.01	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.504	0.323	0.083	0.525	0.465	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	177	304	214	379	0	0	-1
normalized size	1	1.00	0.97	1.67	1.18	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.291	0.166	0.076	0.510	0.423	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	98	127	150	148	122	0	155
normalized size	1	1.00	0.92	1.20	1.42	1.40	1.15	0.00	1.46
time (sec)	N/A	0.089	0.091	0.033	0.482	0.429	0.226	0.000	3.753

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	46	53	26	25
normalized size	1	1.00	1.00	0.96	0.93	1.70	1.96	0.96	0.93
time (sec)	N/A	0.035	0.027	0.004	0.429	0.407	0.153	0.325	3.633

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.116	0.736	0.220	0.000	0.421	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.113	0.776	0.253	0.000	0.402	0.000	0.000	0.000



## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [19] had the largest ratio of [.7333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.00	17	0.353
2	A	5	5	1.00	17	0.294
3	A	4	4	1.00	15	0.267
4	A	4	4	1.00	13	0.308
5	A	0	0	0.00	0	0.000
6	A	0	0	0.00	0	0.000
7	A	4	4	1.00	14	0.286
8	A	4	4	1.00	16	0.250
9	A	13	8	1.00	17	0.471
10	A	11	9	1.00	17	0.529
11	A	10	10	1.00	15	0.667
12	A	3	2	1.00	13	0.154
13	A	0	0	0.00	0	0.000
14	A	0	0	0.00	0	0.000
15	A	3	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	2	1.00	16	0.125
17	A	26	10	1.00	17	0.588
18	A	23	12	1.00	17	0.706
19	A	15	11	1.00	15	0.733
20	A	3	2	1.00	13	0.154
21	A	0	0	0.00	0	0.000
22	A	0	0	0.00	0	0.000
23	A	3	2	1.00	14	0.143
24	A	3	2	1.00	16	0.125
25	A	6	3	1.00	23	0.130
26	A	5	3	1.00	23	0.130
27	A	4	3	1.00	21	0.143
28	A	2	1	1.00	15	0.067
29	A	4	3	1.00	23	0.130
30	A	5	4	1.00	23	0.174
31	A	6	4	1.00	23	0.174
32	A	10	3	1.00	25	0.120
33	A	8	3	1.00	25	0.120
34	A	6	3	1.00	23	0.130
35	A	4	3	1.00	17	0.176
36	A	6	3	1.00	25	0.120
37	A	8	4	1.00	25	0.160
38	A	10	4	1.00	25	0.160
39	A	14	3	1.00	25	0.120
40	A	11	3	1.00	25	0.120
41	A	8	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	4	3	1.00	17	0.176
43	A	8	3	1.00	25	0.120
44	A	11	4	1.00	25	0.160
45	A	14	4	1.00	25	0.160
46	A	6	6	1.00	25	0.240
47	A	5	5	1.00	25	0.200
48	A	4	4	1.00	23	0.174
49	A	5	5	1.00	17	0.294
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	13	8	1.00	25	0.320
53	A	11	9	1.00	25	0.360
54	A	11	11	1.00	23	0.478
55	A	4	3	1.00	17	0.176
56	A	0	0	0.00	0	0.000
57	A	0	0	0.00	0	0.000
58	A	26	10	1.00	25	0.400
59	A	24	13	1.00	25	0.520
60	A	17	12	1.00	23	0.522
61	A	4	3	1.00	17	0.176
62	A	0	0	0.00	0	0.000
63	A	0	0	0.00	0	0.000
64	A	5	4	1.00	17	0.235
65	A	8	4	1.00	19	0.210
66	A	11	4	1.00	19	0.210
67	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	0	0	0.00	0	0.000
69	A	0	0	0.00	0	0.000
70	A	8	3	1.00	25	0.120
71	A	6	3	1.00	25	0.120
72	A	4	3	1.00	23	0.130
73	A	0	0	0.00	0	0.000
74	A	0	0	0.00	0	0.000
75	A	0	0	0.00	0	0.000
76	A	5	5	1.00	24	0.208
77	A	4	4	1.00	24	0.167
78	A	3	3	1.00	22	0.136
79	A	2	2	1.00	21	0.095
80	A	0	0	0.00	0	0.000
81	A	0	0	0.00	0	0.000
82	A	6	6	1.00	24	0.250
83	A	5	5	1.00	24	0.208
84	A	5	5	1.00	22	0.227
85	A	2	2	1.00	21	0.095
86	A	0	0	0.00	0	0.000
87	A	0	0	0.00	0	0.000
88	A	12	9	1.00	24	0.375
89	A	11	10	1.00	24	0.417
90	A	4	3	1.00	22	0.136
91	A	2	2	1.00	21	0.095
92	A	0	0	0.00	0	0.000
93	A	0	0	0.00	0	0.000





# Chapter 3

## Listing of integrals

### 3.1 $\int \frac{x^3}{a+be^{c+dx}} dx$

**Optimal.** Leaf size=110

$$-\frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{ad^4} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} - \frac{x^3 \log\left(\frac{be^{c+dx}}{a} + 1\right)}{ad} + \frac{x^4}{4a}$$

[Out]  $\frac{1}{4}x^4/a - x^3 \ln(1+b \exp(d*x+c)/a)/a/d - 3x^2 \text{polylog}(2, -b \exp(d*x+c)/a)/a/d^2 + 6x \text{polylog}(3, -b \exp(d*x+c)/a)/a/d^3 - 6 \text{polylog}(4, -b \exp(d*x+c)/a)/a/d^4$

**Rubi [A]** time = 0.19, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3x^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6 \text{PolyLog}\left(4, -\frac{be^{c+dx}}{a}\right)}{ad^4} - \frac{x^3 \log\left(\frac{be^{c+dx}}{a} + 1\right)}{ad} + \frac{x^4}{4a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/(a + b * E^{(c + d*x)}), x]$

[Out]  $x^4/(4*a) - (x^3 * \text{Log}[1 + (b * E^{(c + d*x)})/a])/(a*d) - (3*x^2 * \text{PolyLog}[2, -((b * E^{(c + d*x)})/a)])/(a*d^2) + (6*x * \text{PolyLog}[3, -((b * E^{(c + d*x)})/a)])/(a*d^3) - (6 * \text{PolyLog}[4, -((b * E^{(c + d*x)})/a)])/(a*d^4)$

**Rule 2184**

$\text{Int}[\frac{(c_1 + (d_1 * x_1)^{m_1})}{(a_1 + (b_1 * (F_1)^{(g_1 * (e_1) + (f_1 * x_1))))^{n_1}}, x\_Symbol] := \text{Simp}[(c + d*x)^{(m + 1)} / (a*d*(m + 1)), x] - \text{Dist}[\dots]$

$b/a$ , Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_))]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[(((e\_) + (f\_)\*(x\_))^(m\_))\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(p\_)]), x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{x^3}{a + be^{c+dx}} dx &= \frac{x^4}{4a} - \frac{b \int \frac{e^{c+dx} x^3}{a + be^{c+dx}} dx}{a} \\
&= \frac{x^4}{4a} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} + \frac{3 \int x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right) dx}{ad} \\
&= \frac{x^4}{4a} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2 \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6 \int x \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right) dx}{ad^2} \\
&= \frac{x^4}{4a} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2 \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x \text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6 \int \text{Li}_3\left(-\frac{be^{c+dx}}{a}\right) dx}{ad^3} \\
&= \frac{x^4}{4a} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2 \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x \text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6 \text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{bx}{a}\right)}{x} dx, x, e^{c+dx}\right)}{ad^4} \\
&= \frac{x^4}{4a} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2 \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x \text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6 \text{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{ad^4}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 112, normalized size = 1.02

$$\frac{6 \text{Li}_4\left(-\frac{ae^{-c-dx}}{b}\right)}{ad^4} + \frac{6x \text{Li}_3\left(-\frac{ae^{-c-dx}}{b}\right)}{ad^3} + \frac{3x^2 \text{Li}_2\left(-\frac{ae^{-c-dx}}{b}\right)}{ad^2} - \frac{x^3 \log\left(\frac{ae^{-c-dx}}{b} + 1\right)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*E^(c + d\*x)),x]

[Out] -((x^3\*Log[1 + (a\*E^(-c - d\*x))/b])/(a\*d)) + (3\*x^2\*PolyLog[2, -((a\*E^(-c - d\*x))/b)])/(a\*d^2) + (6\*x\*PolyLog[3, -((a\*E^(-c - d\*x))/b)])/(a\*d^3) + (6\*PolyLog[4, -((a\*E^(-c - d\*x))/b)])/(a\*d^4)

**fricas [C]** time = 0.43, size = 120, normalized size = 1.09

$$\frac{d^4 x^4 - 12 d^2 x^2 \text{Li}_2\left(-\frac{be^{(dx+c)} + a}{a}\right) + 4 c^3 \log\left(be^{(dx+c)} + a\right) + 24 dx \text{polylog}\left(3, -\frac{be^{(dx+c)}}{a}\right) - 4 (d^3 x^3 + c^3) \log\left(\frac{be^{(dx+c)}}{a}\right)}{4 ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*exp(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{4}*(d^4*x^4 - 12*d^2*x^2*\text{dilog}(-b*e^{(d*x + c)} + a)/a + 1) + 4*c^3*\log(b*e^{(d*x + c)} + a) + 24*d*x*\text{polylog}(3, -b*e^{(d*x + c)}/a) - 4*(d^3*x^3 + c^3)*\log((b*e^{(d*x + c)} + a)/a) - 24*\text{polylog}(4, -b*e^{(d*x + c)}/a))/(a*d^4)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{be^{(dx+c)} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+b*exp(d*x+c)),x, algorithm="giac")`

[Out] `integrate(x^3/(b*e^(d*x + c) + a), x)`

**maple** [A] time = 0.02, size = 191, normalized size = 1.74

$$\frac{x^4}{4a} - \frac{x^3 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{ad} + \frac{c^3 x}{a d^3} - \frac{3x^2 \text{polylog}\left(2, -\frac{be^{dx+c}}{a}\right)}{a d^2} + \frac{3c^4}{4a d^4} - \frac{c^3 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{a d^4} + \frac{c^3 \ln(b e^{dx+c} + a)}{a d^4} - \frac{c^3 \ln(e^{dx+c})}{a d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a+b*exp(d*x+c)),x)`

[Out]  $\frac{1}{4}*x^4/a + 1/d^3/a*x*c^3 + 3/4/d^4/a*c^4 - x^3*\ln(1+b*\exp(d*x+c)/a)/a/d - 1/d^4/a*\ln(1+b*\exp(d*x+c)/a)*c^3 - 3*x^2*\text{polylog}(2, -b*\exp(d*x+c)/a)/a/d^2 + 6*x*\text{polylog}(3, -b*\exp(d*x+c)/a)/a/d^3 - 6*\text{polylog}(4, -b*\exp(d*x+c)/a)/a/d^4 - 1/d^4*c^3/a*\ln(\exp(d*x+c)) + 1/d^4*c^3/a*\ln(a+b*\exp(d*x+c))$

**maxima** [A] time = 0.98, size = 94, normalized size = 0.85

$$\frac{x^4}{4a} - \frac{d^3 x^3 \log\left(\frac{be^{(dx+c)}}{a} + 1\right) + 3d^2 x^2 \text{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 6dx \text{Li}_3\left(-\frac{be^{(dx+c)}}{a}\right) + 6 \text{Li}_4\left(-\frac{be^{(dx+c)}}{a}\right)}{ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+b*exp(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{4}*x^4/a - (d^3*x^3*\log(b*e^{(d*x + c)}/a + 1) + 3*d^2*x^2*\text{dilog}(-b*e^{(d*x + c)}/a) - 6*d*x*\text{polylog}(3, -b*e^{(d*x + c)}/a) + 6*\text{polylog}(4, -b*e^{(d*x + c)}/a))/(a*d^4)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{a + b e^{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*exp(c + d*x)),x)
```

```
[Out] int(x^3/(a + b*exp(c + d*x)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^3}{a + be^ce^{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*exp(d*x+c)),x)
```

```
[Out] Integral(x**3/(a + b*exp(c)*exp(d*x)), x)
```

## 3.2 $\int \frac{x^2}{a+be^{c+dx}} dx$

**Optimal.** Leaf size=84

$$\frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} - \frac{x^2 \log\left(\frac{be^{c+dx}}{a} + 1\right)}{ad} + \frac{x^3}{3a}$$

[Out]  $1/3*x^3/a - x^2*\ln(1+b*\exp(d*x+c)/a)/a/d - 2*x*polylog(2, -b*\exp(d*x+c)/a)/a/d^2 + 2*polylog(3, -b*\exp(d*x+c)/a)/a/d^3$

**Rubi [A]** time = 0.17, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {2184, 2190, 2531, 2282, 6589}

$$-\frac{2x\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{x^2 \log\left(\frac{be^{c+dx}}{a} + 1\right)}{ad} + \frac{x^3}{3a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(a + b*E^{(c + d*x)}), x]$

[Out]  $x^3/(3*a) - (x^2*\text{Log}[1 + (b*E^{(c + d*x)})/a])/(a*d) - (2*x*\text{PolyLog}[2, -((b*E^{(c + d*x)})/a)])/(a*d^2) + (2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/a)])/(a*d^3)$

### Rule 2184

$\text{Int}[\frac{(c + d*x)^m}{(a + b*(F^{(g*(e + f*x)))^n})}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{m+1}/(a*d*(m+1)), x] - \text{Dist}[b/a, \text{Int}[\frac{(c + d*x)^m*(F^{(g*(e + f*x)))^n}}{(a + b*(F^{(g*(e + f*x)))^n})}, x], x] /;$   $\text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\amp; \ \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[\frac{(F^{(g*(e + f*x)))^n*(c + d*x)^m}{(a + b*(F^{(g*(e + f*x)))^n})}, x\_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]}{(b*f*g*n*\text{Log}[F])}, x] - \text{Dist}[\frac{(d*m)}{(b*f*g*n*\text{Log}[F])}, \text{Int}[\frac{(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]}{x}, x], x] /;$   $\text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\amp; \ \text{IGtQ}[m, 0]$

### Rule 2282

$\text{Int}[u, x\_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$   $\text{FunctionOfExponentialQ}[u, x] \ \&\amp; \ !\text{MatchQ}[u, (w_*)*(a_*)*(v_*)^n)^m] /;$   $\text{FreeQ}[$

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.) \*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{a + be^{c+dx}} dx &= \frac{x^3}{3a} - \frac{b \int \frac{e^{c+dx} x^2}{a + be^{c+dx}} dx}{a} \\
 &= \frac{x^3}{3a} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} + \frac{2 \int x \log\left(1 + \frac{be^{c+dx}}{a}\right) dx}{ad} \\
 &= \frac{x^3}{3a} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{2x \operatorname{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{2 \int \operatorname{Li}_2\left(-\frac{be^{c+dx}}{a}\right) dx}{ad^2} \\
 &= \frac{x^3}{3a} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{2x \operatorname{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{2 \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{bx}{a}\right)}{x} dx, x, e^{c+dx}\right)}{ad^3} \\
 &= \frac{x^3}{3a} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{2x \operatorname{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{2 \operatorname{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 83, normalized size = 0.99

$$\frac{2 \operatorname{Li}_3\left(-\frac{ae^{-c-dx}}{b}\right)}{ad^3} + \frac{2x \operatorname{Li}_2\left(-\frac{ae^{-c-dx}}{b}\right)}{ad^2} - \frac{x^2 \log\left(\frac{ae^{-c-dx}}{b} + 1\right)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*E^(c + d\*x)),x]

[Out] -((x^2\*Log[1 + (a\*E^(-c - d\*x))/b])/(a\*d)) + (2\*x\*PolyLog[2, -((a\*E^(-c - d\*x))/b)))/(a\*d^2) + (2\*PolyLog[3, -((a\*E^(-c - d\*x))/b)))/(a\*d^3)

**fricas** [C] time = 0.42, size = 100, normalized size = 1.19

$$\frac{d^3 x^3 - 6 dx \operatorname{Li}_2\left(-\frac{be^{(dx+c)+a}}{a} + 1\right) - 3c^2 \log\left(be^{(dx+c)} + a\right) - 3(d^2 x^2 - c^2) \log\left(\frac{be^{(dx+c)+a}}{a}\right) + 6 \operatorname{polylog}\left(3, -\frac{be^{(dx+c)}}{a}\right)}{3ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(d\*x+c)),x, algorithm="fricas")

[Out] 1/3\*(d^3\*x^3 - 6\*d\*x\*dilog(-(b\*e^(d\*x + c) + a)/a + 1) - 3\*c^2\*log(b\*e^(d\*x + c) + a) - 3\*(d^2\*x^2 - c^2)\*log((b\*e^(d\*x + c) + a)/a) + 6\*polylog(3, -b\*e^(d\*x + c)/a))/(a\*d^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{be^{(dx+c)} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(d\*x+c)),x, algorithm="giac")

[Out] integrate(x^2/(b\*e^(d\*x + c) + a), x)

**maple** [B] time = 0.02, size = 166, normalized size = 1.98

$$\frac{x^3}{3a} - \frac{x^2 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{ad} - \frac{c^2 x}{a d^2} - \frac{2c^3}{3a d^3} + \frac{c^2 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{a d^3} - \frac{c^2 \ln\left(be^{dx+c} + a\right)}{a d^3} + \frac{c^2 \ln\left(e^{dx+c}\right)}{a d^3} - \frac{2x \operatorname{polylog}\left(2, -\frac{be^{dx+c}}{a}\right)}{a d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*exp(d\*x+c)+a),x)

[Out] 1/3\*x^3/a-1/d^2/a\*x\*c^2-2/3/d^3/a\*c^3-x^2\*ln(1/a\*b\*exp(d\*x+c)+1)/a/d+1/d^3/a\*ln(1/a\*b\*exp(d\*x+c)+1)\*c^2-2\*x\*polylog(2,-1/a\*b\*exp(d\*x+c))/a/d^2+2\*polylog(3,-1/a\*b\*exp(d\*x+c))/a/d^3+1/d^3\*c^2/a\*ln(exp(d\*x+c))-1/d^3\*c^2/a\*ln(b\*exp(d\*x+c)+a)

**maxima** [A] time = 0.97, size = 72, normalized size = 0.86

$$\frac{x^3}{3a} - \frac{d^2 x^2 \log\left(\frac{be^{(dx+c)}}{a} + 1\right) + 2 dx \operatorname{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 2 \operatorname{Li}_3\left(-\frac{be^{(dx+c)}}{a}\right)}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{3}x^3/a - (d^2x^2 \log(b e^{(d x + c)}/a + 1) + 2 d x \operatorname{dilog}(-b e^{(d x + c)}/a) - 2 \operatorname{polylog}(3, -b e^{(d x + c)}/a)) / (a d^3)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{a + b e^{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b\*exp(c + d\*x)),x)

[Out] int(x^2/(a + b\*exp(c + d\*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b e^{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*exp(d\*x+c)),x)

[Out] Integral(x\*\*2/(a + b\*exp(c)\*exp(d\*x)), x)

### 3.3 $\int \frac{x}{a+be^{c+dx}} dx$

Optimal. Leaf size=58

$$-\frac{\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} - \frac{x \log\left(\frac{be^{c+dx}}{a} + 1\right)}{ad} + \frac{x^2}{2a}$$

[Out]  $1/2*x^2/a-x*\ln(1+b*\exp(d*x+c)/a)/a/d-\operatorname{polylog}(2,-b*\exp(d*x+c)/a)/a/d^2$

**Rubi [A]** time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2184, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{ad^2} - \frac{x \log\left(\frac{be^{c+dx}}{a} + 1\right)}{ad} + \frac{x^2}{2a}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*E^(c + d\*x)), x]

[Out]  $x^2/(2*a) - (x*\operatorname{Log}[1 + (b*E^{(c + d*x)})/a])/(a*d) - \operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/a)]/(a*d^2)$

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```



Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x}{a + be^{c+dx}} dx &= \frac{x^2}{2a} - \frac{b \int \frac{e^{c+dx} x}{a + be^{c+dx}} dx}{a} \\
 &= \frac{x^2}{2a} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} + \frac{\int \log\left(1 + \frac{be^{c+dx}}{a}\right) dx}{ad} \\
 &= \frac{x^2}{2a} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} + \frac{\text{Subst}\left(\int \frac{\log\left(1 + \frac{bx}{a}\right)}{x} dx, x, e^{c+dx}\right)}{ad^2} \\
 &= \frac{x^2}{2a} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 53, normalized size = 0.91

$$\frac{\text{Li}_2\left(-\frac{ae^{-c-dx}}{b}\right)}{ad^2} - \frac{x \log\left(\frac{ae^{-c-dx}}{b} + 1\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*E^(c + d*x)), x]
```

```
[Out] -((x*Log[1 + (a*E^(-c - d*x))/b])/(a*d)) + PolyLog[2, -((a*E^(-c - d*x))/b)]/(a*d^2)
```

**fricas [A]** time = 0.46, size = 72, normalized size = 1.24

$$\frac{d^2 x^2 + 2c \log\left(be^{(dx+c)} + a\right) - 2(dx+c) \log\left(\frac{be^{(dx+c)+a}}{a}\right) - 2 \text{Li}_2\left(-\frac{be^{(dx+c)+a}}{a} + 1\right)}{2ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*x^2 + 2*c*log(b*e^(d*x + c) + a) - 2*(d*x + c)*log((b*e^(d*x + c) + a)/a) - 2*dilog(-(b*e^(d*x + c) + a)/a + 1))/(a*d^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{be^{(dx+c)} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c)),x, algorithm="giac")

[Out] integrate(x/(b\*e^(d\*x + c) + a), x)

**maple** [B] time = 0.02, size = 133, normalized size = 2.29

$$\frac{x^2}{2a} + \frac{cx}{ad} - \frac{x \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{ad} + \frac{c^2}{2ad^2} - \frac{c \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{ad^2} + \frac{c \ln\left(b e^{dx+c} + a\right)}{ad^2} - \frac{c \ln\left(e^{dx+c}\right)}{ad^2} - \frac{\text{polylog}\left(2, -\frac{be^{dx+c}}{a}\right)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*exp(d\*x+c)+a),x)

[Out] 1/2\*x^2/a+1/d/a\*x\*c+1/2/d^2/a\*c^2-x\*ln(1/a\*b\*exp(d\*x+c)+1)/a/d-1/d^2/a\*ln(1/a\*b\*exp(d\*x+c)+1)\*c-polylog(2,-1/a\*b\*exp(d\*x+c))/a/d^2-1/d^2\*c/a\*ln(exp(d\*x+c))+1/d^2\*c/a\*ln(b\*exp(d\*x+c)+a)

**maxima** [A] time = 0.97, size = 48, normalized size = 0.83

$$\frac{x^2}{2a} - \frac{dx \log\left(\frac{be^{(dx+c)}}{a} + 1\right) + \text{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*x^2/a - (d\*x\*log(b\*e^(d\*x + c)/a + 1) + dilog(-b\*e^(d\*x + c)/a))/(a\*d^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{a + be^{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*exp(c + d\*x)),x)

[Out] int(x/(a + b\*exp(c + d\*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + be^ce^{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c)),x)

[Out] Integral(x/(a + b\*exp(c)\*exp(d\*x)), x)

### 3.4 $\int \frac{1}{a+be^{c+dx}} dx$

Optimal. Leaf size=26

$$\frac{x}{a} - \frac{\log(a + be^{c+dx})}{ad}$$

[Out] x/a-ln(a+b\*exp(d\*x+c))/a/d

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2282, 36, 29, 31}

$$\frac{x}{a} - \frac{\log(a + be^{c+dx})}{ad}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(c + d\*x))^(-1), x]

[Out] x/a - Log[a + b\*E^(c + d\*x)]/(a\*d)

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{a + be^{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, e^{c+dx}\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, e^{c+dx}\right)}{ad} - \frac{b \text{Subst}\left(\int \frac{1}{a+bx} dx, x, e^{c+dx}\right)}{ad} \\ &= \frac{x}{a} - \frac{\log(a + be^{c+dx})}{ad} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{a} - \frac{\log(a + be^{c+dx})}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(c + d\*x))^(-1), x]

[Out] x/a - Log[a + b\*E^(c + d\*x)]/(a\*d)

**fricas [A]** time = 0.45, size = 24, normalized size = 0.92

$$\frac{dx - \log\left(be^{(dx+c)} + a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c)),x, algorithm="fricas")

[Out] (d\*x - log(b\*e^(d\*x + c) + a))/(a\*d)

**giac [A]** time = 0.33, size = 31, normalized size = 1.19

$$\frac{\frac{dx+c}{a} - \frac{\log(|be^{(dx+c)}+a|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c)),x, algorithm="giac")

[Out] ((d\*x + c)/a - log(abs(b\*e^(d\*x + c) + a))/a)/d

**maple** [A] time = 0.01, size = 35, normalized size = 1.35

$$-\frac{\ln(b e^{dx+c} + a)}{ad} + \frac{\ln(e^{dx+c})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(d\*x+c)+a),x)

[Out] 1/d/a\*ln(exp(d\*x+c))-ln(b\*exp(d\*x+c)+a)/a/d

**maxima** [A] time = 0.90, size = 32, normalized size = 1.23

$$\frac{dx + c}{ad} - \frac{\log(b e^{(dx+c)} + a)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c)),x, algorithm="maxima")

[Out] (d\*x + c)/(a\*d) - log(b\*e^(d\*x + c) + a)/(a\*d)

**mupad** [B] time = 3.50, size = 24, normalized size = 0.92

$$-\frac{\ln(a + b e^{dx} e^c) - dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*exp(c + d\*x)),x)

[Out] -(log(a + b\*exp(d\*x)\*exp(c)) - d\*x)/(a\*d)

**sympy** [A] time = 0.12, size = 17, normalized size = 0.65

$$\frac{x}{a} - \frac{\log\left(\frac{a}{b} + e^{c+dx}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c)),x)

[Out] x/a - log(a/b + exp(c + d\*x))/(a\*d)

$$3.5 \quad \int \frac{1}{(a+be^{c+dx})x} dx$$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1}{x(a+be^{c+dx})}, x\right)$$

[Out] Unintegrable(1/(a+b\*exp(d\*x+c))/x,x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(a+be^{c+dx})x} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*E^(c + d\*x))\*x), x]

[Out] Defer[Int][1/((a + b\*E^(c + d\*x))\*x), x]

Rubi steps

$$\int \frac{1}{(a+be^{c+dx})x} dx = \int \frac{1}{(a+be^{c+dx})x} dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+be^{c+dx})x} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*E^(c + d\*x))\*x), x]

[Out] Integrate[1/((a + b\*E^(c + d\*x))\*x), x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx e^{(dx+c)} + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))/x,x, algorithm="fricas")

[Out] integral(1/(b\*x\*e^(d\*x + c) + a\*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))/x,x, algorithm="giac")

[Out] integrate(1/((b\*e^(d\*x + c) + a)\*x), x)

maple [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(d\*x+c)+a)/x,x)

[Out] int(1/(b\*exp(d\*x+c)+a)/x,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))/x,x, algorithm="maxima")

[Out] integrate(1/((b\*e^(d\*x + c) + a)\*x), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x(a + be^{c+dx})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*exp(c + d\*x))),x)

[Out] int(1/(x\*(a + b\*exp(c + d\*x))), x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + be^c e^{dx})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))/x,x)

[Out] Integral(1/(x\*(a + b\*exp(c)\*exp(d\*x))), x)

$$3.6 \quad \int \frac{1}{(a+be^{c+dx})x^2} dx$$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1}{x^2(a+be^{c+dx})}, x\right)$$

[Out] Unintegrable(1/(a+b\*exp(d\*x+c))/x^2, x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(a+be^{c+dx})x^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*E^(c + d\*x))\*x^2), x]

[Out] Defer[Int][1/((a + b\*E^(c + d\*x))\*x^2), x]

Rubi steps

$$\int \frac{1}{(a+be^{c+dx})x^2} dx = \int \frac{1}{(a+be^{c+dx})x^2} dx$$

Mathematica [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+be^{c+dx})x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*E^(c + d\*x))\*x^2), x]

[Out] Integrate[1/((a + b\*E^(c + d\*x))\*x^2), x]

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^2e^{(dx+c)} + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))/x^2,x, algorithm="fricas")

[Out] integral(1/(b\*x^2\*e^(d\*x + c) + a\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))/x^2,x, algorithm="giac")

[Out] integrate(1/((b\*e^(d\*x + c) + a)\*x^2), x)

**maple** [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(b e^{dx+c} + a) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(d\*x+c)+a)/x^2,x)

[Out] int(1/(b\*exp(d\*x+c)+a)/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))/x^2,x, algorithm="maxima")

[Out] integrate(1/((b\*e^(d\*x + c) + a)\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 (a + b e^{c+dx})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*exp(c + d\*x))),x)

[Out] int(1/(x^2\*(a + b\*exp(c + d\*x))), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b e^c e^{dx})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))/x\*\*2,x)

[Out] Integral(1/(x\*\*2\*(a + b\*exp(c)\*exp(d\*x))), x)

$$3.7 \quad \int \frac{1}{a+be^{c-dx}} dx$$

Optimal. Leaf size=26

$$\frac{\log(a+be^{c-dx})}{ad} + \frac{x}{a}$$

[Out] x/a+ln(a+b\*exp(-d\*x+c))/a/d

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2282, 36, 29, 31}

$$\frac{\log(a+be^{c-dx})}{ad} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(c - d\*x))^(-1), x]

[Out] x/a + Log[a + b\*E^(c - d\*x)]/(a\*d)

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_)\*(x\_))(-1), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{a + be^{c-dx}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, e^{c-dx}\right)}{d} \\
 &= -\frac{\text{Subst}\left(\int \frac{1}{x} dx, x, e^{c-dx}\right)}{ad} + \frac{b \text{Subst}\left(\int \frac{1}{a+bx} dx, x, e^{c-dx}\right)}{ad} \\
 &= \frac{x}{a} + \frac{\log(a + be^{c-dx})}{ad}
 \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 21, normalized size = 0.81

$$\frac{\log(ae^{dx} + be^c)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(c - d\*x))^(-1),x]

[Out] Log[b\*E^c + a\*E^(d\*x)]/(a\*d)

**fricas** [A] time = 0.43, size = 23, normalized size = 0.88

$$\frac{dx + \log(be^{-dx+c} + a)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c)),x, algorithm="fricas")

[Out] (d\*x + log(b\*e^(-d\*x + c) + a))/(a\*d)

**giac** [A] time = 0.34, size = 33, normalized size = 1.27

$$\frac{\frac{dx-c}{a} + \frac{\log(|be^{-dx+c}+a|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c)),x, algorithm="giac")

[Out] ((d\*x - c)/a + log(abs(b\*e^(-d\*x + c) + a))/a)/d

**maple [A]** time = 0.01, size = 37, normalized size = 1.42

$$\frac{\ln(b e^{-dx+c} + a)}{ad} - \frac{\ln(e^{-dx+c})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*exp(-d\*x+c)),x)

[Out] -1/d/a\*ln(exp(-d\*x+c))+ln(a+b\*exp(-d\*x+c))/a/d

**maxima [A]** time = 0.88, size = 34, normalized size = 1.31

$$\frac{dx - c}{ad} + \frac{\log(b e^{-dx+c} + a)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c)),x, algorithm="maxima")

[Out] (d\*x - c)/(a\*d) + log(b\*e^(-d\*x + c) + a)/(a\*d)

**mupad [B]** time = 3.47, size = 23, normalized size = 0.88

$$\frac{\ln(a + b e^{-dx} e^c) + dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*exp(c - d\*x)),x)

[Out] (log(a + b\*exp(-d\*x)\*exp(c)) + d\*x)/(a\*d)

**sympy [A]** time = 0.13, size = 17, normalized size = 0.65

$$\frac{x}{a} + \frac{\log\left(\frac{a}{b} + e^{c-dx}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c)),x)

[Out] x/a + log(a/b + exp(c - d\*x))/(a\*d)

### 3.8 $\int \frac{1}{a+be^{-c-dx}} dx$

Optimal. Leaf size=28

$$\frac{\log(a + be^{-c-dx})}{ad} + \frac{x}{a}$$

[Out] x/a+ln(a+b\*exp(-d\*x-c))/a/d

**Rubi [A]** time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2282, 36, 29, 31}

$$\frac{\log(a + be^{-c-dx})}{ad} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(-c - d\*x))^(-1), x]

[Out] x/a + Log[a + b\*E^(-c - d\*x)]/(a\*d)

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rubi steps



$$\begin{aligned} \int \frac{1}{a + be^{-c-dx}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, e^{-c-dx}\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \frac{1}{x} dx, x, e^{-c-dx}\right)}{ad} + \frac{b \text{Subst}\left(\int \frac{1}{a+bx} dx, x, e^{-c-dx}\right)}{ad} \\ &= \frac{x}{a} + \frac{\log(a + be^{-c-dx})}{ad} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 19, normalized size = 0.68

$$\frac{\log(ae^{c+dx} + b)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(-c - d\*x))^(-1), x]

[Out] Log[b + a\*E^(c + d\*x)]/(a\*d)

**fricas [A]** time = 0.43, size = 25, normalized size = 0.89

$$\frac{dx + \log\left( be^{(-dx-c)} + a \right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c)),x, algorithm="fricas")

[Out] (d\*x + log(b\*e^(-d\*x - c) + a))/(a\*d)

**giac [A]** time = 0.38, size = 33, normalized size = 1.18

$$\frac{\frac{dx+c}{a} + \frac{\log(|be^{(-dx-c)}+a|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c)),x, algorithm="giac")

[Out] ((d\*x + c)/a + log(abs(b\*e^(-d\*x - c) + a))/a)/d

**maple** [A] time = 0.01, size = 41, normalized size = 1.46

$$\frac{\ln(b e^{-dx-c} + a)}{ad} - \frac{\ln(e^{-dx-c})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*exp(-d\*x-c)),x)

[Out] -1/d/a\*ln(exp(-d\*x-c))+ln(a+b\*exp(-d\*x-c))/a/d

**maxima** [A] time = 0.88, size = 34, normalized size = 1.21

$$\frac{dx + c}{ad} + \frac{\log(b e^{(-dx-c)} + a)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c)),x, algorithm="maxima")

[Out] (d\*x + c)/(a\*d) + log(b\*e^(-d\*x - c) + a)/(a\*d)

**mupad** [B] time = 0.05, size = 25, normalized size = 0.89

$$\frac{\ln(a + b e^{-c} e^{-dx}) + dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*exp(-c - d\*x)),x)

[Out] (log(a + b\*exp(-c)\*exp(-d\*x)) + d\*x)/(a\*d)

**sympy** [A] time = 0.13, size = 19, normalized size = 0.68

$$\frac{x}{a} + \frac{\log\left(\frac{a}{b} + e^{-c-dx}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c)),x)

[Out] x/a + log(a/b + exp(-c - d\*x))/(a\*d)

$$3.9 \quad \int \frac{x^3}{(a+be^{c+dx})^2} dx$$

**Optimal.** Leaf size=217

$$\frac{6\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^4} - \frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^4} + \frac{6x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{3x^2 \log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^2d^2} - \frac{x^3 \log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^2d^2}$$

[Out]  $-x^3/a^2/d+x^3/a/d/(a+b*\exp(d*x+c))+1/4*x^4/a^2+3*x^2*\ln(1+b*\exp(d*x+c)/a)/a^2/d^2-x^3*\ln(1+b*\exp(d*x+c)/a)/a^2/d+6*x*polylog(2,-b*\exp(d*x+c)/a)/a^2/d^3-3*x^2*polylog(2,-b*\exp(d*x+c)/a)/a^2/d^2-6*polylog(3,-b*\exp(d*x+c)/a)/a^2/d^4+6*x*polylog(3,-b*\exp(d*x+c)/a)/a^2/d^3-6*polylog(4,-b*\exp(d*x+c)/a)/a^2/d^4$

**Rubi [A]** time = 0.51, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {2185, 2184, 2190, 2531, 6609, 2282, 6589, 2191}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{6x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^3} + \frac{6x\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{6\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^4} - \frac{6\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{a^2d^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*E^(c + d\*x))^2,x]

[Out]  $-(x^3/(a^2*d)) + x^3/(a*d*(a + b*E^(c + d*x))) + x^4/(4*a^2) + (3*x^2*\text{Log}[1 + (b*E^(c + d*x))/a])/(a^2*d^2) - (x^3*\text{Log}[1 + (b*E^(c + d*x))/a])/(a^2*d) + (6*x*\text{PolyLog}[2, -((b*E^(c + d*x))/a)])/(a^2*d^3) - (3*x^2*\text{PolyLog}[2, -((b*E^(c + d*x))/a)])/(a^2*d^2) - (6*\text{PolyLog}[3, -((b*E^(c + d*x))/a)])/(a^2*d^4) + (6*x*\text{PolyLog}[3, -((b*E^(c + d*x))/a)])/(a^2*d^3) - (6*\text{PolyLog}[4, -((b*E^(c + d*x))/a)])/(a^2*d^4)$

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2185

Int[((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/a, Int[(c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n), x], x]

$(f*x))^n)^{p+1}, x], x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m*(F^{(g*(e + f*x)))^n})^p, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{ILtQ}[p, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(((F\_)^{(g\_)*((e\_)+(f\_)*(x\_)))^{(n\_)*((c\_)+(d\_)*(x\_))^{(m\_))}/((a\_)+(b\_)*((F\_)^{(g\_)*((e\_)+(f\_)*(x\_)))^{(n\_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 2191

$\text{Int}[((F\_)^{(g\_)*((e\_)+(f\_)*(x\_)))^{(n\_)*((a\_)+(b\_)*((F\_)^{(g\_)*((e\_)+(f\_)*(x\_)))^{(n\_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*(a + b*(F^{(g*(e + f*x)))^n})^{p+1})/(b*f*g*n*(p+1)*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*(p+1)*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*(a + b*(F^{(g*(e + f*x)))^n})^{p+1}, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, m, n, p\}, x] \&\& \text{NeQ}[p, -1]$

### Rule 2282

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_))^{(m\_)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& \text{!MatchQ}[u, E^{((c\_)*((a\_)+(b\_)*x))}*(F\_)[v_] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e\_)*((F\_)^{(c\_)*((a\_)+(b\_)*(x\_)))^{(n\_)}], x\_Symbol] \rightarrow -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})]]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c\_)*((a\_)+(b\_)*(x\_))^{(p\_)}]/((d\_)+(e\_)*(x\_)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_)))]^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + be^{c+dx})^2} dx &= \frac{\int \frac{x^3}{a+be^{c+dx}} dx}{a} - \frac{b \int \frac{e^{c+dx} x^3}{(a+be^{c+dx})^2} dx}{a} \\
&= \frac{x^3}{ad(a + be^{c+dx})} + \frac{x^4}{4a^2} - \frac{b \int \frac{e^{c+dx} x^3}{a+be^{c+dx}} dx}{a^2} - \frac{3 \int \frac{x^2}{a+be^{c+dx}} dx}{ad} \\
&= -\frac{x^3}{a^2 d} + \frac{x^3}{ad(a + be^{c+dx})} + \frac{x^4}{4a^2} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} + \frac{3 \int x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right) dx}{a^2 d} + \frac{(3b) \int}{a^2 d} \\
&= -\frac{x^3}{a^2 d} + \frac{x^3}{ad(a + be^{c+dx})} + \frac{x^4}{4a^2} + \frac{3x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d^2} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} - \frac{3x^2 \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2 d^2} \\
&= -\frac{x^3}{a^2 d} + \frac{x^3}{ad(a + be^{c+dx})} + \frac{x^4}{4a^2} + \frac{3x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d^2} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} + \frac{6x \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2 d^3} \\
&= -\frac{x^3}{a^2 d} + \frac{x^3}{ad(a + be^{c+dx})} + \frac{x^4}{4a^2} + \frac{3x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d^2} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} + \frac{6x \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2 d^3} \\
&= -\frac{x^3}{a^2 d} + \frac{x^3}{ad(a + be^{c+dx})} + \frac{x^4}{4a^2} + \frac{3x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d^2} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} + \frac{6x \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2 d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 158, normalized size = 0.73

$$\frac{24(dx-1)\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{d^4} - \frac{24\text{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{d^4} - \frac{12x(dx-2)\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{d^3} + \frac{12x^2 \log\left(\frac{be^{c+dx}}{a} + 1\right)}{d^2} + \frac{4ax^3}{ad+bde^{c+dx}} - \frac{4x^3 \log\left(\frac{be^{c+dx}}{a} + 1\right)}{d} - \frac{4x^3}{d} + x^4$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*E^(c + d\*x))^2,x]

[Out]  $\left(\frac{-4x^3}{d} + \frac{4ax^3}{(ad + bde^{(c+dx)})} + x^4 + \frac{12x^2 \operatorname{Log}\left[1 + \frac{bE^{(c+dx)}}{a}\right]}{d^2} - \frac{4x^3 \operatorname{Log}\left[1 + \frac{bE^{(c+dx)}}{a}\right]}{d} - \frac{12x(-2+dx) \operatorname{PolyLog}\left[2, -\frac{bE^{(c+dx)}}{a}\right]}{d^3} + \frac{24(-1+dx) \operatorname{PolyLog}\left[3, -\frac{bE^{(c+dx)}}{a}\right]}{d^4} - \frac{24 \operatorname{PolyLog}\left[4, -\frac{bE^{(c+dx)}}{a}\right]}{d^4}\right) / (4a^2)$

**fricas** [C] time = 0.46, size = 330, normalized size = 1.52

---

$ad^4x^4 - ac^4 - 4ac^3 - 12(ad^2x^2 - 2adx + (bd^2x^2 - 2bdx)e^{(dx+c)})\operatorname{Li}_2\left(-\frac{be^{(dx+c)+a}}{a} + 1\right) + (bd^4x^4 - 4bd^3x^3 - bc^4 - 4$

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*exp(d\*x+c))^2,x, algorithm="fricas")

[Out]  $\frac{1}{4}(ad^4x^4 - ac^4 - 4ac^3 - 12(ad^2x^2 - 2adx + (bd^2x^2 - 2bdx)e^{(dx+c)})\operatorname{Li}_2\left(-\frac{be^{(dx+c)+a}}{a} + 1\right) + (bd^4x^4 - 4bd^3x^3 - bc^4 - 4ac^3 - 4ac^2 + (b^2c^3 + 3b^2c^2 + b^2c) e^{(dx+c)} + 4(ac^3 + 3ac^2 + (b^2c^3 + 3b^2c^2)e^{(dx+c)})\operatorname{Log}(be^{(dx+c)} + a) - 4(ad^3x^3 - 3ad^2x^2 + ac^3 + 3ac^2 + (bd^3x^3 - 3bd^2x^2 + b^2c^3 + 3b^2c^2)e^{(dx+c)})\operatorname{Log}\left(\frac{be^{(dx+c)} + a}{a}\right) - 24(be^{(dx+c)} + a)\operatorname{polylog}\left(4, -\frac{be^{(dx+c)}}{a}\right) + 24(adx + (bdx - b)e^{(dx+c)} - a)\operatorname{polylog}\left(3, -\frac{be^{(dx+c)}}{a}\right)) / (a^2bd^4e^{(dx+c)} + a^3d^4)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(be^{(dx+c)} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*exp(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(x^3/(b\*exp(d\*x + c) + a)^2, x)

**maple** [A] time = 0.14, size = 382, normalized size = 1.76

$$\frac{x^4}{4a^2} + \frac{x^3}{(be^{dx+c} + a)ad} - \frac{x^3 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{a^2d} - \frac{x^3}{a^2d} + \frac{c^3x}{a^2d^3} - \frac{3x^2 \operatorname{polylog}\left(2, -\frac{be^{dx+c}}{a}\right)}{a^2d^2} + \frac{3x^2 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{a^2d^2} + \frac{3c^4}{4a^2d^4} - \frac{c^3 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{4a^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*exp(d\*x+c)+a)^2,x)

```
[Out] x^3/a/d/(b*exp(d*x+c)+a)+3/4/a^2/d^4*c^4-6*polylog(3,-1/a*b*exp(d*x+c))/a^2/d^4+3/a^2/d^4*c^2*ln(b*exp(d*x+c)+a)-3/a^2/d^4*c^2*ln(exp(d*x+c))+1/a^2/d^4*c^3*ln(b*exp(d*x+c)+a)-1/a^2/d^4*c^3*ln(exp(d*x+c))+1/4*x^4/a^2-x^3/a^2/d-1/a^2/d^4*c^3*ln(1/a*b*exp(d*x+c)+1)+2/a^2/d^4*c^3-6*polylog(4,-1/a*b*exp(d*x+c))/a^2/d^4-3/a^2/d^4*c^2*ln(1/a*b*exp(d*x+c)+1)+6*x*polylog(3,-1/a*b*exp(d*x+c))/a^2/d^3+3*x^2*ln(1/a*b*exp(d*x+c)+1)/a^2/d^2+6*x*polylog(2,-1/a*b*exp(d*x+c))/a^2/d^3-3*x^2*polylog(2,-1/a*b*exp(d*x+c))/a^2/d^2-x^3*ln(1/a*b*exp(d*x+c)+1)/a^2/d+1/a^2/d^3*c^3*x+3/a^2/d^3*c^2*x
```

**maxima** [A] time = 1.03, size = 195, normalized size = 0.90

$$\frac{x^3}{abde^{dx+c} + a^2d} + \frac{d^4x^4 - 4d^3x^3}{4a^2d^4} - \frac{d^3x^3 \log\left(\frac{be^{dx+c}}{a} + 1\right) + 3d^2x^2 \operatorname{Li}_2\left(-\frac{be^{dx+c}}{a}\right) - 6dx \operatorname{Li}_3\left(-\frac{be^{dx+c}}{a}\right) + 6 \operatorname{Li}_4\left(-\frac{be^{dx+c}}{a}\right)}{a^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] x^3/(a*b*d*e^(d*x + c) + a^2*d) + 1/4*(d^4*x^4 - 4*d^3*x^3)/(a^2*d^4) - (d^3*x^3*log(b*e^(d*x + c)/a + 1) + 3*d^2*x^2*dilog(-b*e^(d*x + c)/a) - 6*d*x*polylog(3, -b*e^(d*x + c)/a) + 6*polylog(4, -b*e^(d*x + c)/a))/(a^2*d^4) + 3*(d^2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*x*dilog(-b*e^(d*x + c)/a) - 2*polylog(3, -b*e^(d*x + c)/a))/(a^2*d^4)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(a + be^{c+dx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*exp(c + d*x))^2,x)
```

```
[Out] int(x^3/(a + b*exp(c + d*x))^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3}{a^2d + abde^{c+dx}} + \frac{\int \left(-\frac{3x^2}{a+be^c e^{dx}}\right) dx + \int \frac{dx^3}{a+be^c e^{dx}} dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*exp(d*x+c))**2,x)
```

```
[Out] x**3/(a**2*d + a*b*d*exp(c + d*x)) + (Integral(-3*x**2/(a + b*exp(c)*exp(d*x)), x) + Integral(d*x**3/(a + b*exp(c)*exp(d*x)), x))/(a*d)
```

$$3.10 \quad \int \frac{x^2}{(a+be^{c+dx})^2} dx$$

**Optimal.** Leaf size=165

$$\frac{2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} + \frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{2x\log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^2d^2} - \frac{x^2\log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^2d} - \frac{x^2}{a^2d} + \frac{x^3}{3a^2} + \frac{x^2}{ad(a+be^{c+dx})}$$

[Out]  $-x^2/a^2/d+x^2/a/d/(a+b*\exp(d*x+c))+1/3*x^3/a^2+2*x*\ln(1+b*\exp(d*x+c)/a)/a^2/d^2-x^2*\ln(1+b*\exp(d*x+c)/a)/a^2/d+2*polylog(2,-b*\exp(d*x+c)/a)/a^2/d^3-2*x*polylog(2,-b*\exp(d*x+c)/a)/a^2/d^2+2*polylog(3,-b*\exp(d*x+c)/a)/a^2/d^3$

**Rubi [A]** time = 0.40, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$ , Rules used = {2185, 2184, 2190, 2531, 2282, 6589, 2191, 2279, 2391}

$$-\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^3} + \frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^3} + \frac{2x\log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^2d^2} - \frac{x^2\log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*E^(c + d\*x))^2,x]

[Out]  $-(x^2/(a^2*d)) + x^2/(a*d*(a + b*E^(c + d*x))) + x^3/(3*a^2) + (2*x*Log[1 + (b*E^(c + d*x))/a])/(a^2*d^2) - (x^2*Log[1 + (b*E^(c + d*x))/a])/(a^2*d) + (2*PolyLog[2, -((b*E^(c + d*x))/a)])/(a^2*d^3) - (2*x*PolyLog[2, -((b*E^(c + d*x))/a)])/(a^2*d^2) + (2*PolyLog[3, -((b*E^(c + d*x))/a)])/(a^2*d^3)$

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n]/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2185

Int[((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/a, Int[(c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x))))^n]^(p + 1), x], x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x))))^n\*(a + b\*(F^(g\*(e + f\*x))))^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && ILtQ[p, 0] && IGtQ[m, 0]



Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((a_) + (b_)*((F_)^((g_)*(
(e_) + (f_)*(x_)))^(n_)))^(p_))*((c_) + (d_)*(x_))^(m_), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*(c_) + (d_)*(x_)))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(a + be^{c+dx})^2} dx &= \frac{\int \frac{x^2}{a+be^{c+dx}} dx}{a} - \frac{b \int \frac{e^{c+dx} x^2}{(a+be^{c+dx})^2} dx}{a} \\
 &= \frac{x^2}{ad(a + be^{c+dx})} + \frac{x^3}{3a^2} - \frac{b \int \frac{e^{c+dx} x^2}{a+be^{c+dx}} dx}{a^2} - \frac{2 \int \frac{x}{a+be^{c+dx}} dx}{ad} \\
 &= -\frac{x^2}{a^2 d} + \frac{x^2}{ad(a + be^{c+dx})} + \frac{x^3}{3a^2} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} + \frac{2 \int x \log\left(1 + \frac{be^{c+dx}}{a}\right) dx}{a^2 d} + \frac{(2b) \int \frac{e^c}{a+be^{c+dx}} dx}{a^2} \\
 &= -\frac{x^2}{a^2 d} + \frac{x^2}{ad(a + be^{c+dx})} + \frac{x^3}{3a^2} + \frac{2x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d^2} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} - \frac{2x \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2 d^2} \\
 &= -\frac{x^2}{a^2 d} + \frac{x^2}{ad(a + be^{c+dx})} + \frac{x^3}{3a^2} + \frac{2x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d^2} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} - \frac{2x \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2 d^2} \\
 &= -\frac{x^2}{a^2 d} + \frac{x^2}{ad(a + be^{c+dx})} + \frac{x^3}{3a^2} + \frac{2x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d^2} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} + \frac{2 \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2 d^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 113, normalized size = 0.68

$$\frac{\frac{d^2 x^2 (adx + b(dx-3)e^{c+dx})}{a + be^{c+dx}} + (6 - 6dx) \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right) + 6 \text{Li}_3\left(-\frac{be^{c+dx}}{a}\right) - 3dx(dx-2) \log\left(\frac{be^{c+dx}}{a} + 1\right)}{3a^2 d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*E^(c + d\*x))^2, x]

[Out] ((d^2\*x^2\*(a\*d\*x + b\*E^(c + d\*x))\*(-3 + d\*x))/(a + b\*E^(c + d\*x)) - 3\*d\*x\*(-2 + d\*x)\*Log[1 + (b\*E^(c + d\*x))/a] + (6 - 6\*d\*x)\*PolyLog[2, -(b\*E^(c + d\*x))/a] + 6\*PolyLog[3, -(b\*E^(c + d\*x))/a])/(3\*a^2\*d^3)

**fricas** [C] time = 0.48, size = 263, normalized size = 1.59

$$ad^3x^3 + ac^3 + 3ac^2 - 6\left(adx + (bdx - b)e^{(dx+c)} - a\right)\text{Li}_2\left(-\frac{be^{(dx+c)}+a}{a} + 1\right) + (bd^3x^3 - 3bd^2x^2 + bc^3 + 3bc^2)e^{(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(d\*x+c))^2,x, algorithm="fricas")

[Out] 1/3\*(a\*d^3\*x^3 + a\*c^3 + 3\*a\*c^2 - 6\*(a\*d\*x + (b\*d\*x - b)\*e^(d\*x + c) - a)\*dilog(-(b\*e^(d\*x + c) + a)/a + 1) + (b\*d^3\*x^3 - 3\*b\*d^2\*x^2 + b\*c^3 + 3\*b\*c^2)\*e^(d\*x + c) - 3\*(a\*c^2 + 2\*a\*c + (b\*c^2 + 2\*b\*c)\*e^(d\*x + c))\*log(b\*e^(d\*x + c) + a) - 3\*(a\*d^2\*x^2 - a\*c^2 - 2\*a\*d\*x - 2\*a\*c + (b\*d^2\*x^2 - b\*c^2 - 2\*b\*d\*x - 2\*b\*c)\*e^(d\*x + c))\*log((b\*e^(d\*x + c) + a)/a) + 6\*(b\*e^(d\*x + c) + a)\*polylog(3, -b\*e^(d\*x + c)/a))/(a^2\*b\*d^3\*e^(d\*x + c) + a^3\*d^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(be^{(dx+c)} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(x^2/(b\*e^(d\*x + c) + a)^2, x)

**maple** [B] time = 0.11, size = 324, normalized size = 1.96

$$\frac{x^3}{3a^2} + \frac{x^2}{(be^{dx+c} + a)ad} - \frac{x^2 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{a^2d} - \frac{c^2x}{a^2d^2} - \frac{x^2}{a^2d} - \frac{2c^3}{3a^2d^3} + \frac{c^2 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{a^2d^3} - \frac{c^2 \ln(be^{dx+c} + a)}{a^2d^3} + \frac{c^2 \ln(e^{dx+c})}{a^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*exp(d\*x+c)+a)^2,x)

[Out] x^2/a/d/(b\*exp(d\*x+c)+a)-1/a^2/d^3\*c^2\*ln(b\*exp(d\*x+c)+a)+1/a^2/d^3\*c^2\*ln(exp(d\*x+c))+1/3\*x^3/a^2-1/a^2/d^2\*c^2\*x-2/3/a^2/d^3\*c^3-x^2\*ln(1/a\*b\*exp(d\*x+c)+1)/a^2/d+1/a^2/d^3\*ln(1/a\*b\*exp(d\*x+c)+1)\*c^2-2\*x\*polylog(2,-1/a\*b\*exp(d\*x+c))/a^2/d^2+2\*polylog(3,-1/a\*b\*exp(d\*x+c))/a^2/d^3-2/a^2/d^3\*c\*ln(b\*exp(d\*x+c)+a)+2/a^2/d^3\*c\*ln(exp(d\*x+c))-x^2/a^2/d-2/a^2/d^2\*c\*x-1/a^2/d^3\*c^2+2\*x\*ln(1/a\*b\*exp(d\*x+c)+1)/a^2/d^2+2/a^2/d^3\*ln(1/a\*b\*exp(d\*x+c)+1)\*c+2\*polylog(2,-1/a\*b\*exp(d\*x+c))/a^2/d^3

**maxima** [A] time = 1.02, size = 149, normalized size = 0.90

$$\frac{x^2}{abde^{(dx+c)} + a^2d} + \frac{d^3x^3 - 3d^2x^2}{3a^2d^3} - \frac{d^2x^2 \log\left(\frac{be^{(dx+c)}}{a} + 1\right) + 2dx \operatorname{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 2\operatorname{Li}_3\left(-\frac{be^{(dx+c)}}{a}\right)}{a^2d^3} + \frac{2\left(dx \log\left(\frac{be^{(dx+c)}}{a} + 1\right) + \operatorname{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)\right)}{a^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(d\*x+c))^2,x, algorithm="maxima")

[Out] x^2/(a\*b\*d\*e^(d\*x + c) + a^2\*d) + 1/3\*(d^3\*x^3 - 3\*d^2\*x^2)/(a^2\*d^3) - (d^2\*x^2\*log(b\*e^(d\*x + c)/a + 1) + 2\*d\*x\*dilog(-b\*e^(d\*x + c)/a) - 2\*polylog(3, -b\*e^(d\*x + c)/a))/(a^2\*d^3) + 2\*(d\*x\*log(b\*e^(d\*x + c)/a + 1) + dilog(-b\*e^(d\*x + c)/a))/(a^2\*d^3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a + b e^{c+dx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b\*exp(c + d\*x))^2,x)

[Out] int(x^2/(a + b\*exp(c + d\*x))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{a^2d + abde^{c+dx}} + \frac{\int\left(-\frac{2x}{a+be^c e^{dx}}\right) dx + \int \frac{dx^2}{a+be^c e^{dx}} dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*exp(d\*x+c))\*\*2,x)

[Out] x\*\*2/(a\*\*2\*d + a\*b\*d\*exp(c + d\*x)) + (Integral(-2\*x/(a + b\*exp(c)\*exp(d\*x)), x) + Integral(d\*x\*\*2/(a + b\*exp(c)\*exp(d\*x)), x))/(a\*d)

$$3.11 \quad \int \frac{x}{(a+be^{c+dx})^2} dx$$

**Optimal.** Leaf size=107

$$-\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{\log(a+be^{c+dx})}{a^2d^2} - \frac{x \log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^2d} - \frac{x}{a^2d} + \frac{x^2}{2a^2} + \frac{x}{ad(a+be^{c+dx})}$$

[Out]  $-x/a^2/d+x/a/d/(a+b*\exp(d*x+c))+1/2*x^2/a^2+\ln(a+b*\exp(d*x+c))/a^2/d^2-x*\ln(1+b*\exp(d*x+c)/a)/a^2/d-\text{polylog}(2,-b*\exp(d*x+c)/a)/a^2/d^2$

**Rubi [A]** time = 0.20, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 15, number of rules / integrand size = 0.667, Rules used = {2185, 2184, 2190, 2279, 2391, 2191, 2282, 36, 29, 31}

$$-\frac{\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{\log(a+be^{c+dx})}{a^2d^2} - \frac{x \log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^2d} - \frac{x}{a^2d} + \frac{x^2}{2a^2} + \frac{x}{ad(a+be^{c+dx})}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*E^(c + d\*x))^2, x]

[Out]  $-(x/(a^2*d)) + x/(a*d*(a + b*E^(c + d*x))) + x^2/(2*a^2) + \text{Log}[a + b*E^(c + d*x)]/(a^2*d^2) - (x*\text{Log}[1 + (b*E^(c + d*x))/a])/(a^2*d) - \text{PolyLog}[2, -(b*E^(c + d*x))/a]/(a^2*d^2)$

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2185

```
Int[((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int((((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2191

```
Int((((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Log[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a + b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]
```

### Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{x}{(a + be^{c+dx})^2} dx &= \frac{\int \frac{x}{a+be^{c+dx}} dx}{a} - \frac{b \int \frac{e^{c+dx} x}{(a+be^{c+dx})^2} dx}{a} \\
 &= \frac{x}{ad(a + be^{c+dx})} + \frac{x^2}{2a^2} - \frac{b \int \frac{e^{c+dx} x}{a+be^{c+dx}} dx}{a^2} - \frac{\int \frac{1}{a+be^{c+dx}} dx}{ad} \\
 &= \frac{x}{ad(a + be^{c+dx})} + \frac{x^2}{2a^2} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, e^{c+dx}\right)}{ad^2} + \frac{\int \log\left(1 + \frac{be^{c+dx}}{a}\right) dx}{a^2 d} \\
 &= \frac{x}{ad(a + be^{c+dx})} + \frac{x^2}{2a^2} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} - \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, e^{c+dx}\right)}{a^2 d^2} + \frac{\text{Subst}\left(\int \frac{\log\left(1 + \frac{bx}{a}\right)}{x} dx, x, e^{c+dx}\right)}{a^2 d^2} \\
 &= -\frac{x}{a^2 d} + \frac{x}{ad(a + be^{c+dx})} + \frac{x^2}{2a^2} + \frac{\log(a + be^{c+dx})}{a^2 d^2} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^2 d} - \frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2 d^2}
 \end{aligned}$$

**Mathematica** [A] time = 0.12, size = 85, normalized size = 0.79

$$\frac{-2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right) + \frac{dx(adx+b(dx-2)e^{c+dx})}{a+be^{c+dx}} - 2(dx-1)\log\left(\frac{be^{c+dx}}{a} + 1\right)}{2a^2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*E^(c + d\*x))^2, x]

[Out] ((d\*x\*(a\*d\*x + b\*E^(c + d\*x))\*(-2 + d\*x))/(a + b\*E^(c + d\*x)) - 2\*(-1 + d\*x)\*Log[1 + (b\*E^(c + d\*x))/a] - 2\*PolyLog[2, -((b\*E^(c + d\*x))/a)])/(2\*a^2\*d^2)

**fricas** [A] time = 0.43, size = 176, normalized size = 1.64

$$\frac{ad^2x^2 - ac^2 - 2ac - 2\left(be^{(dx+c)} + a\right)\text{Li}_2\left(-\frac{be^{(dx+c)}+a}{a} + 1\right) + \left(bd^2x^2 - bc^2 - 2bdx - 2bc\right)e^{(dx+c)} + 2\left(ac + (bc + b)e^{(dx+c)}\right)}{2\left(a^2bd^2e^{(dx+c)} + a^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c))^2,x, algorithm="fricas")

[Out]  $\frac{1}{2}*(a*d^2*x^2 - a*c^2 - 2*a*c - 2*(b*e^{(d*x + c)} + a)*\text{dilog}(-(b*e^{(d*x + c)} + a)/a + 1) + (b*d^2*x^2 - b*c^2 - 2*b*d*x - 2*b*c)*e^{(d*x + c)} + 2*(a*c + (b*c + b)*e^{(d*x + c)} + a)*\log(b*e^{(d*x + c)} + a) - 2*(a*d*x + a*c + (b*d*x + b*c)*e^{(d*x + c)})*\log((b*e^{(d*x + c)} + a)/a))/(a^2*b*d^2*e^{(d*x + c)} + a^3*d^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(be^{dx+c} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(x/(b\*e^{(d\*x + c)} + a)^2, x)

**maple** [B] time = 0.03, size = 231, normalized size = 2.16

$$-\frac{bx e^{dx+c}}{(b e^{dx+c} + a) a^2 d} + \frac{x^2}{2a^2} - \frac{bc e^{dx+c}}{(b e^{dx+c} + a) a^2 d^2} + \frac{cx}{a^2 d} - \frac{x \ln\left(\frac{b e^{dx+c} + a}{a}\right)}{a^2 d} - \frac{c}{(b e^{dx+c} + a) a d^2} + \frac{c^2}{2a^2 d^2} - \frac{c \ln\left(\frac{b e^{dx+c} + a}{a}\right)}{a^2 d^2} + \frac{c \ln\left(\frac{b e^{dx+c} + a}{a}\right)}{a^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*exp(d\*x+c)+a)^2,x)

[Out]  $\frac{1}{2}*x^2/a^2 + 1/d/a^2*x*c + 1/2/d^2/a^2*c^2 + \ln(b*\exp(d*x+c)+a)/a^2/d^2 - 1/d/a^2*b*\exp(d*x+c)/(b*\exp(d*x+c)+a)*x - 1/d^2/a^2*b*\exp(d*x+c)/(b*\exp(d*x+c)+a)*c - 1/d^2/a^2*\text{dilog}((b*\exp(d*x+c)+a)/a) - 1/d/a^2*\ln((b*\exp(d*x+c)+a)/a)*x - 1/d^2/a^2*\ln((b*\exp(d*x+c)+a)/a)*c - 1/d^2*c/a^2*\ln(\exp(d*x+c)) + 1/d^2*c/a^2*\ln(b*\exp(d*x+c)+a) - 1/d^2*c/a/(b*\exp(d*x+c)+a)$

**maxima** [A] time = 1.01, size = 95, normalized size = 0.89

$$\frac{x}{abde^{(dx+c)} + a^2d} + \frac{x^2}{2a^2} - \frac{x}{a^2d} - \frac{dx \log\left(\frac{be^{(dx+c)}}{a} + 1\right) + \text{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{a^2d^2} + \frac{\log\left(be^{(dx+c)} + a\right)}{a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c))^2,x, algorithm="maxima")



[Out]  $x/(a*b*d*e^{(d*x + c)} + a^2*d) + 1/2*x^2/a^2 - x/(a^2*d) - (d*x*\log(b*e^{(d*x + c)}/a + 1) + \text{dilog}(-b*e^{(d*x + c)}/a))/(a^2*d^2) + \log(b*e^{(d*x + c)} + a)/(a^2*d^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b e^{c+dx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*exp(c + d*x))^2,x)`

[Out] `int(x/(a + b*exp(c + d*x))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{a^2d + abde^{c+dx}} + \frac{\int \frac{dx}{a+be^ce^{dx}} dx + \int \left(-\frac{1}{a+be^ce^{dx}}\right) dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*exp(d*x+c))**2,x)`

[Out] `x/(a**2*d + a*b*d*exp(c + d*x)) + (Integral(d*x/(a + b*exp(c)*exp(d*x)), x) + Integral(-1/(a + b*exp(c)*exp(d*x)), x))/(a*d)`

$$3.12 \quad \int \frac{1}{(a+be^{c+dx})^2} dx$$

Optimal. Leaf size=46

$$-\frac{\log(a+be^{c+dx})}{a^2d} + \frac{x}{a^2} + \frac{1}{ad(a+be^{c+dx})}$$

[Out] 1/a/d/(a+b\*exp(d\*x+c))+x/a^2-ln(a+b\*exp(d\*x+c))/a^2/d

**Rubi [A]** time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2282, 44}

$$-\frac{\log(a+be^{c+dx})}{a^2d} + \frac{x}{a^2} + \frac{1}{ad(a+be^{c+dx})}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(c + d\*x))^(-2), x]

[Out] 1/(a\*d\*(a + b\*E^(c + d\*x))) + x/a^2 - Log[a + b\*E^(c + d\*x)]/(a^2\*d)

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + be^{c+dx})^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, e^{c+dx}\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right) dx, x, e^{c+dx}\right)}{d} \\ &= \frac{1}{ad(a + be^{c+dx})} + \frac{x}{a^2} - \frac{\log(a + be^{c+dx})}{a^2d} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 40, normalized size = 0.87

$$\frac{\frac{a}{a+be^{c+dx}} - \log(a + be^{c+dx}) + dx}{a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(c + d\*x))^(-2), x]

[Out] (a/(a + b\*E^(c + d\*x)) + d\*x - Log[a + b\*E^(c + d\*x)])/(a^2\*d)

**fricas** [A] time = 0.44, size = 60, normalized size = 1.30

$$\frac{bdxe^{(dx+c)} + adx - (be^{(dx+c)} + a) \log(be^{(dx+c)} + a) + a}{a^2bde^{(dx+c)} + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^2,x, algorithm="fricas")

[Out] (b\*d\*x\*e^(d\*x + c) + a\*d\*x - (b\*e^(d\*x + c) + a)\*log(b\*e^(d\*x + c) + a) + a)/(a^2\*b\*d\*e^(d\*x + c) + a^3\*d)

**giac** [A] time = 0.44, size = 51, normalized size = 1.11

$$\frac{b \left( \frac{\log\left(\left| -\frac{a}{be^{(dx+c)}+a} + 1 \right| \right)}{a^2b} + \frac{1}{(be^{(dx+c)}+a)ab} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^2,x, algorithm="giac")

[Out]  $b \cdot (\log(\text{abs}(-a/(b \cdot e^{(d \cdot x + c) + a) + 1)))/(a^2 \cdot b) + 1/((b \cdot e^{(d \cdot x + c) + a}) \cdot a \cdot b))/d$

**maple** [A] time = 0.01, size = 54, normalized size = 1.17

$$\frac{1}{(b e^{dx+c} + a) a d} - \frac{\ln(b e^{dx+c} + a)}{a^2 d} + \frac{\ln(e^{dx+c})}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*exp(d*x+c)+a)^2,x)`

[Out]  $1/d/a^2 \cdot \ln(\exp(d \cdot x + c)) - \ln(b \cdot \exp(d \cdot x + c) + a)/a^2/d + 1/a/d/(b \cdot \exp(d \cdot x + c) + a)$

**maxima** [A] time = 1.01, size = 51, normalized size = 1.11

$$\frac{1}{(a b e^{(dx+c)} + a^2) d} + \frac{dx + c}{a^2 d} - \frac{\log(b e^{(dx+c)} + a)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(d*x+c))^2,x, algorithm="maxima")`

[Out]  $1/((a \cdot b \cdot e^{(d \cdot x + c) + a^2}) \cdot d) + (d \cdot x + c)/(a^2 \cdot d) - \log(b \cdot e^{(d \cdot x + c) + a})/(a^2 \cdot d)$

**mupad** [B] time = 3.56, size = 66, normalized size = 1.43

$$\frac{\frac{x}{a} + \frac{b x e^{c+dx}}{a^2} - \frac{b e^{c+dx}}{a^2 d}}{a + b e^{c+dx}} - \frac{\ln(a + b e^{dx} e^c)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*exp(c + d*x))^2,x)`

[Out]  $(x/a + (b \cdot x \cdot \exp(c + d \cdot x))/a^2 - (b \cdot \exp(c + d \cdot x))/(a^2 \cdot d))/(a + b \cdot \exp(c + d \cdot x)) - \log(a + b \cdot \exp(d \cdot x) \cdot \exp(c))/(a^2 \cdot d)$

**sympy** [A] time = 0.15, size = 39, normalized size = 0.85

$$\frac{1}{a^2 d + a b d e^{c+dx}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{c+dx}\right)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(d*x+c))**2,x)`

[Out]  $1/(a**2 \cdot d + a \cdot b \cdot d \cdot \exp(c + d \cdot x)) + x/a**2 - \log(a/b + \exp(c + d \cdot x))/(a**2 \cdot d)$

$$3.13 \quad \int \frac{1}{(a+be^{c+dx})^2 x} dx$$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1}{x(a+be^{c+dx})^2}, x\right)$$

[Out] Unintegrable(1/(a+b\*exp(d\*x+c))^2/x, x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(a+be^{c+dx})^2 x} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*E^(c + d\*x))^2\*x), x]

[Out] Defer[Int][1/((a + b\*E^(c + d\*x))^2\*x), x]

Rubi steps

$$\int \frac{1}{(a+be^{c+dx})^2 x} dx = \int \frac{1}{(a+be^{c+dx})^2} dx$$

Mathematica [A] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+be^{c+dx})^2 x} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*E^(c + d\*x))^2\*x), x]

[Out] Integrate[1/((a + b\*E^(c + d\*x))^2\*x), x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2xe^{(2dx+2c)} + 2abxe^{(dx+c)} + a^2x'}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^2/x,x, algorithm="fricas")

[Out] integral(1/(b^2\*x\*e^(2\*d\*x + 2\*c) + 2\*a\*b\*x\*e^(d\*x + c) + a^2\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^2/x,x, algorithm="giac")

[Out] integrate(1/((b\*e^(d\*x + c) + a)^2\*x), x)

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(d\*x+c)+a)^2/x,x)

[Out] int(1/(b\*exp(d\*x+c)+a)^2/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{abdx e^{(dx+c)} + a^2 dx} + \int \frac{dx + 1}{abdx^2 e^{(dx+c)} + a^2 dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^2/x,x, algorithm="maxima")

[Out] 1/(a\*b\*d\*x\*e^(d\*x + c) + a^2\*d\*x) + integrate((d\*x + 1)/(a\*b\*d\*x^2\*e^(d\*x + c) + a^2\*d\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x(a + be^{c+dx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*exp(c + d*x))^2),x)`

[Out] `int(1/(x*(a + b*exp(c + d*x))^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{a^2 dx + ab dx e^{c+dx}} + \frac{\int \frac{dx}{ax^2 + bx^2 e^c e^{dx}} dx + \int \frac{1}{ax^2 + bx^2 e^c e^{dx}} dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(d*x+c))**2/x,x)`

[Out] `1/(a**2*d*x + a*b*d*x*exp(c + d*x)) + (Integral(d*x/(a*x**2 + b*x**2*exp(c)*exp(d*x)), x) + Integral(1/(a*x**2 + b*x**2*exp(c)*exp(d*x)), x))/(a*d)`

$$3.14 \quad \int \frac{1}{(a+be^{c+dx})^2 x^2} dx$$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1}{x^2 (a + be^{c+dx})^2}, x\right)$$

[Out] Unintegrable(1/(a+b\*exp(d\*x+c))^2/x^2,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(a + be^{c+dx})^2 x^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*E^(c + d\*x))^2\*x^2), x]

[Out] Defer[Int][1/((a + b\*E^(c + d\*x))^2\*x^2), x]

Rubi steps

$$\int \frac{1}{(a + be^{c+dx})^2 x^2} dx = \int \frac{1}{(a + be^{c+dx})^2 x^2} dx$$

Mathematica [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + be^{c+dx})^2 x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*E^(c + d\*x))^2\*x^2), x]

[Out] Integrate[1/((a + b\*E^(c + d\*x))^2\*x^2), x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2 x^2 e^{(2dx+2c)} + 2 abx^2 e^{(dx+c)} + a^2 x^2}, x\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^2/x^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^2\*e^(2\*d\*x + 2\*c) + 2\*a\*b\*x^2\*e^(d\*x + c) + a^2\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^2/x^2,x, algorithm="giac")

[Out] integrate(1/((b\*e^(d\*x + c) + a)^2\*x^2), x)

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(d\*x+c)+a)^2/x^2,x)

[Out] int(1/(b\*exp(d\*x+c)+a)^2/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{abdx^2e^{dx+c} + a^2dx^2} + \int \frac{dx + 2}{abdx^3e^{dx+c} + a^2dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^2/x^2,x, algorithm="maxima")

[Out] 1/(a\*b\*d\*x^2\*e^(d\*x + c) + a^2\*d\*x^2) + integrate((d\*x + 2)/(a\*b\*d\*x^3\*e^(d\*x + c) + a^2\*d\*x^3), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 (a + be^{c+dx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b*exp(c + d*x))^2),x)`

[Out] `int(1/(x^2*(a + b*exp(c + d*x))^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{a^2 dx^2 + ab dx^2 e^{c+dx}} + \frac{\int \frac{dx}{ax^3 + bx^3 e^{c+dx}} dx + \int \frac{2}{ax^3 + bx^3 e^{c+dx}} dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(d*x+c))**2/x**2,x)`

[Out] `1/(a**2*d*x**2 + a*b*d*x**2*exp(c + d*x)) + (Integral(d*x/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x) + Integral(2/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x))/(a*d)`

$$3.15 \quad \int \frac{1}{(a+be^{c-dx})^2} dx$$

Optimal. Leaf size=48

$$\frac{\log(a+be^{c-dx})}{a^2d} + \frac{x}{a^2} - \frac{1}{ad(a+be^{c-dx})}$$

[Out] -1/a/d/(a+b\*exp(-d\*x+c))+x/a^2+ln(a+b\*exp(-d\*x+c))/a^2/d

**Rubi** [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2282, 44}

$$\frac{\log(a+be^{c-dx})}{a^2d} + \frac{x}{a^2} - \frac{1}{ad(a+be^{c-dx})}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(c - d\*x))^(-2),x]

[Out] -(1/(a\*d\*(a + b\*E^(c - d\*x)))) + x/a^2 + Log[a + b\*E^(c - d\*x)]/(a^2\*d)

#### Rule 44

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + be^{c-dx})^2} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, e^{c-dx}\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right) dx, x, e^{c-dx}\right)}{d} \\
&= -\frac{1}{ad(a + be^{c-dx})} + \frac{x}{a^2} + \frac{\log(a + be^{c-dx})}{a^2d}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 42, normalized size = 0.88

$$\frac{\frac{be^c}{ae^{dx}+be^c} + \log(ae^{dx} + be^c)}{a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(c - d\*x))^(-2), x]

[Out] ((b\*E^c)/(b\*E^c + a\*E^(d\*x)) + Log[b\*E^c + a\*E^(d\*x)])/(a^2\*d)

**fricas [A]** time = 0.43, size = 65, normalized size = 1.35

$$\frac{bdxe^{(-dx+c)} + adx + (be^{(-dx+c)} + a) \log(be^{(-dx+c)} + a) - a}{a^2bde^{(-dx+c)} + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c))^2,x, algorithm="fricas")

[Out] (b\*d\*x\*e^(-d\*x + c) + a\*d\*x + (b\*e^(-d\*x + c) + a)\*log(b\*e^(-d\*x + c) + a) - a)/(a^2\*b\*d\*e^(-d\*x + c) + a^3\*d)

**giac [A]** time = 0.24, size = 54, normalized size = 1.12

$$-\frac{b \left( \frac{\log\left(\left|-\frac{a}{be^{(-dx+c)}+a}+1\right|\right)}{a^2b} + \frac{1}{(be^{(-dx+c)}+a)ab} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c))^2,x, algorithm="giac")

[Out]  $-b*(\log(\text{abs}(-a/(b*e^{(-d*x + c)} + a) + 1)))/(a^2*b) + 1/((b*e^{(-d*x + c)} + a)*a*b))/d$

**maple** [A] time = 0.01, size = 58, normalized size = 1.21

$$-\frac{1}{(b e^{-dx+c} + a) a d} + \frac{\ln(b e^{-dx+c} + a)}{a^2 d} - \frac{\ln(e^{-dx+c})}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*exp(-d*x+c)+a)^2,x)`

[Out]  $-1/d/a^2*\ln(\exp(-d*x+c))+\ln(b*\exp(-d*x+c)+a)/a^2/d-1/a/d/(b*\exp(-d*x+c)+a)$

**maxima** [A] time = 1.09, size = 55, normalized size = 1.15

$$-\frac{1}{(a b e^{-dx+c} + a^2) d} + \frac{dx - c}{a^2 d} + \frac{\log(b e^{-dx+c} + a)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(-d*x+c))^2,x, algorithm="maxima")`

[Out]  $-1/((a*b*e^{(-d*x + c)} + a^2)*d) + (d*x - c)/(a^2*d) + \log(b*e^{(-d*x + c)} + a)/(a^2*d)$

**mupad** [B] time = 0.16, size = 68, normalized size = 1.42

$$\frac{\frac{x}{a} + \frac{b x e^{c-dx}}{a^2} + \frac{b e^{c-dx}}{a^2 d}}{a + b e^{c-dx}} + \frac{\ln(a + b e^{-dx} e^c)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*exp(c - d*x))^2,x)`

[Out]  $(x/a + (b*x*\exp(c - d*x))/a^2 + (b*\exp(c - d*x))/(a^2*d))/(a + b*\exp(c - d*x)) + \log(a + b*\exp(-d*x)*\exp(c))/(a^2*d)$

**sympy** [A] time = 0.16, size = 39, normalized size = 0.81

$$-\frac{1}{a^2 d + a b d e^{c-dx}} + \frac{x}{a^2} + \frac{\log\left(\frac{a}{b} + e^{c-dx}\right)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(-d*x+c))**2,x)`

[Out]  $-1/(a**2*d + a*b*d*\exp(c - d*x)) + x/a**2 + \log(a/b + \exp(c - d*x))/(a**2*d)$

$$3.16 \quad \int \frac{1}{(a+be^{-c-dx})^2} dx$$

Optimal. Leaf size=52

$$\frac{\log(a+be^{-c-dx})}{a^2d} + \frac{x}{a^2} - \frac{1}{ad(a+be^{-c-dx})}$$

[Out] -1/a/d/(a+b\*exp(-d\*x-c))+x/a^2+ln(a+b\*exp(-d\*x-c))/a^2/d

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2282, 44}

$$\frac{\log(a+be^{-c-dx})}{a^2d} + \frac{x}{a^2} - \frac{1}{ad(a+be^{-c-dx})}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(-c - d\*x))^(-2),x]

[Out] -(1/(a\*d\*(a + b\*E^(-c - d\*x)))) + x/a^2 + Log[a + b\*E^(-c - d\*x)]/(a^2\*d)

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + be^{-c-dx})^2} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, e^{-c-dx}\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right) dx, x, e^{-c-dx}\right)}{d} \\ &= -\frac{1}{ad(a + be^{-c-dx})} + \frac{x}{a^2} + \frac{\log(a + be^{-c-dx})}{a^2d} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 35, normalized size = 0.67

$$\frac{\frac{b}{ae^{c+dx}+b} + \log(ae^{c+dx} + b)}{a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(-c - d\*x))^(-2), x]

[Out] (b/(b + a\*E^(c + d\*x)) + Log[b + a\*E^(c + d\*x)])/(a^2\*d)

**fricas [A]** time = 0.49, size = 73, normalized size = 1.40

$$\frac{bdxe^{(-dx-c)} + adx + (be^{(-dx-c)} + a) \log (be^{(-dx-c)} + a) - a}{a^2bde^{(-dx-c)} + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c))^2,x, algorithm="fricas")

[Out] (b\*d\*x\*e^(-d\*x - c) + a\*d\*x + (b\*e^(-d\*x - c) + a)\*log(b\*e^(-d\*x - c) + a) - a)/(a^2\*b\*d\*e^(-d\*x - c) + a^3\*d)

**giac [A]** time = 0.22, size = 58, normalized size = 1.12

$$-\frac{b \left( \frac{\log\left(-\frac{a}{be^{(-dx-c)}+a} + 1\right)}{a^2b} + \frac{1}{(be^{(-dx-c)}+a)ab} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c))^2,x, algorithm="giac")

[Out]  $-b*(\log(\text{abs}(-a/(b*e^{(-d*x - c)} + a) + 1)))/(a^2*b) + 1/((b*e^{(-d*x - c)} + a)*a*b))/d$

**maple** [A] time = 0.01, size = 64, normalized size = 1.23

$$-\frac{1}{(b e^{-dx-c} + a) a d} + \frac{\ln(b e^{-dx-c} + a)}{a^2 d} - \frac{\ln(e^{-dx-c})}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*exp(-d*x-c)+a)^2,x)`

[Out]  $-1/d/a^2*\ln(\exp(-d*x-c))+\ln(b*\exp(-d*x-c)+a)/a^2/d-1/a/d/(b*\exp(-d*x-c)+a)$

**maxima** [A] time = 0.95, size = 57, normalized size = 1.10

$$-\frac{1}{(a b e^{(-dx-c)} + a^2) d} + \frac{dx + c}{a^2 d} + \frac{\log(b e^{(-dx-c)} + a)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(-d*x-c))^2,x, algorithm="maxima")`

[Out]  $-1/((a*b*e^{(-d*x - c)} + a^2)*d) + (d*x + c)/(a^2*d) + \log(b*e^{(-d*x - c)} + a)/(a^2*d)$

**mupad** [B] time = 3.50, size = 76, normalized size = 1.46

$$\frac{\frac{x}{a} + \frac{b e^{-c-dx}}{a^2 d} + \frac{b x e^{-c-dx}}{a^2}}{a + b e^{-c-dx}} + \frac{\ln(a + b e^{-c} e^{-dx})}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*exp(- c - d*x))^2,x)`

[Out]  $(x/a + (b*\exp(- c - d*x))/(a^2*d) + (b*x*\exp(- c - d*x))/a^2)/(a + b*\exp(- c - d*x)) + \log(a + b*\exp(-c)*\exp(-d*x))/(a^2*d)$

**sympy** [A] time = 0.17, size = 42, normalized size = 0.81

$$-\frac{1}{a^2 d + a b d e^{-c-dx}} + \frac{x}{a^2} + \frac{\log\left(\frac{a}{b} + e^{-c-dx}\right)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(-d*x-c))**2,x)`

[Out]  $-1/(a**2*d + a*b*d*\exp(-c - d*x)) + x/a**2 + \log(a/b + \exp(-c - d*x))/(a**2*d)$



$$3.17 \quad \int \frac{x^3}{(a+be^{c+dx})^3} dx$$

**Optimal.** Leaf size=333

$$\frac{3\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^4} - \frac{9\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^4} - \frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^4} + \frac{9x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{3x\log\left(\frac{be^{c+dx}}{a} + 1\right)}{a^3d^3} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^4}$$

[Out]  $3/2*x^2/a^3/d^2 - 3/2*x^2/a^2/d^2/(a+b*\exp(d*x+c)) - 3/2*x^3/a^3/d + 1/2*x^3/a/d/(a+b*\exp(d*x+c))^2 + x^3/a^2/d/(a+b*\exp(d*x+c)) + 1/4*x^4/a^3 - 3*x*\ln(1+b*\exp(d*x+c)/a)/a^3/d^3 + 9/2*x^2*\ln(1+b*\exp(d*x+c)/a)/a^3/d^2 - x^3*\ln(1+b*\exp(d*x+c)/a)/a^3/d - 3*polylog(2, -b*\exp(d*x+c)/a)/a^3/d^4 + 9*x*polylog(2, -b*\exp(d*x+c)/a)/a^3/d^3 - 3*x^2*polylog(2, -b*\exp(d*x+c)/a)/a^3/d^2 - 9*polylog(3, -b*\exp(d*x+c)/a)/a^3/d^4 + 6*x*polylog(3, -b*\exp(d*x+c)/a)/a^3/d^3 - 6*polylog(4, -b*\exp(d*x+c)/a)/a^3/d^4$

**Rubi [A]** time = 1.08, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$ , Rules used = {2185, 2184, 2190, 2531, 6609, 2282, 6589, 2191, 2279, 2391}

$$\frac{3x^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{9x\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{6x\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{a^3d^4} - \frac{9\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a}\right)}{a^3d^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*E^(c + d\*x))^3, x]

[Out]  $(3*x^2)/(2*a^3*d^2) - (3*x^2)/(2*a^2*d^2*(a + b*E^(c + d*x))) - (3*x^3)/(2*a^3*d) + x^3/(2*a*d*(a + b*E^(c + d*x))^2) + x^3/(a^2*d*(a + b*E^(c + d*x))) + x^4/(4*a^3) - (3*x*\text{Log}[1 + (b*E^(c + d*x))/a])/(a^3*d^3) + (9*x^2*\text{Log}[1 + (b*E^(c + d*x))/a])/(2*a^3*d^2) - (x^3*\text{Log}[1 + (b*E^(c + d*x))/a])/(a^3*d) - (3*\text{PolyLog}[2, -((b*E^(c + d*x))/a)])/(a^3*d^4) + (9*x*\text{PolyLog}[2, -((b*E^(c + d*x))/a)])/(a^3*d^3) - (3*x^2*\text{PolyLog}[2, -((b*E^(c + d*x))/a)])/(a^3*d^2) - (9*\text{PolyLog}[3, -((b*E^(c + d*x))/a)])/(a^3*d^4) + (6*x*\text{PolyLog}[3, -((b*E^(c + d*x))/a)])/(a^3*d^3) - (6*\text{PolyLog}[4, -((b*E^(c + d*x))/a)])/(a^3*d^4)$

**Rule 2184**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x)))^n/(a + b\*(F^(g\*(e + f\*x)))^n), x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2185

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Dist[1/a, Int[(c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x)))^n\*(a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && ILtQ[p, 0] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2191

Int[((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1))/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + be^{c+dx})^3} dx &= \frac{\int \frac{x^3}{(a+be^{c+dx})^2} dx}{a} - \frac{b \int \frac{e^{c+dx} x^3}{(a+be^{c+dx})^3} dx}{a} \\
&= \frac{x^3}{2ad(a + be^{c+dx})^2} + \frac{\int \frac{x^3}{a+be^{c+dx}} dx}{a^2} - \frac{b \int \frac{e^{c+dx} x^3}{(a+be^{c+dx})^2} dx}{a^2} - \frac{3 \int \frac{x^2}{(a+be^{c+dx})^2} dx}{2ad} \\
&= \frac{x^3}{2ad(a + be^{c+dx})^2} + \frac{x^3}{a^2 d(a + be^{c+dx})} + \frac{x^4}{4a^3} - \frac{b \int \frac{e^{c+dx} x^3}{a+be^{c+dx}} dx}{a^3} - \frac{3 \int \frac{x^2}{a+be^{c+dx}} dx}{2a^2 d} - \frac{3 \int \frac{x^2}{a+be^{c+dx}} dx}{a^2 d} \\
&= -\frac{3x^2}{2a^2 d^2(a + be^{c+dx})} - \frac{3x^3}{2a^3 d} + \frac{x^3}{2ad(a + be^{c+dx})^2} + \frac{x^3}{a^2 d(a + be^{c+dx})} + \frac{x^4}{4a^3} - \frac{x^3 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d} \\
&= \frac{3x^2}{2a^3 d^2} - \frac{3x^2}{2a^2 d^2(a + be^{c+dx})} - \frac{3x^3}{2a^3 d} + \frac{x^3}{2ad(a + be^{c+dx})^2} + \frac{x^3}{a^2 d(a + be^{c+dx})} + \frac{x^4}{4a^3} + \frac{9x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d} \\
&= \frac{3x^2}{2a^3 d^2} - \frac{3x^2}{2a^2 d^2(a + be^{c+dx})} - \frac{3x^3}{2a^3 d} + \frac{x^3}{2ad(a + be^{c+dx})^2} + \frac{x^3}{a^2 d(a + be^{c+dx})} + \frac{x^4}{4a^3} - \frac{3x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d} \\
&= \frac{3x^2}{2a^3 d^2} - \frac{3x^2}{2a^2 d^2(a + be^{c+dx})} - \frac{3x^3}{2a^3 d} + \frac{x^3}{2ad(a + be^{c+dx})^2} + \frac{x^3}{a^2 d(a + be^{c+dx})} + \frac{x^4}{4a^3} - \frac{3x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 241, normalized size = 0.72

$$\frac{2a^2 x^3}{d(a+be^{c+dx})^2} + \frac{12(2dx-3)\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{d^4} - \frac{24\text{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{d^4} - \frac{12x \log\left(\frac{be^{c+dx}}{a}+1\right)}{d^3} - \frac{6ax^2}{d^2(a+be^{c+dx})} + \frac{18x^2 \log\left(\frac{be^{c+dx}}{a}+1\right)}{d^2} - \frac{12(d^2x^2-3dx+1)\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{d^4}$$


---


$$4a^3$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*E^(c + d\*x))^3,x]

```
[Out] ((6*x^2)/d^2 - (6*a*x^2)/(d^2*(a + b*E^(c + d*x))) - (6*x^3)/d + (2*a^2*x^3)/(d*(a + b*E^(c + d*x))^2) + (4*a*x^3)/(a*d + b*d*E^(c + d*x)) + x^4 - (12*x*Log[1 + (b*E^(c + d*x))/a])/d^3 + (18*x^2*Log[1 + (b*E^(c + d*x))/a])/d^2 - (4*x^3*Log[1 + (b*E^(c + d*x))/a])/d - (12*(1 - 3*d*x + d^2*x^2)*PolyLog[2, -((b*E^(c + d*x))/a)])/d^4 + (12*(-3 + 2*d*x)*PolyLog[3, -((b*E^(c + d*x))/a)])/d^4 - (24*PolyLog[4, -((b*E^(c + d*x))/a)])/d^4)/(4*a^3)
```

**fricas** [C] time = 0.45, size = 702, normalized size = 2.11

$$a^2 d^4 x^4 - a^2 c^4 - 6 a^2 c^3 - 6 a^2 c^2 - 12 (a^2 d^2 x^2 - 3 a^2 d x + a^2 + (b^2 d^2 x^2 - 3 b^2 d x + b^2) e^{(2 d x + 2 c)} + 2 (a b d^2 x^2 - 3 a b d x + a^2 b)) e^{(d x + c)} + 2 (a b d^2 x^2 - 3 a b d x + a^2 b) e^{(d x + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*d^4*x^4 - a^2*c^4 - 6*a^2*c^3 - 6*a^2*c^2 - 12*(a^2*d^2*x^2 - 3*a^2*d*x + a^2 + (b^2*d^2*x^2 - 3*b^2*d*x + b^2)*e^(2*d*x + 2*c) + 2*(a*b*d^2*x^2 - 3*a*b*d*x + a*b)*e^(d*x + c))*dilog(-(b*e^(d*x + c) + a)/a + 1) + (b^2*d^4*x^4 - 6*b^2*d^3*x^3 - b^2*c^4 + 6*b^2*d^2*x^2 - 6*b^2*c^3 - 6*b^2*c^2)*e^(2*d*x + 2*c) + 2*(a*b*d^4*x^4 - 4*a*b*d^3*x^3 - a*b*c^4 + 3*a*b*d^2*x^2 - 6*a*b*c^3 - 6*a*b*c^2)*e^(d*x + c) + 2*(2*a^2*c^3 + 9*a^2*c^2 + 6*a^2*c + (2*b^2*c^3 + 9*b^2*c^2 + 6*b^2*c)*e^(2*d*x + 2*c) + 2*(2*a*b*c^3 + 9*a*b*c^2 + 6*a*b*c)*e^(d*x + c))*log(b*e^(d*x + c) + a) - 2*(2*a^2*d^3*x^3 - 9*a^2*d^2*x^2 + 2*a^2*c^3 + 9*a^2*c^2 + 6*a^2*d*x + 6*a^2*c + (2*b^2*d^3*x^3 - 9*b^2*d^2*x^2 + 2*b^2*c^3 + 9*b^2*c^2 + 6*b^2*d*x + 6*b^2*c)*e^(2*d*x + 2*c) + 2*(2*a*b*d^3*x^3 - 9*a*b*d^2*x^2 + 2*a*b*c^3 + 9*a*b*c^2 + 6*a*b*d*x + 6*a*b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a) - 24*(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) + a^2)*polylog(4, -b*e^(d*x + c)/a) + 12*(2*a^2*d*x - 3*a^2 + (2*b^2*d*x - 3*b^2)*e^(2*d*x + 2*c) + 2*(2*a*b*d*x - 3*a*b)*e^(d*x + c))*polylog(3, -b*e^(d*x + c)/a))/(a^3*b^2*d^4*e^(2*d*x + 2*c) + 2*a^4*b*d^4*e^(d*x + c) + a^5*d^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b e^{(d x + c)} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*e^(d*x + c) + a)^3, x)
```

**maple** [A] time = 0.17, size = 548, normalized size = 1.65

$$\frac{x^4}{4a^3} - \frac{x^3 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{a^3 d} - \frac{3x^3}{2a^3 d} + \frac{(2bdx e^{dx+c} + 3adx - 3b e^{dx+c} - 3a)x^2}{2(b e^{dx+c} + a)^2 a^2 d^2} + \frac{c^3 x}{a^3 d^3} - \frac{3x^2 \operatorname{polylog}\left(2, -\frac{be^{dx+c}}{a}\right)}{a^3 d^2} + \frac{9x^2 \ln\left(\frac{be^{dx+c}}{a} + 1\right)}{2a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(b*exp(d*x+c)+a)^3,x)`

[Out]  $\frac{1}{2}x^2(2xb d \exp(dx+c) + 3a d x - 3b \exp(dx+c) - 3a)/d^2/a^2/(b \exp(dx+c) + a)^2 + 9x \operatorname{polylog}(2, -1/a b \exp(dx+c))/a^3/d^3 - 3x^2 \operatorname{polylog}(2, -1/a b \exp(dx+c))/a^3/d^2 + 6x \operatorname{polylog}(3, -1/a b \exp(dx+c))/a^3/d^3 + 3/a^3/d^4 c^3 + 3/2/a^3/d^4 c^2 + 3/4/a^3/d^4 c^4 + 1/4x^4/a^3 - 3x \ln(1/a b \exp(dx+c) + 1)/a^3/d^3 + 9/2x^2 \ln(1/a b \exp(dx+c) + 1)/a^3/d^2 - x^3 \ln(1/a b \exp(dx+c) + 1)/a^3/d + 9/2/a^3/d^3 c^2 x + 3/a^3/d^3 c x + 1/a^3/d^3 c^3 x + 9/2/a^3/d^4 c^2 \ln(b \exp(dx+c) + a) - 9/2/a^3/d^4 c^2 \ln(\exp(dx+c)) + 1/a^3/d^4 c^3 \ln(b \exp(dx+c) + a) - 1/a^3/d^4 c^3 \ln(\exp(dx+c)) + 3/a^3/d^4 c \ln(b \exp(dx+c) + a) - 3/a^3/d^4 c \ln(\exp(dx+c)) - 9/2/a^3/d^4 \ln(1/a b \exp(dx+c) + 1) c^2 - 3/a^3/d^4 \ln(1/a b \exp(dx+c) + 1) c - 1/a^3/d^4 \ln(1/a b \exp(dx+c) + 1) c^3 + 3/2x^2/a^3/d^2 - 3/2x^3/a^3/d - 3 \operatorname{polylog}(2, -1/a b \exp(dx+c))/a^3/d^4 - 9 \operatorname{polylog}(3, -1/a b \exp(dx+c))/a^3/d^4 - 6 \operatorname{polylog}(4, -1/a b \exp(dx+c))/a^3/d^4$

**maxima** [A] time = 1.08, size = 303, normalized size = 0.91

$$\frac{3ad^3x^3 - 3ax^2 + (2bdx^3e^c - 3bx^2e^c)e^{(dx)}}{2(a^2b^2d^2e^{2dx+2c} + 2a^3bd^2e^{dx+c} + a^4d^2)} + \frac{d^4x^4 - 6d^3x^3 + 6d^2x^2}{4a^3d^4} - \frac{d^3x^3 \log\left(\frac{be^{dx+c}}{a} + 1\right) + 3d^2x^2 \operatorname{Li}_2\left(-\frac{be^{dx+c}}{a}\right)}{a^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2}(3a d x^3 - 3a x^2 + (2b d x^3 e^c - 3b x^2 e^c) e^{(dx)})/(a^2 b^2 d^2 e^{(2dx+2c)} + 2a^3 b d^2 e^{(dx+c)} + a^4 d^2) + \frac{1}{4}(d^4 x^4 - 6d^3 x^3 + 6d^2 x^2)/(a^3 d^4) - \frac{(d^3 x^3 \log(b e^{(dx+c)}/a + 1) + 3d^2 x^2 \operatorname{dilog}(-b e^{(dx+c)}/a) - 6d x \operatorname{polylog}(3, -b e^{(dx+c)}/a) + 6 \operatorname{polylog}(4, -b e^{(dx+c)}/a))/(a^3 d^4) + 9/2(d^2 x^2 \log(b e^{(dx+c)}/a + 1) + 2d x \operatorname{dilog}(-b e^{(dx+c)}/a) - 2 \operatorname{polylog}(3, -b e^{(dx+c)}/a))/(a^3 d^4) - 3(d x \log(b e^{(dx+c)}/a + 1) + \operatorname{dilog}(-b e^{(dx+c)}/a))/(a^3 d^4)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(a + b e^{c+dx})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a + b*exp(c + d*x))^3, x)`

[Out] `int(x^3/(a + b*exp(c + d*x))^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3adx^3 - 3ax^2 + (2bdx^3 - 3bx^2)e^{c+dx}}{2a^4d^2 + 4a^3bd^2e^{c+dx} + 2a^2b^2d^2e^{2c+2dx}} + \frac{\int \frac{6x}{a+be^ce^{dx}} dx + \int \left(-\frac{9dx^2}{a+be^ce^{dx}}\right) dx + \int \frac{2d^2x^3}{a+be^ce^{dx}} dx}{2a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(a+b*exp(d*x+c))**3, x)`

[Out] `(3*a*d*x**3 - 3*a*x**2 + (2*b*d*x**3 - 3*b*x**2)*exp(c + d*x))/(2*a**4*d**2 + 4*a**3*b*d**2*exp(c + d*x) + 2*a**2*b**2*d**2*exp(2*c + 2*d*x)) + (Integral(6*x/(a + b*exp(c)*exp(d*x)), x) + Integral(-9*d*x**2/(a + b*exp(c)*exp(d*x)), x) + Integral(2*d**2*x**3/(a + b*exp(c)*exp(d*x)), x))/(2*a**2*d**2)`

$$3.18 \quad \int \frac{x^2}{(a+be^{c+dx})^3} dx$$

**Optimal.** Leaf size=243

$$\frac{3\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{\log(a+be^{c+dx})}{a^3d^3} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{3x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d^2} - \frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d} + \frac{x}{a^3d^2}$$

[Out]  $x/a^3/d^2 - x/a^2/d^2/(a+b*\exp(d*x+c)) - 3/2*x^2/a^3/d + 1/2*x^2/a/d/(a+b*\exp(d*x+c))^2 + x^2/a^2/d/(a+b*\exp(d*x+c)) + 1/3*x^3/a^3 - \ln(a+b*\exp(d*x+c))/a^3/d^3 + 3*x*\ln(1+b*\exp(d*x+c)/a)/a^3/d^2 - x^2*\ln(1+b*\exp(d*x+c)/a)/a^3/d + 3*\text{polylog}(2, -b*\exp(d*x+c)/a)/a^3/d^3 - 2*x*\text{polylog}(2, -b*\exp(d*x+c)/a)/a^3/d^2 + 2*\text{polylog}(3, -b*\exp(d*x+c)/a)/a^3/d^3$

**Rubi [A]** time = 0.76, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$ , Rules used = {2185, 2184, 2190, 2531, 2282, 6589, 2191, 2279, 2391, 36, 29, 31}

$$-\frac{2x\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{x}{a^2d^2(a+be^{c+dx})} + \frac{3x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(a + b*E^{(c + d*x)})^3, x]$

[Out]  $x/(a^3*d^2) - x/(a^2*d^2*(a + b*E^{(c + d*x)})) - (3*x^2)/(2*a^3*d) + x^2/(2*a*d*(a + b*E^{(c + d*x)})^2) + x^2/(a^2*d*(a + b*E^{(c + d*x)})) + x^3/(3*a^3) - \text{Log}[a + b*E^{(c + d*x)}]/(a^3*d^3) + (3*x*\text{Log}[1 + (b*E^{(c + d*x)})/a])/(a^3*d^2) - (x^2*\text{Log}[1 + (b*E^{(c + d*x)})/a])/(a^3*d) + (3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/a)])/(a^3*d^3) - (2*x*\text{PolyLog}[2, -((b*E^{(c + d*x)})/a)])/(a^3*d^2) + (2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/a)])/(a^3*d^3)$

**Rule 29**

$\text{Int}[(x_)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

**Rule 31**

$\text{Int}[(a_) + (b_)*(x_)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$



Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2184

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_)))))^(n_.), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(((c + d*x)^m*(F^(g*(e + f*x))))^n)/(a + b*(F^(g*(e + f*x))))^n, x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2185

```
Int[(((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
f*x))))^n^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x))))^n*
(a + b*(F^(g*(e + f*x))))^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x))))^n^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x))))^n^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a + be^{c+dx})^3} dx &= \frac{\int \frac{x^2}{(a+be^{c+dx})^2} dx}{a} - \frac{b \int \frac{e^{c+dx} x^2}{(a+be^{c+dx})^3} dx}{a} \\
&= \frac{x^2}{2ad(a + be^{c+dx})^2} + \frac{\int \frac{x^2}{a+be^{c+dx}} dx}{a^2} - \frac{b \int \frac{e^{c+dx} x^2}{(a+be^{c+dx})^2} dx}{a^2} - \frac{\int \frac{x}{(a+be^{c+dx})^2} dx}{ad} \\
&= \frac{x^2}{2ad(a + be^{c+dx})^2} + \frac{x^2}{a^2 d(a + be^{c+dx})} + \frac{x^3}{3a^3} - \frac{b \int \frac{e^{c+dx} x^2}{a+be^{c+dx}} dx}{a^3} - \frac{\int \frac{x}{a+be^{c+dx}} dx}{a^2 d} - \frac{2 \int \frac{x}{a+be^{c+dx}} dx}{a^2 d} \\
&= -\frac{x}{a^2 d^2(a + be^{c+dx})} - \frac{3x^2}{2a^3 d} + \frac{x^2}{2ad(a + be^{c+dx})^2} + \frac{x^2}{a^2 d(a + be^{c+dx})} + \frac{x^3}{3a^3} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d} \\
&= -\frac{x}{a^2 d^2(a + be^{c+dx})} - \frac{3x^2}{2a^3 d} + \frac{x^2}{2ad(a + be^{c+dx})^2} + \frac{x^2}{a^2 d(a + be^{c+dx})} + \frac{x^3}{3a^3} + \frac{3x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d^2} \\
&= -\frac{x}{a^2 d^2(a + be^{c+dx})} - \frac{3x^2}{2a^3 d} + \frac{x^2}{2ad(a + be^{c+dx})^2} + \frac{x^2}{a^2 d(a + be^{c+dx})} + \frac{x^3}{3a^3} + \frac{3x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d^2} \\
&= \frac{x}{a^3 d^2} - \frac{x}{a^2 d^2(a + be^{c+dx})} - \frac{3x^2}{2a^3 d} + \frac{x^2}{2ad(a + be^{c+dx})^2} + \frac{x^2}{a^2 d(a + be^{c+dx})} + \frac{x^3}{3a^3} - \frac{\log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 203, normalized size = 0.84

$$\frac{3a^2 x^2}{d(a+be^{c+dx})^2} - \frac{6(2dx-3)\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{d^3} + \frac{12\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{d^3} - \frac{6\log\left(\frac{be^{c+dx}}{a}+1\right)}{d^3} - \frac{6ax}{d^2(a+be^{c+dx})} + \frac{18x\log\left(\frac{be^{c+dx}}{a}+1\right)}{d^2} + \frac{6ax^2}{ad+bde^{c+dx}} - \frac{6x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*E^(c + d\*x))^3,x]

[Out] ((6\*x)/d^2 - (6\*a\*x)/(d^2\*(a + b\*E^(c + d\*x))) - (9\*x^2)/d + (3\*a^2\*x^2)/(d\*(a + b\*E^(c + d\*x))^2) + (6\*a\*x^2)/(a\*d + b\*d\*E^(c + d\*x)) + 2\*x^3 - (6\*Log[1 + (b\*E^(c + d\*x))/a])/d^3 + (18\*x\*Log[1 + (b\*E^(c + d\*x))/a])/d^2 - (6\*

$x^2 \cdot \text{Log}[1 + (b \cdot E^{(c + d \cdot x)})/a]/d - (6 \cdot (-3 + 2 \cdot d \cdot x) \cdot \text{PolyLog}[2, -((b \cdot E^{(c + d \cdot x)})/a)])/d^3 + (12 \cdot \text{PolyLog}[3, -((b \cdot E^{(c + d \cdot x)})/a)])/d^3)/(6 \cdot a^3)$

**fricas** [C] time = 0.41, size = 521, normalized size = 2.14

$$2 a^2 d^3 x^3 + 2 a^2 c^3 + 9 a^2 c^2 + 6 a^2 c - 6 (2 a^2 d x - 3 a^2 + (2 b^2 d x - 3 b^2) e^{(2 d x + 2 c)} + 2 (2 a b d x - 3 a b) e^{(d x + c)}) \text{Li}_2 \left( -\frac{b e^{(d x + c)}}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(d\*x+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{6} \cdot (2 \cdot a^2 \cdot d^3 \cdot x^3 + 2 \cdot a^2 \cdot c^3 + 9 \cdot a^2 \cdot c^2 + 6 \cdot a^2 \cdot c - 6 \cdot (2 \cdot a^2 \cdot d \cdot x - 3 \cdot a^2 + (2 \cdot b^2 \cdot d \cdot x - 3 \cdot b^2) \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot (2 \cdot a \cdot b \cdot d \cdot x - 3 \cdot a \cdot b) \cdot e^{(d \cdot x + c)}) \cdot \text{dilog}(-\frac{b \cdot e^{(d \cdot x + c)}}{a} + 1) + (2 \cdot b^2 \cdot d^3 \cdot x^3 - 9 \cdot b^2 \cdot d^2 \cdot x^2 + 2 \cdot b^2 \cdot c^3 + 9 \cdot b^2 \cdot c^2 + 6 \cdot b^2 \cdot d \cdot x + 6 \cdot b^2 \cdot c) \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot (2 \cdot a \cdot b \cdot d^3 \cdot x^3 - 6 \cdot a \cdot b \cdot d^2 \cdot x^2 + 2 \cdot a \cdot b \cdot c^3 + 9 \cdot a \cdot b \cdot c^2 + 3 \cdot a \cdot b \cdot d \cdot x + 6 \cdot a \cdot b \cdot c) \cdot e^{(d \cdot x + c)} - 6 \cdot (a^2 \cdot c^2 + 3 \cdot a^2 \cdot c + a^2 + (b^2 \cdot c^2 + 3 \cdot b^2 \cdot c + b^2) \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot (a \cdot b \cdot c^2 + 3 \cdot a \cdot b \cdot c + a \cdot b) \cdot e^{(d \cdot x + c)}) \cdot \log(b \cdot e^{(d \cdot x + c)} + a) - 6 \cdot (a^2 \cdot d^2 \cdot x^2 - a^2 \cdot c^2 - 3 \cdot a^2 \cdot d \cdot x - 3 \cdot a^2 \cdot c + (b^2 \cdot d^2 \cdot x^2 - b^2 \cdot c^2 - 3 \cdot b^2 \cdot d \cdot x - 3 \cdot b^2 \cdot c) \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot (a \cdot b \cdot d^2 \cdot x^2 - a \cdot b \cdot c^2 - 3 \cdot a \cdot b \cdot d \cdot x - 3 \cdot a \cdot b \cdot c) \cdot e^{(d \cdot x + c)}) \cdot \log((b \cdot e^{(d \cdot x + c)} + a)/a) + 12 \cdot (b^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot a \cdot b \cdot e^{(d \cdot x + c)} + a^2) \cdot \text{polylog}(3, -b \cdot e^{(d \cdot x + c)}/a))/(a^3 \cdot b^2 \cdot d^3 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot a^4 \cdot b \cdot d^3 \cdot e^{(d \cdot x + c)} + a^5 \cdot d^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b e^{(d x + c)} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*exp(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(x^2/(b\*e^(d\*x + c) + a)^3, x)

**maple** [A] time = 0.14, size = 385, normalized size = 1.58

$$\frac{x^3}{3a^3} - \frac{x^2 \ln\left(\frac{b e^{dx+c}}{a} + 1\right)}{a^3 d} - \frac{c^2 x}{a^3 d^2} - \frac{3x^2}{2a^3 d} + \frac{(2bdx e^{dx+c} + 3adx - 2b e^{dx+c} - 2a)x}{2(b e^{dx+c} + a)^2 a^2 d^2} - \frac{2c^3}{3a^3 d^3} + \frac{c^2 \ln\left(\frac{b e^{dx+c}}{a} + 1\right)}{a^3 d^3} - \frac{c^2 \ln(b e^{dx+c})}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*exp(d\*x+c)+a)^3,x)

[Out]  $\frac{1}{2}x(2bdx \exp(dx+c) + 3a^2dx - 2b \exp(dx+c) - 2a)/d^2/a^2/(b \exp(dx+c) + a)^2 - \ln(b \exp(dx+c) + a)/a^3/d^3 + 1/a^3/d^3 \ln(\exp(dx+c)) - 1/a^3/d^3 c^2 \ln(b \exp(dx+c) + a) + 1/a^3/d^3 c^2 \ln(\exp(dx+c)) + 1/3 x^3/a^3 - 1/a^3/d^2 c^2 x - 2/3/a^3/d^3 c^3 - x^2 \ln(1/a^3 b \exp(dx+c) + 1)/a^3/d + 1/a^3/d^3 \ln(1/a^3 b \exp(dx+c) + 1) * c^2 - 2x \operatorname{polylog}(2, -1/a^3 b \exp(dx+c))/a^3/d^2 + 2 \operatorname{polylog}(3, -1/a^3 b \exp(dx+c))/a^3/d^3 - 3/a^3/d^3 c \ln(b \exp(dx+c) + a) + 3/a^3/d^3 c \ln(\exp(dx+c)) - 3/2 x^2/a^3/d - 3/a^3/d^2 c x - 3/2/a^3/d^3 c^2 + 3x \ln(1/a^3 b \exp(dx+c) + 1)/a^3/d^2 + 3/a^3/d^3 \ln(1/a^3 b \exp(dx+c) + 1) * c + 3 \operatorname{polylog}(2, -1/a^3 b \exp(dx+c))/a^3/d^3$

**maxima** [A] time = 1.06, size = 234, normalized size = 0.96

$$\frac{3 adx^2 - 2 ax + 2 (bdx^2 e^c - bxe^c) e^{(dx)}}{2 (a^2 b^2 d^2 e^{(2dx+2c)} + 2 a^3 b d^2 e^{(dx+c)} + a^4 d^2)} + \frac{x}{a^3 d^2} + \frac{2 d^3 x^3 - 9 d^2 x^2}{6 a^3 d^3} - \frac{d^2 x^2 \log\left(\frac{be^{(dx+c)}}{a} + 1\right) + 2 dx \operatorname{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{a^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a+b*exp(d*x+c))^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2} * (3a^2 dx^2 - 2a^2 x + 2(b^2 dx^2 e^c - b^2 x e^c) e^{(dx)}) / (a^2 b^2 d^2 e^{(2dx+2c)} + 2a^3 b d^2 e^{(dx+c)} + a^4 d^2) + x / (a^3 d^2) + 1/6 * (2d^3 x^3 - 9d^2 x^2) / (a^3 d^3) - (d^2 x^2 \log(b e^{(dx+c)} / a + 1) + 2d^2 x \operatorname{dilog}(-b e^{(dx+c)} / a) - 2 \operatorname{polylog}(3, -b e^{(dx+c)} / a)) / (a^3 d^3) + 3 * (d^2 x \log(b e^{(dx+c)} / a + 1) + \operatorname{dilog}(-b e^{(dx+c)} / a)) / (a^3 d^3) - \log(b e^{(dx+c)} / a + 1) / (a^3 d^3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + b e^{c+dx})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*exp(c + d*x))^3,x)`

[Out] `int(x^2/(a + b*exp(c + d*x))^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3 adx^2 - 2 ax + (2bdx^2 - 2bx) e^{c+dx}}{2a^4 d^2 + 4a^3 b d^2 e^{c+dx} + 2a^2 b^2 d^2 e^{2c+2dx}} + \frac{\int \left(-\frac{3dx}{a+be^c e^{dx}}\right) dx + \int \frac{d^2 x^2}{a+be^c e^{dx}} dx + \int \frac{1}{a+be^c e^{dx}} dx}{a^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+b*exp(d*x+c))**3,x)`

```
[Out] (3*a*d*x**2 - 2*a*x + (2*b*d*x**2 - 2*b*x)*exp(c + d*x))/(2*a**4*d**2 + 4*a
**3*b*d**2*exp(c + d*x) + 2*a**2*b**2*d**2*exp(2*c + 2*d*x)) + (Integral(-3
*d*x/(a + b*exp(c)*exp(d*x)), x) + Integral(d**2*x**2/(a + b*exp(c)*exp(d*x
)), x) + Integral(1/(a + b*exp(c)*exp(d*x)), x))/(a**2*d**2)
```

$$3.19 \quad \int \frac{x}{(a+be^{c+dx})^3} dx$$

**Optimal.** Leaf size=159

$$-\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{3\log(a+be^{c+dx})}{2a^3d^2} - \frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d} - \frac{3x}{2a^3d} + \frac{x^2}{2a^3} - \frac{1}{2a^2d^2(a+be^{c+dx})} + \frac{x}{a^2d(a+be^{c+dx})} + \frac{x^2}{2ad(a+be^{c+dx})}$$

[Out]  $-1/2/a^2/d^2/(a+b*\exp(d*x+c))-3/2*x/a^3/d+1/2*x/a/d/(a+b*\exp(d*x+c))^2+x/a^2/d/(a+b*\exp(d*x+c))+1/2*x^2/a^3+3/2*\ln(a+b*\exp(d*x+c))/a^3/d^2-x*\ln(1+b*\exp(d*x+c)/a)/a^3/d-\text{polylog}(2,-b*\exp(d*x+c)/a)/a^3/d^2$

**Rubi [A]** time = 0.33, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$ , Rules used = {2185, 2184, 2190, 2279, 2391, 2191, 2282, 36, 29, 31, 44}

$$-\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2} - \frac{1}{2a^2d^2(a+be^{c+dx})} + \frac{3\log(a+be^{c+dx})}{2a^3d^2} + \frac{x}{a^2d(a+be^{c+dx})} - \frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d} - \frac{3x}{2a^3d} + \frac{x^2}{2a^3}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*E^(c + d\*x))^3,x]

[Out]  $-1/(2*a^2*d^2*(a + b*E^(c + d*x))) - (3*x)/(2*a^3*d) + x/(2*a*d*(a + b*E^(c + d*x))^2) + x/(a^2*d*(a + b*E^(c + d*x))) + x^2/(2*a^3) + (3*\text{Log}[a + b*E^(c + d*x)])/(2*a^3*d^2) - (x*\text{Log}[1 + (b*E^(c + d*x))/a])/(a^3*d) - \text{PolyLog}[2, -((b*E^(c + d*x))/a)]/(a^3*d^2)$

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^-1, x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 44

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2184

```
Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))
^((n_))), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2185

```
Int[((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))^(p_)*((c_) +
(d_)*(x_))^(m_), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((a_) + (b_)*((F_)^((g_)*
(e_) + (f_)*(x_))))^(n_))^(p_)*((c_) + (d_)*(x_))^(m_), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```



Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + be^{c+dx})^3} dx &= \frac{\int \frac{x}{(a+be^{c+dx})^2} dx}{a} - \frac{b \int \frac{e^{c+dx} x}{(a+be^{c+dx})^3} dx}{a} \\
&= \frac{x}{2ad(a + be^{c+dx})^2} + \frac{\int \frac{x}{a+be^{c+dx}} dx}{a^2} - \frac{b \int \frac{e^{c+dx} x}{(a+be^{c+dx})^2} dx}{a^2} - \frac{\int \frac{1}{(a+be^{c+dx})^2} dx}{2ad} \\
&= \frac{x}{2ad(a + be^{c+dx})^2} + \frac{x}{a^2 d(a + be^{c+dx})} + \frac{x^2}{2a^3} - \frac{b \int \frac{e^{c+dx} x}{a+be^{c+dx}} dx}{a^3} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, e^{c+dx}\right)}{2ad^2} \\
&= \frac{x}{2ad(a + be^{c+dx})^2} + \frac{x}{a^2 d(a + be^{c+dx})} + \frac{x^2}{2a^3} - \frac{x \log\left(1 + \frac{be^{c+dx}}{a}\right)}{a^3 d} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, e^{c+dx}\right)}{a^2 d^2} \\
&= -\frac{1}{2a^2 d^2 (a + be^{c+dx})} - \frac{x}{2a^3 d} + \frac{x}{2ad(a + be^{c+dx})^2} + \frac{x}{a^2 d(a + be^{c+dx})} + \frac{x^2}{2a^3} + \frac{\log(a + be^{c+dx})}{2a^3 d^2} \\
&= -\frac{1}{2a^2 d^2 (a + be^{c+dx})} - \frac{3x}{2a^3 d} + \frac{x}{2ad(a + be^{c+dx})^2} + \frac{x}{a^2 d(a + be^{c+dx})} + \frac{x^2}{2a^3} + \frac{3 \log(a + be^{c+dx})}{2a^3 d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 120, normalized size = 0.75

$$\frac{dx \left( dx - 2 \log \left( \frac{be^{c+dx}}{a} + 1 \right) \right) - 2 \operatorname{Li}_2 \left( -\frac{be^{c+dx}}{a} \right)}{a} + \frac{adx}{(a+be^{c+dx})^2} + \frac{2dx-1}{a+be^{c+dx}} + \frac{3 \log \left( \frac{be^{c+dx}}{a} + 1 \right) - 3dx}{a}$$


---


$$2a^2d^2$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*E^(c + d\*x))^3, x]

[Out] ((a\*d\*x)/(a + b\*E^(c + d\*x))^2 + (-1 + 2\*d\*x)/(a + b\*E^(c + d\*x)) + (-3\*d\*x + 3\*Log[1 + (b\*E^(c + d\*x))/a])/a + (d\*x\*(d\*x - 2\*Log[1 + (b\*E^(c + d\*x))/a]) - 2\*PolyLog[2, -((b\*E^(c + d\*x))/a)])/a)/(2\*a^2\*d^2)

**fricas [B]** time = 0.47, size = 338, normalized size = 2.13

$$\frac{a^2d^2x^2 - a^2c^2 - 3a^2c - a^2 - 2(b^2e^{2dx+2c} + 2abe^{dx+c} + a^2)\operatorname{Li}_2\left(-\frac{be^{(dx+c)+a}}{a} + 1\right) + (b^2d^2x^2 - b^2c^2 - 3b^2dx - 3b^2c)}{2a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c))^3,x, algorithm="fricas")

[Out] 1/2\*(a^2\*d^2\*x^2 - a^2\*c^2 - 3\*a^2\*c - a^2 - 2\*(b^2\*e^(2\*d\*x + 2\*c) + 2\*a\*b\*e^(d\*x + c) + a^2)\*dilog(-(b\*e^(d\*x + c) + a)/a + 1) + (b^2\*d^2\*x^2 - b^2\*c^2 - 3\*b^2\*d\*x - 3\*b^2\*c)\*e^(2\*d\*x + 2\*c) + (2\*a\*b\*d^2\*x^2 - 2\*a\*b\*c^2 - 4\*a\*b\*d\*x - 6\*a\*b\*c - a\*b)\*e^(d\*x + c) + (2\*a^2\*c + 3\*a^2 + (2\*b^2\*c + 3\*b^2)\*e^(2\*d\*x + 2\*c) + 2\*(2\*a\*b\*c + 3\*a\*b)\*e^(d\*x + c))\*log(b\*e^(d\*x + c) + a) - 2\*(a^2\*d\*x + a^2\*c + (b^2\*d\*x + b^2\*c)\*e^(2\*d\*x + 2\*c) + 2\*(a\*b\*d\*x + a\*b\*c)\*e^(d\*x + c))\*log((b\*e^(d\*x + c) + a)/a)/(a^3\*b^2\*d^2\*e^(2\*d\*x + 2\*c) + 2\*a^4\*b\*d^2\*e^(d\*x + c) + a^5\*d^2)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(be^{(dx+c)} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(x/(b\*e^(d\*x + c) + a)^3, x)

**maple [B]** time = 0.04, size = 393, normalized size = 2.47

$$\frac{bx e^{dx+c}}{(b e^{dx+c} + a)^2 a^2 d} - \frac{b^2 x e^{2dx+2c}}{2(b e^{dx+c} + a)^2 a^3 d} - \frac{bc e^{dx+c}}{(b e^{dx+c} + a)^2 a^2 d^2} - \frac{b^2 c e^{2dx+2c}}{2(b e^{dx+c} + a)^2 a^3 d^2} - \frac{bx e^{dx+c}}{(b e^{dx+c} + a) a^3 d} + \frac{x^2}{2a^3} - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*exp(d\*x+c)+a)^3,x)

[Out]  $\frac{1}{2}x^2/a^3 + 1/d/a^3 x + 1/2/d^2/a^3 c^2 + 3/2 \ln(b \exp(dx+c)+a)/a^3/d^2 - 1/d/a^3 b \exp(dx+c)/(b \exp(dx+c)+a) x - 1/d^2/a^3 b \exp(dx+c)/(b \exp(dx+c)+a) c - 1/2/a^2/d^2/(b \exp(dx+c)+a) - 1/2/d/a^3 b^2 \exp(dx+c)^2/(b \exp(dx+c)+a)^2 x - 1/2/d^2/a^3 b^2 \exp(dx+c)^2/(b \exp(dx+c)+a)^2 c - 1/d/a^2 b \exp(dx+c)/(b \exp(dx+c)+a)^2 x - 1/d^2/a^2 b \exp(dx+c)/(b \exp(dx+c)+a)^2 c - 1/d^2/a^3 \operatorname{dilog}((b \exp(dx+c)+a)/a) - 1/d/a^3 \ln((b \exp(dx+c)+a)/a) x - 1/d^2/a^3 \ln((b \exp(dx+c)+a)/a) c - 1/d^2 c/a^3 \ln(\exp(dx+c)) + 1/d^2 c/a^3 \ln(b \exp(dx+c)+a) - 1/d^2 c/a^2/(b \exp(dx+c)+a) - 1/2/d^2 c/a/(b \exp(dx+c)+a)^2$

**maxima [A]** time = 1.05, size = 149, normalized size = 0.94

$$\frac{3 a d x + (2 b d x e^c - b e^c) e^{(d x)} - a}{2(a^2 b^2 d^2 e^{2(dx+c)} + 2 a^3 b d^2 e^{(dx+c)} + a^4 d^2)} + \frac{x^2}{2 a^3} - \frac{3 x}{2 a^3 d} - \frac{d x \log\left(\frac{b e^{(d x+c)}}{a} + 1\right) + \operatorname{Li}_2\left(-\frac{b e^{(d x+c)}}{a}\right)}{a^3 d^2} + \frac{3 \log\left(b e^{(d x+c)} + a\right)}{2 a^3 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c))^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}*(3*a*d*x + (2*b*d*x*e^c - b*e^c)*e^{(d*x)} - a)/(a^2*b^2*d^2*e^{(2*d*x + 2*c)} + 2*a^3*b*d^2*e^{(d*x + c)} + a^4*d^2) + 1/2*x^2/a^3 - 3/2*x/(a^3*d) - (d*x*\log(b*e^{(d*x + c)}/a + 1) + \operatorname{dilog}(-b*e^{(d*x + c)}/a))/(a^3*d^2) + 3/2*\log(b*e^{(d*x + c)} + a)/(a^3*d^2)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b e^{c+dx})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*exp(c + d\*x))^3,x)

[Out] int(x/(a + b\*exp(c + d\*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3adx - a + (2bdx - b)e^{c+dx}}{2a^4d^2 + 4a^3bd^2e^{c+dx} + 2a^2b^2d^2e^{2c+2dx}} + \frac{\int \frac{2dx}{a+be^ce^{dx}} dx + \int \left(-\frac{3}{a+be^ce^{dx}}\right) dx}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*exp(d\*x+c))\*\*3,x)

[Out] (3\*a\*d\*x - a + (2\*b\*d\*x - b)\*exp(c + d\*x))/(2\*a\*\*4\*d\*\*2 + 4\*a\*\*3\*b\*d\*\*2\*exp(c + d\*x) + 2\*a\*\*2\*b\*\*2\*d\*\*2\*exp(2\*c + 2\*d\*x)) + (Integral(2\*d\*x/(a + b\*exp(c)\*exp(d\*x)), x) + Integral(-3/(a + b\*exp(c)\*exp(d\*x)), x))/(2\*a\*\*2\*d)

$$3.20 \quad \int \frac{1}{(a+be^{c+dx})^3} dx$$

Optimal. Leaf size=69

$$-\frac{\log(a+be^{c+dx})}{a^3d} + \frac{x}{a^3} + \frac{1}{a^2d(a+be^{c+dx})} + \frac{1}{2ad(a+be^{c+dx})^2}$$

[Out] 1/2/a/d/(a+b\*exp(d\*x+c))^2+1/a^2/d/(a+b\*exp(d\*x+c))+x/a^3-ln(a+b\*exp(d\*x+c))/a^3/d

**Rubi** [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2282, 44}

$$\frac{1}{a^2d(a+be^{c+dx})} - \frac{\log(a+be^{c+dx})}{a^3d} + \frac{x}{a^3} + \frac{1}{2ad(a+be^{c+dx})^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(c + d\*x))^(-3), x]

[Out] 1/(2\*a\*d\*(a + b\*E^(c + d\*x))^2) + 1/(a^2\*d\*(a + b\*E^(c + d\*x))) + x/a^3 - Log[a + b\*E^(c + d\*x)]/(a^3\*d)

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + be^{c+dx})^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3} dx, x, e^{c+dx}\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right) dx, x, e^{c+dx}\right)}{d} \\
&= \frac{1}{2ad(a + be^{c+dx})^2} + \frac{1}{a^2d(a + be^{c+dx})} + \frac{x}{a^3} - \frac{\log(a + be^{c+dx})}{a^3d}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 62, normalized size = 0.90

$$\frac{\frac{a^2}{(a+be^{c+dx})^2} + \frac{2a}{a+be^{c+dx}} - 2\log(a + be^{c+dx}) + 2dx}{2a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(c + d\*x))^(-3), x]

[Out] (a^2/(a + b\*E^(c + d\*x))^2 + (2\*a)/(a + b\*E^(c + d\*x)) + 2\*d\*x - 2\*Log[a + b\*E^(c + d\*x)])/(2\*a^3\*d)

**fricas [A]** time = 0.43, size = 127, normalized size = 1.84

$$\frac{2b^2dxe^{2dx+2c} + 2a^2dx + 3a^2 + 2(2abdx + ab)e^{(dx+c)} - 2(b^2e^{2dx+2c} + 2abe^{(dx+c)} + a^2)\log(be^{(dx+c)} + a)}{2(a^3b^2de^{2dx+2c} + 2a^4bde^{(dx+c)} + a^5d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3,x, algorithm="fricas")

[Out] 1/2\*(2\*b^2\*d\*x\*e^(2\*d\*x + 2\*c) + 2\*a^2\*d\*x + 3\*a^2 + 2\*(2\*a\*b\*d\*x + a\*b)\*e^(d\*x + c) - 2\*(b^2\*e^(2\*d\*x + 2\*c) + 2\*a\*b\*e^(d\*x + c) + a^2)\*log(b\*e^(d\*x + c) + a))/(a^3\*b^2\*d\*e^(2\*d\*x + 2\*c) + 2\*a^4\*b\*d\*e^(d\*x + c) + a^5\*d)

**giac [A]** time = 0.22, size = 65, normalized size = 0.94

$$\frac{\frac{2(dx+c)}{a^3} - \frac{2\log(|be^{(dx+c)}+a|)}{a^3} + \frac{2abe^{(dx+c)}+3a^2}{(be^{(dx+c)}+a)^2 a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3,x, algorithm="giac")

[Out]  $\frac{1}{2} \cdot \frac{2(d*x + c)/a^3 - 2 \cdot \log(\text{abs}(b \cdot e^{(d*x + c)} + a))}{a^3} + \frac{(2 \cdot a \cdot b \cdot e^{(d*x + c)} + 3 \cdot a^2)}{((b \cdot e^{(d*x + c)} + a)^2 \cdot a^3)} / d$

**maple [A]** time = 0.01, size = 74, normalized size = 1.07

$$\frac{1}{2(b e^{dx+c} + a)^2 a d} + \frac{1}{(b e^{dx+c} + a) a^2 d} - \frac{\ln(b e^{dx+c} + a)}{a^3 d} + \frac{\ln(e^{dx+c})}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(d\*x+c)+a)^3,x)

[Out]  $\frac{1}{d \cdot a^3} \cdot \ln(\exp(d*x+c)) - \frac{\ln(b \cdot \exp(d*x+c) + a)}{a^3 \cdot d} + \frac{1}{a^2 \cdot d} \cdot \frac{1}{(b \cdot \exp(d*x+c) + a)} + \frac{1}{2 \cdot a \cdot d} \cdot \frac{1}{(b \cdot \exp(d*x+c) + a)^2}$

**maxima [A]** time = 0.91, size = 84, normalized size = 1.22

$$\frac{2 b e^{(dx+c)} + 3 a}{2(a^2 b^2 e^{2(dx+c)} + 2 a^3 b e^{(dx+c)} + a^4) d} + \frac{dx + c}{a^3 d} - \frac{\log(b e^{(dx+c)} + a)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3,x, algorithm="maxima")

[Out]  $\frac{1}{2} \cdot \frac{2 \cdot b \cdot e^{(d*x + c)} + 3 \cdot a}{((a^2 \cdot b^2 \cdot e^{(2 \cdot d*x + 2 \cdot c)} + 2 \cdot a^3 \cdot b \cdot e^{(d*x + c)} + a^4) \cdot d)} + \frac{(d*x + c)}{(a^3 \cdot d)} - \frac{\log(b \cdot e^{(d*x + c)} + a)}{(a^3 \cdot d)}$

**mupad [B]** time = 3.74, size = 121, normalized size = 1.75

$$\frac{\frac{x}{a} + \frac{b^2 x e^{2c+2dx}}{a^3} + \frac{2 b x e^{c+dx}}{a^2} - \frac{3 b^2 e^{2c+2dx}}{2 a^3 d} - \frac{2 b e^{c+dx}}{a^2 d}}{a^2 + 2 e^{c+dx} a b + e^{2c+2dx} b^2} - \frac{\ln(a + b e^{dx} e^c)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*exp(c + d\*x))^3,x)

[Out]  $(x/a + (b^2 \cdot x \cdot \exp(2 \cdot c + 2 \cdot d \cdot x)))/a^3 + (2 \cdot b \cdot x \cdot \exp(c + d \cdot x))/a^2 - (3 \cdot b^2 \cdot \exp(2 \cdot c + 2 \cdot d \cdot x))/(2 \cdot a^3 \cdot d) - (2 \cdot b \cdot \exp(c + d \cdot x))/(a^2 \cdot d)/(a^2 + b^2 \cdot \exp(2 \cdot c + 2 \cdot d \cdot x) + 2 \cdot a \cdot b \cdot \exp(c + d \cdot x)) - \log(a + b \cdot \exp(d \cdot x) \cdot \exp(c))/(a^3 \cdot d)$

**sympy [A]** time = 0.19, size = 76, normalized size = 1.10

$$\frac{3a + 2be^{c+dx}}{2a^4d + 4a^3bde^{c+dx} + 2a^2b^2de^{2c+2dx}} + \frac{x}{a^3} - \frac{\log\left(\frac{a}{b} + e^{c+dx}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(d*x+c))**3,x)
```

```
[Out] (3*a + 2*b*exp(c + d*x))/(2*a**4*d + 4*a**3*b*d*exp(c + d*x) + 2*a**2*b**2*d*exp(2*c + 2*d*x)) + x/a**3 - log(a/b + exp(c + d*x))/(a**3*d)
```



$$3.21 \quad \int \frac{1}{(a+be^{c+dx})^3 x} dx$$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1}{x(a+be^{c+dx})^3}, x\right)$$

[Out] Unintegrable(1/(a+b\*exp(d\*x+c))^3/x, x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(a+be^{c+dx})^3 x} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*E^(c + d\*x))^3\*x), x]

[Out] Defer[Int][1/((a + b\*E^(c + d\*x))^3\*x), x]

Rubi steps

$$\int \frac{1}{(a+be^{c+dx})^3 x} dx = \int \frac{1}{(a+be^{c+dx})^3} dx$$

Mathematica [A] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+be^{c+dx})^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*E^(c + d\*x))^3\*x), x]

[Out] Integrate[1/((a + b\*E^(c + d\*x))^3\*x), x]

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3 x e^{(3 dx+3 c)} + 3 a b^2 x e^{(2 dx+2 c)} + 3 a^2 b x e^{(dx+c)} + a^3 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3/x,x, algorithm="fricas")

[Out] integral(1/(b^3\*x\*e^(3\*d\*x + 3\*c) + 3\*a\*b^2\*x\*e^(2\*d\*x + 2\*c) + 3\*a^2\*b\*x\*e^(d\*x + c) + a^3\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3/x,x, algorithm="giac")

[Out] integrate(1/((b\*e^(d\*x + c) + a)^3\*x), x)

**maple** [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(d\*x+c)+a)^3/x,x)

[Out] int(1/(b\*exp(d\*x+c)+a)^3/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3 adx + (2 bdx e^c + b e^c) e^{dx} + a}{2 (a^2 b^2 d^2 x^2 e^{2 dx+2c} + 2 a^3 b d^2 x^2 e^{dx+c} + a^4 d^2 x^2)} + \int \frac{2 d^2 x^2 + 3 dx + 2}{2 (a^2 b d^2 x^3 e^{dx+c} + a^3 d^2 x^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3/x,x, algorithm="maxima")

[Out] 1/2\*(3\*a\*d\*x + (2\*b\*d\*x\*e^c + b\*e^c)\*e^(d\*x) + a)/(a^2\*b^2\*d^2\*x^2\*e^(2\*d\*x + 2\*c) + 2\*a^3\*b\*d^2\*x^2\*e^(d\*x + c) + a^4\*d^2\*x^2) + integrate(1/2\*(2\*d^2\*x^2 + 3\*d\*x + 2)/(a^2\*b\*d^2\*x^3\*e^(d\*x + c) + a^3\*d^2\*x^3), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x(a + b e^{c+dx})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*exp(c + d*x))^3), x)`

[Out] `int(1/(x*(a + b*exp(c + d*x))^3), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3adx + a + (2bdx + b)e^{c+dx}}{2a^4d^2x^2 + 4a^3bd^2x^2e^{c+dx} + 2a^2b^2d^2x^2e^{2c+2dx}} + \frac{\int \frac{3dx}{ax^3+bx^3e^ce^{dx}} dx + \int \frac{2d^2x^2}{ax^3+bx^3e^ce^{dx}} dx + \int \frac{2}{ax^3+bx^3e^ce^{dx}} dx}{2a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(d*x+c))**3/x, x)`

[Out] `(3*a*d*x + a + (2*b*d*x + b)*exp(c + d*x))/(2*a**4*d**2*x**2 + 4*a**3*b*d**2*x**2*exp(c + d*x) + 2*a**2*b**2*d**2*x**2*exp(2*c + 2*d*x)) + (Integral(3*d*x/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x) + Integral(2*d**2*x**2/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x) + Integral(2/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x))/(2*a**2*d**2)`

$$3.22 \quad \int \frac{1}{(a+be^{c+dx})^3 x^2} dx$$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1}{x^2 (a + be^{c+dx})^3}, x\right)$$

[Out] Unintegrable(1/(a+b\*exp(d\*x+c))^3/x^2,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(a + be^{c+dx})^3 x^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*E^(c + d\*x))^3\*x^2), x]

[Out] Defer[Int][1/((a + b\*E^(c + d\*x))^3\*x^2), x]

Rubi steps

$$\int \frac{1}{(a + be^{c+dx})^3 x^2} dx = \int \frac{1}{(a + be^{c+dx})^3 x^2} dx$$

Mathematica [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + be^{c+dx})^3 x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*E^(c + d\*x))^3\*x^2), x]

[Out] Integrate[1/((a + b\*E^(c + d\*x))^3\*x^2), x]

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3 x^2 e^{(3 dx+3 c)} + 3 a b^2 x^2 e^{(2 dx+2 c)} + 3 a^2 b x^2 e^{(dx+c)} + a^3 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3/x^2,x, algorithm="fricas")

[Out] integral(1/(b^3\*x^2\*e^(3\*d\*x + 3\*c) + 3\*a\*b^2\*x^2\*e^(2\*d\*x + 2\*c) + 3\*a^2\*b\*x^2\*e^(d\*x + c) + a^3\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3/x^2,x, algorithm="giac")

[Out] integrate(1/((b\*e^(d\*x + c) + a)^3\*x^2), x)

**maple** [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(be^{dx+c} + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(d\*x+c)+a)^3/x^2,x)

[Out] int(1/(b\*exp(d\*x+c)+a)^3/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3 adx + 2 (bdxe^c + be^c)e^{(dx)} + 2 a}{2 (a^2 b^2 d^2 x^3 e^{(2 dx+2 c)} + 2 a^3 b d^2 x^3 e^{(dx+c)} + a^4 d^2 x^3)} + \int \frac{d^2 x^2 + 3 dx + 3}{a^2 b d^2 x^4 e^{(dx+c)} + a^3 d^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(d\*x+c))^3/x^2,x, algorithm="maxima")

[Out] 1/2\*(3\*a\*d\*x + 2\*(b\*d\*x\*e^c + b\*e^c)\*e^(d\*x) + 2\*a)/(a^2\*b^2\*d^2\*x^3\*e^(2\*d\*x + 2\*c) + 2\*a^3\*b\*d^2\*x^3\*e^(d\*x + c) + a^4\*d^2\*x^3) + integrate((d^2\*x^2 + 3\*d\*x + 3)/(a^2\*b\*d^2\*x^4\*e^(d\*x + c) + a^3\*d^2\*x^4), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 (a + b e^{c+dx})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b*exp(c + d*x))^3), x)`

[Out] `int(1/(x^2*(a + b*exp(c + d*x))^3), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3adx + 2a + (2bdx + 2b)e^{c+dx}}{2a^4d^2x^3 + 4a^3bd^2x^3e^{c+dx} + 2a^2b^2d^2x^3e^{2c+2dx}} + \frac{\int \frac{3dx}{ax^4+bx^4e^ce^{dx}} dx + \int \frac{d^2x^2}{ax^4+bx^4e^ce^{dx}} dx + \int \frac{3}{ax^4+bx^4e^ce^{dx}} dx}{a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(d*x+c))**3/x**2,x)`

[Out] `(3*a*d*x + 2*a + (2*b*d*x + 2*b)*exp(c + d*x))/(2*a**4*d**2*x**3 + 4*a**3*b*d**2*x**3*exp(c + d*x) + 2*a**2*b**2*d**2*x**3*exp(2*c + 2*d*x)) + (Integral(3*d*x/(a*x**4 + b*x**4*exp(c)*exp(d*x)), x) + Integral(d**2*x**2/(a*x**4 + b*x**4*exp(c)*exp(d*x)), x) + Integral(3/(a*x**4 + b*x**4*exp(c)*exp(d*x)), x))/(a**2*d**2)`

$$3.23 \quad \int \frac{1}{(a+be^{c-dx})^3} dx$$

**Optimal.** Leaf size=72

$$\frac{\log(a+be^{c-dx})}{a^3d} + \frac{x}{a^3} - \frac{1}{a^2d(a+be^{c-dx})} - \frac{1}{2ad(a+be^{c-dx})^2}$$

[Out]  $-1/2/a/d/(a+b*\exp(-d*x+c))^{-2}-1/a^2/d/(a+b*\exp(-d*x+c))+x/a^3+\ln(a+b*\exp(-d*x+c))/a^3/d$

**Rubi [A]** time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2282, 44}

$$-\frac{1}{a^2d(a+be^{c-dx})} + \frac{\log(a+be^{c-dx})}{a^3d} + \frac{x}{a^3} - \frac{1}{2ad(a+be^{c-dx})^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(c - d\*x))^(-3), x]

[Out]  $-1/(2*a*d*(a + b*E^(c - d*x))^2) - 1/(a^2*d*(a + b*E^(c - d*x))) + x/a^3 + \text{Log}[a + b*E^(c - d*x)]/(a^3*d)$

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + be^{c-dx})^3} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3} dx, x, e^{c-dx}\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right) dx, x, e^{c-dx}\right)}{d} \\ &= -\frac{1}{2ad(a + be^{c-dx})^2} - \frac{1}{a^2d(a + be^{c-dx})} + \frac{x}{a^3} + \frac{\log(a + be^{c-dx})}{a^3d} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 62, normalized size = 0.86

$$\frac{\frac{be^c(4ae^{dx} + 3be^c)}{(ae^{dx} + be^c)^2} + 2 \log(ae^{dx} + be^c)}{2a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(c - d\*x))^(-3), x]

[Out] ((b\*E^c\*(3\*b\*E^c + 4\*a\*E^(d\*x)))/(b\*E^c + a\*E^(d\*x))^2 + 2\*Log[b\*E^c + a\*E^(d\*x)])/(2\*a^3\*d)

**fricas [A]** time = 0.44, size = 132, normalized size = 1.83

$$\frac{2b^2dxe^{(-2dx+2c)} + 2a^2dx - 3a^2 + 2(2abdx - ab)e^{(-dx+c)} + 2(2abe^{(-dx+c)} + b^2e^{(-2dx+2c)} + a^2)\log(be^{(-dx+c)} + a)}{2(2a^4bde^{(-dx+c)} + a^3b^2de^{(-2dx+2c)} + a^5d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c))^3, x, algorithm="fricas")

[Out] 1/2\*(2\*b^2\*d\*x\*e^(-2\*d\*x + 2\*c) + 2\*a^2\*d\*x - 3\*a^2 + 2\*(2\*a\*b\*d\*x - a\*b)\*e^(-d\*x + c) + 2\*(2\*a\*b\*e^(-d\*x + c) + b^2\*e^(-2\*d\*x + 2\*c) + a^2)\*log(b\*e^(-d\*x + c) + a))/(2\*a^4\*b\*d\*e^(-d\*x + c) + a^3\*b^2\*d\*e^(-2\*d\*x + 2\*c) + a^5\*d)

**giac [A]** time = 0.21, size = 71, normalized size = 0.99

$$\frac{\frac{2(dx-c)}{a^3} + \frac{2 \log(|be^{(-dx+c)} + a|)}{a^3} - \frac{2abe^{(-dx+c)} + 3a^2}{(be^{(-dx+c)} + a)^2 a^3}}{2d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c))^3,x, algorithm="giac")

[Out]  $1/2*(2*(d*x - c)/a^3 + 2*\log(\text{abs}(b*e^{(-d*x + c)} + a)))/a^3 - (2*a*b*e^{(-d*x + c)} + 3*a^2)/((b*e^{(-d*x + c)} + a)^{2*a^3})/d$

**maple** [A] time = 0.01, size = 79, normalized size = 1.10

$$-\frac{1}{2(b e^{-dx+c} + a)^2 a d} - \frac{1}{(b e^{-dx+c} + a) a^2 d} + \frac{\ln(b e^{-dx+c} + a)}{a^3 d} - \frac{\ln(e^{-dx+c})}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(-d\*x+c)+a)^3,x)

[Out]  $-1/d/a^3*\ln(\exp(-d*x+c))+\ln(b*\exp(-d*x+c)+a)/a^3/d-1/a^2/d/(b*\exp(-d*x+c)+a)-1/2/a/d/(b*\exp(-d*x+c)+a)^2$

**maxima** [A] time = 0.90, size = 88, normalized size = 1.22

$$-\frac{2 b e^{(-dx+c)} + 3 a}{2(2 a^3 b e^{(-dx+c)} + a^2 b^2 e^{(-2 dx+2c)} + a^4) d} + \frac{dx - c}{a^3 d} + \frac{\log(b e^{(-dx+c)} + a)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x+c))^3,x, algorithm="maxima")

[Out]  $-1/2*(2*b*e^{(-d*x + c)} + 3*a)/((2*a^3*b*e^{(-d*x + c)} + a^2*b^2*e^{(-2*d*x + 2*c)} + a^4)*d) + (d*x - c)/(a^3*d) + \log(b*e^{(-d*x + c)} + a)/(a^3*d)$

**mupad** [B] time = 3.65, size = 124, normalized size = 1.72

$$\frac{\frac{x}{a} + \frac{b^2 x e^{2c-2dx}}{a^3} + \frac{2 b x e^{c-dx}}{a^2} + \frac{3 b^2 e^{2c-2dx}}{2 a^3 d} + \frac{2 b e^{c-dx}}{a^2 d}}{a^2 + 2 e^{c-dx} a b + e^{2c-2dx} b^2} + \frac{\ln(a + b e^{-dx} e^c)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*exp(c - d\*x))^3,x)

[Out]  $(x/a + (b^2*x*\exp(2*c - 2*d*x))/a^3 + (2*b*x*\exp(c - d*x))/a^2 + (3*b^2*\exp(2*c - 2*d*x))/(2*a^3*d) + (2*b*\exp(c - d*x))/(a^2*d))/(a^2 + b^2*\exp(2*c - 2*d*x) + 2*a*b*\exp(c - d*x)) + \log(a + b*\exp(-d*x)*\exp(c))/(a^3*d)$

**sympy** [A] time = 0.20, size = 78, normalized size = 1.08

$$\frac{-3a - 2be^{c-dx}}{2a^4d + 4a^3bde^{c-dx} + 2a^2b^2de^{2c-2dx}} + \frac{x}{a^3} + \frac{\log\left(\frac{a}{b} + e^{c-dx}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(-d*x+c))**3,x)
```

```
[Out] (-3*a - 2*b*exp(c - d*x))/(2*a**4*d + 4*a**3*b*d*exp(c - d*x) + 2*a**2*b**2*d*exp(2*c - 2*d*x)) + x/a**3 + log(a/b + exp(c - d*x))/(a**3*d)
```

$$3.24 \quad \int \frac{1}{(a+be^{-c-dx})^3} dx$$

**Optimal.** Leaf size=78

$$\frac{\log(a+be^{-c-dx})}{a^3d} + \frac{x}{a^3} - \frac{1}{a^2d(a+be^{-c-dx})} - \frac{1}{2ad(a+be^{-c-dx})^2}$$

[Out]  $-1/2/a/d/(a+b*\exp(-d*x-c))^{-2}-1/a^2/d/(a+b*\exp(-d*x-c))+x/a^3+\ln(a+b*\exp(-d*x-c))/a^3/d$

**Rubi [A]** time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2282, 44}

$$-\frac{1}{a^2d(a+be^{-c-dx})} + \frac{\log(a+be^{-c-dx})}{a^3d} + \frac{x}{a^3} - \frac{1}{2ad(a+be^{-c-dx})^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^(-c - d\*x))^(-3), x]

[Out]  $-1/(2*a*d*(a + b*E^(-c - d*x))^2) - 1/(a^2*d*(a + b*E^(-c - d*x))) + x/a^3 + \text{Log}[a + b*E^(-c - d*x)]/(a^3*d)$

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + be^{-c-dx})^3} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3} dx, x, e^{-c-dx}\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right) dx, x, e^{-c-dx}\right)}{d} \\ &= -\frac{1}{2ad(a + be^{-c-dx})^2} - \frac{1}{a^2d(a + be^{-c-dx})} + \frac{x}{a^3} + \frac{\log(a + be^{-c-dx})}{a^3d} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 54, normalized size = 0.69

$$\frac{\frac{b(4ae^{c+dx}+3b)}{(ae^{c+dx}+b)^2} + 2 \log(ae^{c+dx} + b)}{2a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^(-c - d\*x))^(-3), x]

[Out] ((b\*(3\*b + 4\*a\*E^(c + d\*x)))/(b + a\*E^(c + d\*x))^2 + 2\*Log[b + a\*E^(c + d\*x)])/(2\*a^3\*d)

**fricas [A]** time = 0.42, size = 140, normalized size = 1.79

$$\frac{2b^2dx e^{(-2dx-2c)} + 2a^2dx - 3a^2 + 2(2abdx - ab)e^{(-dx-c)} + 2(2abe^{(-dx-c)} + b^2e^{(-2dx-2c)} + a^2) \log(be^{(-dx-c)} + a)}{2(2a^4bde^{(-dx-c)} + a^3b^2de^{(-2dx-2c)} + a^5d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c))^3, x, algorithm="fricas")

[Out] 1/2\*(2\*b^2\*d\*x\*e^(-2\*d\*x - 2\*c) + 2\*a^2\*d\*x - 3\*a^2 + 2\*(2\*a\*b\*d\*x - a\*b)\*e^(-d\*x - c) + 2\*(2\*a\*b\*e^(-d\*x - c) + b^2\*e^(-2\*d\*x - 2\*c) + a^2)\*log(b\*e^(-d\*x - c) + a))/(2\*a^4\*b\*d\*e^(-d\*x - c) + a^3\*b^2\*d\*e^(-2\*d\*x - 2\*c) + a^5\*d)

**giac [A]** time = 0.18, size = 75, normalized size = 0.96

$$\frac{\frac{2(dx+c)}{a^3} + \frac{2 \log(|be^{(-dx-c)}+a|)}{a^3} - \frac{2abe^{(-dx-c)}+3a^2}{(be^{(-dx-c)}+a)^2 a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c))^3,x, algorithm="giac")

[Out]  $\frac{1}{2} \cdot \frac{2 \cdot (d \cdot x + c) / a^3 + 2 \cdot \log(\text{abs}(b \cdot e^{-d \cdot x - c}) + a)}{a^3} - \frac{(2 \cdot a \cdot b \cdot e^{-d \cdot x - c} + 3 \cdot a^2)}{(b \cdot e^{-d \cdot x - c} + a)^2 \cdot a^3} / d$

**maple** [A] time = 0.01, size = 87, normalized size = 1.12

$$-\frac{1}{2(b e^{-dx-c} + a)^2 a d} - \frac{1}{(b e^{-dx-c} + a) a^2 d} + \frac{\ln(b e^{-dx-c} + a)}{a^3 d} - \frac{\ln(e^{-dx-c})}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*exp(-d\*x-c)+a)^3,x)

[Out]  $-1/d/a^3 \cdot \ln(\exp(-d \cdot x - c)) + \ln(b \cdot \exp(-d \cdot x - c) + a) / a^3 / d - 1/a^2 / d / (b \cdot \exp(-d \cdot x - c) + a) - 1/2/a/d / (b \cdot \exp(-d \cdot x - c) + a)^2$

**maxima** [A] time = 0.87, size = 92, normalized size = 1.18

$$-\frac{2 b e^{-dx-c} + 3 a}{2(2 a^3 b e^{-dx-c} + a^2 b^2 e^{-2 dx-2 c} + a^4) d} + \frac{dx + c}{a^3 d} + \frac{\log(b e^{-dx-c} + a)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c))^3,x, algorithm="maxima")

[Out]  $-1/2 \cdot \frac{2 \cdot b \cdot e^{-d \cdot x - c} + 3 \cdot a}{(2 \cdot a^3 \cdot b \cdot e^{-d \cdot x - c} + a^2 \cdot b^2 \cdot e^{-2 \cdot d \cdot x - 2 \cdot c} + a^4) \cdot d} + \frac{d \cdot x + c}{a^3 \cdot d} + \frac{\log(b \cdot e^{-d \cdot x - c} + a)}{a^3 \cdot d}$

**mupad** [B] time = 3.70, size = 132, normalized size = 1.69

$$\frac{\frac{x}{a} + \frac{2 b e^{-c-d x}}{a^2 d} + \frac{b^2 x e^{-2 c-2 d x}}{a^3} + \frac{3 b^2 e^{-2 c-2 d x}}{2 a^3 d} + \frac{2 b x e^{-c-d x}}{a^2}}{a^2 + 2 e^{-c-d x} a b + e^{-2 c-2 d x} b^2} + \frac{\ln(a + b e^{-c} e^{-d x})}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*exp(- c - d\*x))^3,x)

[Out]  $(x/a + (2 \cdot b \cdot \exp(-c - d \cdot x)) / (a^2 \cdot d) + (b^2 \cdot x \cdot \exp(-2 \cdot c - 2 \cdot d \cdot x)) / a^3 + (3 \cdot b^2 \cdot \exp(-2 \cdot c - 2 \cdot d \cdot x)) / (2 \cdot a^3 \cdot d) + (2 \cdot b \cdot x \cdot \exp(-c - d \cdot x)) / a^2) / (a^2 + b^2 \cdot \exp(-2 \cdot c - 2 \cdot d \cdot x) + 2 \cdot a \cdot b \cdot \exp(-c - d \cdot x)) + \log(a + b \cdot \exp(-c) \cdot \exp(-d \cdot x)) / (a^3 \cdot d)$

sympy [A] time = 0.20, size = 85, normalized size = 1.09

$$\frac{-3a - 2be^{-c-dx}}{2a^4d + 4a^3bde^{-c-dx} + 2a^2b^2de^{-2c-2dx}} + \frac{x}{a^3} + \frac{\log\left(\frac{a}{b} + e^{-c-dx}\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*exp(-d\*x-c))\*\*3,x)

[Out] (-3\*a - 2\*b\*exp(-c - d\*x))/(2\*a\*\*4\*d + 4\*a\*\*3\*b\*d\*exp(-c - d\*x) + 2\*a\*\*2\*b\*  
\*2\*d\*exp(-2\*c - 2\*d\*x)) + x/a\*\*3 + log(a/b + exp(-c - d\*x))/(a\*\*3\*d)

$$3.25 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx)^3 dx$$

**Optimal.** Leaf size=153

$$\frac{a(c+dx)^4}{4d} + \frac{6bd^2(c+dx)(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{3bd(c+dx)^2(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{b(c+dx)^3(F^{eg+fgx})^n}{fgn\log(F)} - \frac{6bd^3(F^{eg+fgx})^n}{f^4g^4n^4\log^4(F)}$$

[Out]  $1/4*a*(d*x+c)^4/d-6*b*d^3*(F^{(f*g*x+e*g)})^n/f^4/g^4/n^4/\ln(F)^4+6*b*d^2*(F^{(f*g*x+e*g)})^n*(d*x+c)/f^3/g^3/n^3/\ln(F)^3-3*b*d*(F^{(f*g*x+e*g)})^n*(d*x+c)^2/f^2/g^2/n^2/\ln(F)^2+b*(F^{(f*g*x+e*g)})^n*(d*x+c)^3/f/g/n/\ln(F)$

**Rubi [A]** time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {2183, 2176, 2194}

$$\frac{a(c+dx)^4}{4d} + \frac{6bd^2(c+dx)(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{3bd(c+dx)^2(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{b(c+dx)^3(F^{eg+fgx})^n}{fgn\log(F)} - \frac{6bd^3(F^{eg+fgx})^n}{f^4g^4n^4\log^4(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]\*(c + d\*x)^3,x]

[Out]  $(a*(c+d*x)^4)/(4*d) - (6*b*d^3*(F^{(e*g+f*g*x)})^n)/(f^4*g^4*n^4*\text{Log}[F]^4) + (6*b*d^2*(F^{(e*g+f*g*x)})^n*(c+d*x))/(f^3*g^3*n^3*\text{Log}[F]^3) - (3*b*d*(F^{(e*g+f*g*x)})^n*(c+d*x)^2)/(f^2*g^2*n^2*\text{Log}[F]^2) + (b*(F^{(e*g+f*g*x)})^n*(c+d*x)^3)/(f*g*n*\text{Log}[F])$

**Rule 2176**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

**Rule 2183**

Int[((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

**Rule 2194**

Int[((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

### Rubi steps

$$\begin{aligned}
 \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx)^3 dx &= \int \left( a(c + dx)^3 + b \left( F^{eg+fgx} \right)^n (c + dx)^3 \right) dx \\
 &= \frac{a(c + dx)^4}{4d} + b \int \left( F^{eg+fgx} \right)^n (c + dx)^3 dx \\
 &= \frac{a(c + dx)^4}{4d} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)^3}{fgn \log(F)} - \frac{(3bd) \int \left( F^{eg+fgx} \right)^n (c + dx)^2 dx}{fgn \log(F)} \\
 &= \frac{a(c + dx)^4}{4d} - \frac{3bd \left( F^{eg+fgx} \right)^n (c + dx)^2}{f^2 g^2 n^2 \log^2(F)} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)^3}{fgn \log(F)} + \frac{(6bd^2) \int \left( F^{eg+fgx} \right)^n (c + dx) dx}{f^2 g^2 n^2 \log^2(F)} \\
 &= \frac{a(c + dx)^4}{4d} + \frac{6bd^2 \left( F^{eg+fgx} \right)^n (c + dx)}{f^3 g^3 n^3 \log^3(F)} - \frac{3bd \left( F^{eg+fgx} \right)^n (c + dx)^2}{f^2 g^2 n^2 \log^2(F)} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)^3}{fgn \log(F)} \\
 &= \frac{a(c + dx)^4}{4d} - \frac{6bd^3 \left( F^{eg+fgx} \right)^n}{f^4 g^4 n^4 \log^4(F)} + \frac{6bd^2 \left( F^{eg+fgx} \right)^n (c + dx)}{f^3 g^3 n^3 \log^3(F)} - \frac{3bd \left( F^{eg+fgx} \right)^n (c + dx)^2}{f^2 g^2 n^2 \log^2(F)} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)^3}{fgn \log(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.28, size = 130, normalized size = 0.85

$$ac^3x + \frac{3}{2}ac^2dx^2 + acd^2x^3 + \frac{1}{4}ad^3x^4 + \frac{b \left( F^{g(e+fx)} \right)^n \left( 6d^2 fgn \log(F)(c + dx) + f^3 g^3 n^3 \log^3(F)(c + dx)^3 - 3df^2 g^2 n^2 \log^2(F)(c + dx)^2 \right)}{f^4 g^4 n^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n\*(c + d\*x)^3,x]

[Out] a\*c^3\*x + (3\*a\*c^2\*d\*x^2)/2 + a\*c\*d^2\*x^3 + (a\*d^3\*x^4)/4 + (b\*(F^(g\*(e + f\*x))))^n\*(-6\*d^3 + 6\*d^2\*f\*g\*n\*(c + d\*x)\*Log[F] - 3\*d\*f^2\*g^2\*n^2\*(c + d\*x)^2\*Log[F]^2 + f^3\*g^3\*n^3\*(c + d\*x)^3\*Log[F]^3)/(f^4\*g^4\*n^4\*Log[F]^4)

**fricas [A]** time = 0.45, size = 267, normalized size = 1.75

$$\frac{(ad^3 f^4 g^4 n^4 x^4 + 4acd^2 f^4 g^4 n^4 x^3 + 6ac^2 d f^4 g^4 n^4 x^2 + 4ac^3 f^4 g^4 n^4 x) \log(F)^4 - 4 \left( 6bd^3 - (bd^3 f^3 g^3 n^3 x^3 + 3bcd^2 f^2 g^2 n^2 x^2 + 3cdf g^2 n^2 x + 3cd^2 f g n^2) \log(F) \right) (c + dx)^3}{f^4 g^4 n^4 \log^4(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)\*(d\*x+c)^3,x, algorithm="fricas")



```
[Out] 1/4*((a*d^3*f^4*g^4*n^4*x^4 + 4*a*c*d^2*f^4*g^4*n^4*x^3 + 6*a*c^2*d*f^4*g^4
*n^4*x^2 + 4*a*c^3*f^4*g^4*n^4*x)*log(F)^4 - 4*(6*b*d^3 - (b*d^3*f^3*g^3*n^
3*x^3 + 3*b*c*d^2*f^3*g^3*n^3*x^2 + 3*b*c^2*d*f^3*g^3*n^3*x + b*c^3*f^3*g^3
*n^3)*log(F)^3 + 3*(b*d^3*f^2*g^2*n^2*x^2 + 2*b*c*d^2*f^2*g^2*n^2*x + b*c^2
*d*f^2*g^2*n^2)*log(F)^2 - 6*(b*d^3*f*g*n*x + b*c*d^2*f*g*n)*log(F))*F^(f*g
*n*x + e*g*n))/(f^4*g^4*n^4*log(F)^4)
```

**giac** [C] time = 0.58, size = 5732, normalized size = 37.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(F^(g*(f*x+e))))^n)*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (((3*pi^2*b*d^3*f
^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F)
) + 2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 9*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*
log(abs(F))*sgn(F) - 9*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 6*b*c*d^2
*f^3*g^3*n^3*x^2*log(abs(F))^3 + 9*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*s
gn(F) - 9*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 6*b*c^2*d*f^3*g^3*n^3*x*
log(abs(F))^3 + 3*pi^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*b*c^3*
f^3*g^3*n^3*log(abs(F)) + 2*b*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*b*d^3*
f^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*b*d^3*f^2*g^2*n^2*x^2 - 6*b*d^3*f^2*g^2*n^2
*x^2*log(abs(F))^2 - 6*pi^2*b*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*b*c*d^2*f
^2*g^2*n^2*x - 12*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b*c^2*d*f^2*
g^2*n^2*sgn(F) + 3*pi^2*b*c^2*d*f^2*g^2*n^2 - 6*b*c^2*d*f^2*g^2*n^2*log(abs
(F))^2 + 12*b*d^3*f*g*n*x*log(abs(F)) + 12*b*c*d^2*f*g*n*log(abs(F)) - 12*b
*d^3)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) -
pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs
(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F)
- pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(
abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log
(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F)
))^3)^2) - 4*(pi^3*b*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*pi*b*d^3*f^3*g^3*n^3*x^
3*log(abs(F))^2*sgn(F) - pi^3*b*d^3*f^3*g^3*n^3*x^3 + 3*pi*b*d^3*f^3*g^3*n^
3*x^3*log(abs(F))^2 + 3*pi^3*b*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 9*pi*b*c*d^2*
f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*b*c*d^2*f^3*g^3*n^3*x^2 + 9*p
i*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 3*pi^3*b*c^2*d*f^3*g^3*n^3*x*sgn(
F) - 9*pi*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*b*c^2*d*f^3*g
^3*n^3*x + 9*pi*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + pi^3*b*c^3*f^3*g^3*n^
3*sgn(F) - 3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*b*c^3*f^3*g^3
*n^3 + 3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi*b*d^3*f^2*g^2*n^2*x^2*lo
g(abs(F))*sgn(F) - 6*pi*b*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 12*pi*b*c*d^2*f
^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*b*c*d^2*f^2*g^2*n^2*x*log(abs(F)) +
6*pi*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*pi*b*c^2*d*f^2*g^2*n^2*log
```

$$\begin{aligned}
& (\text{abs}(F)) - 6\pi * b * d^3 * f * g * n * x * \text{sgn}(F) + 6\pi * b * d^3 * f * g * n * x - 6\pi * b * c * d^2 * f * \\
& g * n * \text{sgn}(F) + 6\pi * b * c * d^2 * f * g * n * (\pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * \\
& f^4 * g^4 * n^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) + \pi * f^4 * g^4 * \\
& 4 * n^4 * \log(\text{abs}(F))^3) / ((\pi^4 * f^4 * g^4 * n^4 * \text{sgn}(F) - 6\pi^2 * f^4 * g^4 * n^4 * \log(\text{abs}( \\
& F))^2 * \text{sgn}(F) - \pi^4 * f^4 * g^4 * n^4 + 6\pi^2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^2 - 2 * f^4 * \\
& * g^4 * n^4 * \log(\text{abs}(F))^4)^2 + 16 * (\pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * f^4 * \\
& 4 * g^4 * n^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) + \pi * f^4 * g^4 * \\
& n^4 * \log(\text{abs}(F))^3)^2) * \cos(-1/2 * \pi * f * g * n * x * \text{sgn}(F) + 1/2 * \pi * f * g * n * x - 1/2 * \pi \\
& * g * n * e * \text{sgn}(F) + 1/2 * \pi * g * n * e) - ((\pi^3 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \text{sgn}(F) - 3 * \pi * \\
& b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b * d^3 * f^3 * g^3 * n^3 * x^3 + 3 \\
& * \pi * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^2 + 3 * \pi^3 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \text{sgn}( \\
& n(F) - 9 * \pi * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 3 * \pi^3 * b * c * d^2 * f \\
& ^3 * g^3 * n^3 * x^2 + 9 * \pi * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^2 + 3 * \pi^3 * b * c^2 * \\
& d * f^3 * g^3 * n^3 * x * \text{sgn}(F) - 9 * \pi * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F))^2 * \text{sgn}(F) - \\
& 3 * \pi^3 * b * c^2 * d * f^3 * g^3 * n^3 * x + 9 * \pi * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F))^2 + \pi \\
& i^3 * b * c^3 * f^3 * g^3 * n^3 * \text{sgn}(F) - 3 * \pi * b * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) \\
& - \pi^3 * b * c^3 * f^3 * g^3 * n^3 + 3 * \pi * b * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F))^2 + 6 * \pi * b * d^3 \\
& 3 * f^2 * g^2 * n^2 * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) - 6 * \pi * b * d^3 * f^2 * g^2 * n^2 * x^2 * \log(\text{abs}(F) \\
& )) + 12 * \pi * b * c * d^2 * f^2 * g^2 * n^2 * x * \log(\text{abs}(F)) * \text{sgn}(F) - 12 * \pi * b * c * d^2 * f^2 * g^2 \\
& * n^2 * x * \log(\text{abs}(F)) + 6 * \pi * b * c^2 * d * f^2 * g^2 * n^2 * \log(\text{abs}(F)) * \text{sgn}(F) - 6 * \pi * b * c \\
& ^2 * d * f^2 * g^2 * n^2 * \log(\text{abs}(F)) - 6 * \pi * b * d^3 * f * g * n * x * \text{sgn}(F) + 6 * \pi * b * d^3 * f * g * n \\
& * x - 6 * \pi * b * c * d^2 * f * g * n * \text{sgn}(F) + 6 * \pi * b * c * d^2 * f * g * n * (\pi^4 * f^4 * g^4 * n^4 * \text{sgn}( \\
& F) - 6 * \pi^2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^4 * f^4 * g^4 * n^4 + 6 * \pi^2 * f^4 * \\
& 4 * g^4 * n^4 * \log(\text{abs}(F))^2 - 2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^4) / ((\pi^4 * f^4 * g^4 * n^4 * \text{sgn}( \\
& F) - 6 * \pi^2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^4 * f^4 * g^4 * n^4 + 6 * \pi^2 \\
& * f^4 * g^4 * n^4 * \log(\text{abs}(F))^2 - 2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^4)^2 + 16 * (\pi^3 * f^4 * \\
& g^4 * n^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * f^4 * g^4 * n^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - \pi^3 * f^4 * \\
& 4 * g^4 * n^4 * \log(\text{abs}(F)) + \pi * f^4 * g^4 * n^4 * \log(\text{abs}(F))^3)^2) + 4 * (3 * \pi^2 * b * d^3 * f^3 \\
& ^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F) \\
& ) + 2 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^3 + 9 * \pi^2 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \\
& \log(\text{abs}(F)) * \text{sgn}(F) - 9 * \pi^2 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F)) + 6 * b * c * d^2 \\
& * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^3 + 9 * \pi^2 * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F)) * \text{sgn}( \\
& F) - 9 * \pi^2 * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F)) + 6 * b * c^2 * d * f^3 * g^3 * n^3 * x * \\
& \log(\text{abs}(F))^3 + 3 * \pi^2 * b * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b * c^3 * \\
& f^3 * g^3 * n^3 * \log(\text{abs}(F)) + 2 * b * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F))^3 - 3 * \pi^2 * b * d^3 * \\
& f^2 * g^2 * n^2 * x^2 * \text{sgn}(F) + 3 * \pi^2 * b * d^3 * f^2 * g^2 * n^2 * x^2 - 6 * b * d^3 * f^2 * g^2 * n^2 \\
& * x^2 * \log(\text{abs}(F))^2 - 6 * \pi^2 * b * c * d^2 * f^2 * g^2 * n^2 * x * \text{sgn}(F) + 6 * \pi^2 * b * c * d^2 * f \\
& ^2 * g^2 * n^2 * x - 12 * b * c * d^2 * f^2 * g^2 * n^2 * x * \log(\text{abs}(F))^2 - 3 * \pi^2 * b * c^2 * d * f^2 * \\
& g^2 * n^2 * \text{sgn}(F) + 3 * \pi^2 * b * c^2 * d * f^2 * g^2 * n^2 - 6 * b * c^2 * d * f^2 * g^2 * n^2 * \log(\text{abs}( \\
& F))^2 + 12 * b * d^3 * f * g * n * x * \log(\text{abs}(F)) + 12 * b * c * d^2 * f * g * n * \log(\text{abs}(F)) - 12 * b \\
& * d^3 * (\pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * f^4 * g^4 * n^4 * \log(\text{abs}(F))^3 * \text{sgn}( \\
& F) - \pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) + \pi * f^4 * g^4 * n^4 * \log(\text{abs}(F))^3) / ((\pi^4 \\
& * f^4 * g^4 * n^4 * \text{sgn}(F) - 6 * \pi^2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^4 * f^4 * g^4 * \\
& 4 * n^4 + 6 * \pi^2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^2 - 2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^4)^2 + \\
& 16 * (\pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * f^4 * g^4 * n^4 * \log(\text{abs}(F))^3 * \text{sgn}
\end{aligned}$$

$$\begin{aligned}
& (F) - \pi^3 f^4 g^4 n^4 \log(\text{abs}(F)) + \pi^3 f^4 g^4 n^4 \log(\text{abs}(F))^3)^2) * \sin( \\
& -1/2 * \pi * f * g * n * x * \text{sgn}(F) + 1/2 * \pi * f * g * n * x - 1/2 * \pi * g * n * e * \text{sgn}(F) + 1/2 * \pi * g * n * \\
& e) * e^{(f * g * n * x * \log(\text{abs}(F)) + g * n * e * \log(\text{abs}(F)))} - 1/2 * I * ((8 * \pi^3 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \text{sgn}(F) + 24 * I * \pi^2 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 2 \\
& 4 * \pi * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 8 * \pi^3 * b * d^3 * f^3 * g^3 * n^3 * x^3 \\
& - 24 * I * \pi^2 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F)) + 24 * \pi * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^2 + 16 * I * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^3 + 24 * \pi^3 * b * c \\
& * d^2 * f^3 * g^3 * n^3 * x^2 * \text{sgn}(F) + 72 * I * \pi^2 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F)) \\
& * \text{sgn}(F) - 72 * \pi * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 24 * \pi^3 * b * c * \\
& d^2 * f^3 * g^3 * n^3 * x^2 - 72 * I * \pi^2 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F)) + 72 * \pi \\
& * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^2 + 48 * I * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^3 + 24 * \pi^3 * b * c^2 * d * f^3 * g^3 * n^3 * x * \text{sgn}(F) + 72 * I * \pi^2 * b * c^2 * d * f^3 * g^3 \\
& * n^3 * x * \log(\text{abs}(F)) * \text{sgn}(F) - 72 * \pi * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F))^2 * \text{sgn}(F) \\
& ) - 24 * \pi^3 * b * c^2 * d * f^3 * g^3 * n^3 * x - 72 * I * \pi^2 * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F)) + 72 * \pi * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F))^2 + 48 * I * b * c^2 * d * f^3 * g^3 * n^3 \\
& * x * \log(\text{abs}(F))^3 + 8 * \pi^3 * b * c^3 * f^3 * g^3 * n^3 * \text{sgn}(F) + 24 * I * \pi^2 * b * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 24 * \pi * b * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 8 \\
& * \pi^3 * b * c^3 * f^3 * g^3 * n^3 - 24 * I * \pi^2 * b * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F)) + 24 * \pi * b \\
& * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F))^2 + 16 * I * b * c^3 * f^3 * g^3 * n^3 * \log(\text{abs}(F))^3 - 24 * \\
& I * \pi^2 * b * d^3 * f^2 * g^2 * n^2 * x^2 * \text{sgn}(F) + 48 * \pi * b * d^3 * f^2 * g^2 * n^2 * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) + 24 * I * \pi^2 * b * d^3 * f^2 * g^2 * n^2 * x^2 - 48 * \pi * b * d^3 * f^2 * g^2 * n^2 * x^2 * \log(\text{abs}(F)) \\
& - 48 * I * b * d^3 * f^2 * g^2 * n^2 * x^2 * \log(\text{abs}(F))^2 - 48 * I * \pi^2 * b * c * d^2 * f^2 * g^2 * n^2 * x * \text{sgn}(F) + 96 * \pi * b * c * d^2 * f^2 * g^2 * n^2 * x * \log(\text{abs}(F)) * \text{sgn}(F) + 48 * I \\
& * \pi^2 * b * c * d^2 * f^2 * g^2 * n^2 * x - 96 * \pi * b * c * d^2 * f^2 * g^2 * n^2 * x * \log(\text{abs}(F)) - 96 * \\
& I * b * c * d^2 * f^2 * g^2 * n^2 * x * \log(\text{abs}(F))^2 - 24 * I * \pi^2 * b * c^2 * d * f^2 * g^2 * n^2 * \text{sgn}(F) \\
& ) + 48 * \pi * b * c^2 * d * f^2 * g^2 * n^2 * \log(\text{abs}(F)) * \text{sgn}(F) + 24 * I * \pi^2 * b * c^2 * d * f^2 * g^2 * n^2 \\
& - 48 * \pi * b * c^2 * d * f^2 * g^2 * n^2 * \log(\text{abs}(F)) - 48 * I * b * c^2 * d * f^2 * g^2 * n^2 * \log(\text{abs}(F))^2 - 48 * \pi * b * d^3 * f * g * n * x * \text{sgn}(F) + 48 * \pi * b * d^3 * f * g * n * x + 96 * I * b * d^3 \\
& * f * g * n * x * \log(\text{abs}(F)) - 48 * \pi * b * c * d^2 * f * g * n * \text{sgn}(F) + 48 * \pi * b * c * d^2 * f * g * n + 9 \\
& 6 * I * b * c * d^2 * f * g * n * \log(\text{abs}(F)) - 96 * I * b * d^3) * e^{(1/2 * I * \pi * f * g * n * x * \text{sgn}(F) - 1/ \\
& 2 * I * \pi * f * g * n * x + 1/2 * I * \pi * g * n * e * \text{sgn}(F) - 1/2 * I * \pi * g * n * e) / (8 * \pi^4 * f^4 * g^4 * n^4 * \\
& 4 * \text{sgn}(F) + 32 * I * \pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) * \text{sgn}(F) - 48 * \pi^2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 32 * I * \pi * f^4 * g^4 * n^4 * \log(\text{abs}(F))^3 * \text{sgn}(F) - 8 * \pi^4 * f^4 * g^4 * n^4 \\
& * g^4 * n^4 - 32 * I * \pi^3 * f^4 * g^4 * n^4 * \log(\text{abs}(F)) + 48 * \pi^2 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^2 + 32 * I * \pi * f^4 * g^4 * n^4 * \log(\text{abs}(F))^3 - 16 * f^4 * g^4 * n^4 * \log(\text{abs}(F))^4) + \\
& (8 * \pi^3 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \text{sgn}(F) - 24 * I * \pi^2 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 24 * \pi * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 8 * \pi^3 * b * d^3 * f^3 * g^3 * n^3 * x^3 \\
& + 24 * I * \pi^2 * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F)) + 24 * \pi \\
& * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^2 - 16 * I * b * d^3 * f^3 * g^3 * n^3 * x^3 * \log(\text{abs}(F))^3 + 24 * \pi^3 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \text{sgn}(F) - 72 * I * \pi^2 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) - 72 * \pi * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 24 * \pi^3 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 + 72 * I * \pi^2 * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F)) + 72 * \pi * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^2 - 48 * I * b * c * d^2 * f^3 * g^3 * n^3 * x^2 * \log(\text{abs}(F))^3 + 24 * \pi^3 * b * c^2 * d * f^3 * g^3 * n^3 * x * \text{sgn}(F) - 72 * I * \pi^2 * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F)) * \text{sgn}(F) - 72 * \pi * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F))^2 * \text{sgn}(F) - 24 * \pi^3 * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F))^3 + 24 * \pi^3 * b * c^2 * d * f^3 * g^3 * n^3 * x * \log(\text{abs}(F))^4)
\end{aligned}$$

$\log(\text{abs}(F))^2 \text{sgn}(F) - 24\pi^3 b^2 c^2 d^3 f^3 g^3 n^3 x + 72I\pi^2 b^2 c^2 d^3 f^3 g^3 n^3 x \log(\text{abs}(F)) + 72\pi^3 b^2 c^2 d^3 f^3 g^3 n^3 x \log(\text{abs}(F))^2 - 48I b^2 c^2 d^3 f^3 g^3 n^3 x \log(\text{abs}(F))^3 + 8\pi^3 b^2 c^3 f^3 g^3 n^3 \text{sgn}(F) - 24I\pi^2 b^2 c^3 f^3 g^3 n^3 \log(\text{abs}(F)) \text{sgn}(F) - 24\pi^3 b^2 c^3 f^3 g^3 n^3 \log(\text{abs}(F))^2 \text{sgn}(F) - 8\pi^3 b^2 c^3 f^3 g^3 n^3 + 24I\pi^2 b^2 c^3 f^3 g^3 n^3 \log(\text{abs}(F)) + 24\pi^3 b^2 c^3 f^3 g^3 n^3 \log(\text{abs}(F))^2 - 16I b^2 c^3 f^3 g^3 n^3 \log(\text{abs}(F))^3 + 24I\pi^2 b^2 d^3 f^2 g^2 n^2 x^2 \text{sgn}(F) + 48\pi^3 b^2 d^3 f^2 g^2 n^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 24I\pi^2 b^2 d^3 f^2 g^2 n^2 x^2 - 48\pi^3 b^2 d^3 f^2 g^2 n^2 x^2 \log(\text{abs}(F)) + 48I b^2 d^3 f^2 g^2 n^2 x^2 \log(\text{abs}(F))^2 + 48I\pi^2 b^2 c^2 d^2 f^2 g^2 n^2 x \text{sgn}(F) + 96\pi^3 b^2 c^2 d^2 f^2 g^2 n^2 x \log(\text{abs}(F)) \text{sgn}(F) - 48I\pi^2 b^2 c^2 d^2 f^2 g^2 n^2 x - 96\pi^3 b^2 c^2 d^2 f^2 g^2 n^2 x \log(\text{abs}(F)) + 96I b^2 c^2 d^2 f^2 g^2 n^2 x \log(\text{abs}(F))^2 + 24I\pi^2 b^2 c^2 d^2 f^2 g^2 n^2 \text{sgn}(F) + 48\pi^3 b^2 c^2 d^2 f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - 24I\pi^2 b^2 c^2 d^2 f^2 g^2 n^2 - 48\pi^3 b^2 c^2 d^2 f^2 g^2 n^2 \log(\text{abs}(F)) + 48I b^2 c^2 d^2 f^2 g^2 n^2 \log(\text{abs}(F))^2 - 48\pi^3 b^2 d^3 f^2 g^2 n^2 \text{sgn}(F) + 48\pi^3 b^2 d^3 f^2 g^2 n^2 \log(\text{abs}(F)) - 96I b^2 d^3 f^2 g^2 n^2 \log(\text{abs}(F)) - 48\pi^3 b^2 c^2 d^2 f^2 g^2 n^2 \text{sgn}(F) + 48\pi^3 b^2 c^2 d^2 f^2 g^2 n^2 \log(\text{abs}(F)) - 96I b^2 c^2 d^2 f^2 g^2 n^2 \log(\text{abs}(F)) + 96I b^2 d^3 e^{-1/2 I \pi f^2 g^2 n^2 \text{sgn}(F)} + 1/2 I \pi f^2 g^2 n^2 - 1/2 I \pi f^2 g^2 n^2 \text{sgn}(F) + 1/2 I \pi f^2 g^2 n^2 e^{-1/2 I \pi f^2 g^2 n^2 \text{sgn}(F)} - 32I\pi^3 f^4 g^4 n^4 \log(\text{abs}(F)) \text{sgn}(F) - 48\pi^3 f^4 g^4 n^4 \log(\text{abs}(F))^2 \text{sgn}(F) + 32I\pi^3 f^4 g^4 n^4 \log(\text{abs}(F))^3 \text{sgn}(F) - 8\pi^3 f^4 g^4 n^4 + 32I\pi^3 f^4 g^4 n^4 \log(\text{abs}(F)) + 48\pi^3 f^4 g^4 n^4 \log(\text{abs}(F))^2 - 32I\pi^3 f^4 g^4 n^4 \log(\text{abs}(F))^3 - 16f^4 g^4 n^4 \log(\text{abs}(F))^4) e^{(f^2 g^2 n^2 \log(\text{abs}(F)) + g^2 n^2 \log(\text{abs}(F)))}$

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(f^2 x + e)g} \right)^n + a \right) (dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*(F^(g\*(f\*x+e)))^n)\*(d\*x+c)^3,x)

[Out] int((a+b\*(F^(g\*(f\*x+e)))^n)\*(d\*x+c)^3,x)

**maxima [A]** time = 1.35, size = 294, normalized size = 1.92

$$\frac{1}{4} ad^3 x^4 + acd^2 x^3 + \frac{3}{2} ac^2 dx^2 + ac^3 x + \frac{(F^{fgx+eg})^n bc^3}{fgn \log(F)} + \frac{3((F^{eg})^n fgnx \log(F) - (F^{eg})^n)(F^{fgx})^n bc^2 d}{f^2 g^2 n^2 \log(F)^2} + \frac{3((F^{eg})^n f^2 g^2 n^2 x^2}{f^2 g^2 n^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)\*(d\*x+c)^3,x, algorithm="maxima")

[Out] 1/4\*a\*d^3\*x^4 + a\*c\*d^2\*x^3 + 3/2\*a\*c^2\*d\*x^2 + a\*c^3\*x + (F^(f\*g\*x + e\*g))^n\*b\*c^3/(f\*g\*n\*log(F)) + 3\*((F^(e\*g))^n\*f\*g\*n\*x\*log(F) - (F^(e\*g))^n)\*(F^(

$$\begin{aligned} & f^2 g^2 n^2 \log(F)^2 + 3((F^{(e*g)})^n f^2 g^2 n^2 x^2 \log(F)^2 - 2(F^{(e*g)})^n f^2 g^2 n^2 x \log(F) + 2(F^{(e*g)})^n (F^{(f*g*x)})^n b^2 c^2 d \\ & \quad / (f^3 g^3 n^3 \log(F)^3) + ((F^{(e*g)})^n f^3 g^3 n^3 x^3 \log(F)^3 - 3(F^{(e*g)})^n f^2 g^2 n^2 x^2 \log(F)^2 + 6(F^{(e*g)})^n f^2 g^2 n^2 x \log(F) - 6(F^{(e*g)})^n \\ & \quad (F^{(f*g*x)})^n b^2 d^3 / (f^4 g^4 n^4 \log(F)^4) \end{aligned}$$

**mupad [B]** time = 3.81, size = 225, normalized size = 1.47

$$\frac{a d^3 x^4}{4} - (F^{f g x} F^{e g})^n \left( \frac{b (-c^3 f^3 g^3 n^3 \ln(F)^3 + 3 c^2 d f^2 g^2 n^2 \ln(F)^2 - 6 c d^2 f g n \ln(F) + 6 d^3)}{f^4 g^4 n^4 \ln(F)^4} - \frac{b d^3 x^3}{f g n \ln(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*(F^(g*(e + f*x))))^n)*(c + d*x)^3,x)`

[Out]  $(a d^3 x^4)/4 - (F^{(f*g*x)} F^{(e*g)})^n ((b (6 d^3 - c^3 f^3 g^3 n^3 \log(F)^3 - 6 c d^2 f^2 g^2 n^2 \log(F)^2) / (f^4 g^4 n^4 \log(F)^4) - (b d^3 x^3) / (f^3 g^3 n^3 \log(F)^3) - (3 b d^2 x (2 d^2 + c^2 f^2 g^2 n^2 \log(F)^2 - 2 c d f^2 g^2 n^2 \log(F))) / (f^3 g^3 n^3 \log(F)^3) + (3 b d^2 x^2 (d - c f^2 g^2 n^2 \log(F))) / (f^2 g^2 n^2 \log(F)^2) + a c^3 x + (3 a c^2 d x^2) / 2 + a c d^2 x^3)$

**sympy [A]** time = 0.28, size = 332, normalized size = 2.17

$$a c^3 x + \frac{3 a c^2 d x^2}{2} + a c d^2 x^3 + \frac{a d^3 x^4}{4} + \left\{ \frac{(b c^3 f^3 g^3 n^3 \log(F)^3 + 3 b c^2 d f^3 g^3 n^3 x \log(F)^3 - 3 b c^2 d f^2 g^2 n^2 \log(F)^2 + 3 b c d^2 f^3 g^3 n^3 x^2 \log(F)^3 - 6 b c d^2 f^2 g^2 n^2 \log(F)^2)}{f^4 g^4 n^4 \log(F)^4}, \frac{b c^3 x + \frac{3 b c^2 d x^2}{2} + b c d^2 x^3 + \frac{b d^3 x^4}{4}}{f^3 g^3 n^3 \log(F)^3} \right\}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F**(g*(f*x+e))))**n)*(d*x+c)**3,x)`

[Out]  $a c^3 x + 3 a c^2 d x^2 / 2 + a c d^2 x^3 + a d^3 x^4 / 4 + \text{Piecewise}(((b c^3 f^3 g^3 n^3 \log(F)^3 + 3 b c^2 d f^3 g^3 n^3 x \log(F)^3 - 3 b c^2 d f^2 g^2 n^2 \log(F)^2 + 3 b c d^2 f^3 g^3 n^3 x^2 \log(F)^3 - 6 b c d^2 f^2 g^2 n^2 \log(F)^2 + 6 b c d^2 f^2 g^2 n^2 x \log(F)^2 + 6 b c d^2 f^2 g^2 n^2 x \log(F) + b d^3 f^3 g^3 n^3 x^3 \log(F)^3 - 3 b d^3 f^2 g^2 n^2 x^2 \log(F)^2 + 6 b d^3 f^2 g^2 n^2 x \log(F) - 6 b d^3) * (F^{(g*(e + f*x))})^n / (f^4 g^4 n^4 \log(F)^4), \text{Ne}(f^4 g^4 n^4 \log(F)^4, 0)), (b c^3 x + 3 b c^2 d x^2 / 2 + b c d^2 x^3 + b d^3 x^4 / 4, \text{True}))$

$$3.26 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx)^2 dx$$

**Optimal.** Leaf size=115

$$\frac{a(c+dx)^3}{3d} - \frac{2bd(c+dx)(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{b(c+dx)^2(F^{eg+fgx})^n}{fgn\log(F)} + \frac{2bd^2(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)}$$

[Out]  $1/3*a*(d*x+c)^3/d+2*b*d^2*(F^(f*g*x+e*g))^n/f^3/g^3/n^3/\ln(F)^3-2*b*d*(F^(f*g*x+e*g))^n*(d*x+c)/f^2/g^2/n^2/\ln(F)^2+b*(F^(f*g*x+e*g))^n*(d*x+c)^2/f/g/n/\ln(F)$

**Rubi [A]** time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {2183, 2176, 2194}

$$\frac{a(c+dx)^3}{3d} - \frac{2bd(c+dx)(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{b(c+dx)^2(F^{eg+fgx})^n}{fgn\log(F)} + \frac{2bd^2(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]\*(c + d\*x)^2,x]

[Out]  $(a*(c+d*x)^3)/(3*d) + (2*b*d^2*(F^(e*g+f*g*x))^n)/(f^3*g^3*n^3*\text{Log}[F]^3) - (2*b*d*(F^(e*g+f*g*x))^n*(c+d*x))/(f^2*g^2*n^2*\text{Log}[F]^2) + (b*(F^(e*g+f*g*x))^n*(c+d*x)^2)/(f*g*n*\text{Log}[F])$

**Rule 2176**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.)+(f\_.)\*(x\_))))^(n\_.)\*((c\_.)+(d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c+d\*x)^m\*(b\*F^(g\*(e+f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c+d\*x)^(m-1)\*(b\*F^(g\*(e+f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

**Rule 2183**

Int[((a\_.)+(b\_.)\*(F\_)^((g\_.)\*((e\_.)+(f\_.)\*(x\_))))^(n\_.))^p, x\_Symbol] :> Int[ExpandIntegrand[(c+d\*x)^m, (a+b\*(F^(g\*(e+f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

**Rule 2194**

`Int[((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]`

### Rubi steps

$$\begin{aligned}
 \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx)^2 dx &= \int \left( a(c + dx)^2 + b \left( F^{eg+fgx} \right)^n (c + dx)^2 \right) dx \\
 &= \frac{a(c + dx)^3}{3d} + b \int \left( F^{eg+fgx} \right)^n (c + dx)^2 dx \\
 &= \frac{a(c + dx)^3}{3d} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)^2}{fgn \log(F)} - \frac{(2bd) \int \left( F^{eg+fgx} \right)^n (c + dx) dx}{fgn \log(F)} \\
 &= \frac{a(c + dx)^3}{3d} - \frac{2bd \left( F^{eg+fgx} \right)^n (c + dx)}{f^2 g^2 n^2 \log^2(F)} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)^2}{fgn \log(F)} + \frac{(2bd^2) \int \left( F^{eg+fgx} \right)^n (c + dx) dx}{f^2 g^2 n^2 \log^2(F)} \\
 &= \frac{a(c + dx)^3}{3d} + \frac{2bd^2 \left( F^{eg+fgx} \right)^n}{f^3 g^3 n^3 \log^3(F)} - \frac{2bd \left( F^{eg+fgx} \right)^n (c + dx)}{f^2 g^2 n^2 \log^2(F)} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)^2}{fgn \log(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 91, normalized size = 0.79

$$ac^2x + acdx^2 + \frac{1}{3}ad^2x^3 + \frac{b \left( F^{g(e+fx)} \right)^n \left( f^2 g^2 n^2 \log^2(F)(c + dx)^2 - 2dfgn \log(F)(c + dx) + 2d^2 \right)}{f^3 g^3 n^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n\*(c + d\*x)^2,x]

[Out] a\*c^2\*x + a\*c\*d\*x^2 + (a\*d^2\*x^3)/3 + (b\*(F^(g\*(e + f\*x))))^n\*(2\*d^2 - 2\*d\*f\*g\*n\*(c + d\*x)\*Log[F] + f^2\*g^2\*n^2\*(c + d\*x)^2\*Log[F]^2)/(f^3\*g^3\*n^3\*Log[F]^3)

**fricas [A]** time = 0.46, size = 166, normalized size = 1.44

$$\frac{(ad^2f^3g^3n^3x^3 + 3acdf^3g^3n^3x^2 + 3ac^2f^3g^3n^3x) \log(F)^3 + 3 \left( 2bd^2 + (bd^2f^2g^2n^2x^2 + 2bcd f^2g^2n^2x + bc^2f^2g^2n^2) \log(F) \right)}{3f^3g^3n^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)\*(d\*x+c)^2,x, algorithm="fricas")

[Out] 1/3\*((a\*d^2\*f^3\*g^3\*n^3\*x^3 + 3\*a\*c\*d\*f^3\*g^3\*n^3\*x^2 + 3\*a\*c^2\*f^3\*g^3\*n^3\*x)\*log(F)^3 + 3\*(2\*b\*d^2 + (b\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*b\*c\*d\*f^2\*g^2\*n^2\*x

$$+ b*c^2*f^2*g^2*n^2)*\log(F)^2 - 2*(b*d^2*f*g*n*x + b*c*d*f*g*n)*\log(F))*F^(f*g*n*x + e*g*n)/(f^3*g^3*n^3*\log(F)^3)$$

**giac** [C] time = 0.44, size = 2726, normalized size = 23.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)\*(d\*x+c)^2,x, algorithm="giac")

[Out]  $\frac{1}{3}a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - ((2*(\pi*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F)))*\text{sgn}(F) - \pi*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F)) + 2*\pi*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F)) + \pi*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b*d^2*f^2*g^2*n^2*\log(\text{abs}(F)) - \pi*b*d^2*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) + \pi*b*d^2*f^2*g^2*n^2*x*\log(\text{abs}(F)) - \pi*b*c*d*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) + \pi*b*c*d*f^2*g^2*n^2*\log(\text{abs}(F)))*(\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)/((\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)^2) - (\pi^2*b*d^2*f^2*g^2*n^2*x^2*\text{sgn}(F) - \pi^2*b*d^2*f^2*g^2*n^2*x^2 + 2*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))^2 + 2*\pi^2*b*c*d*f^2*g^2*n^2*x*\text{sgn}(F) - 2*\pi^2*b*c*d*f^2*g^2*n^2*x + 4*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))^2 + \pi^2*b*c^2*f^2*g^2*n^2*\text{sgn}(F) - \pi^2*b*c^2*f^2*g^2*n^2 + 2*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))^2 - 4*b*d^2*f^2*g^2*n^2*\log(\text{abs}(F)) - 4*b*c*d*f^2*g^2*n^2*\log(\text{abs}(F)) + 4*b*d^2)*(3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)/((\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)^2))*\cos(-1/2*\pi*f*g*n*x*\text{sgn}(F) + 1/2*\pi*f*g*n*x - 1/2*\pi*g*n*e*\text{sgn}(F) + 1/2*\pi*g*n*e) - ((\pi^2*b*d^2*f^2*g^2*n^2*x^2*\text{sgn}(F) - \pi^2*b*d^2*f^2*g^2*n^2*x^2 + 2*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))^2 + 2*\pi^2*b*c*d*f^2*g^2*n^2*x*\text{sgn}(F) - 2*\pi^2*b*c*d*f^2*g^2*n^2*x + 4*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))^2 + \pi^2*b*c^2*f^2*g^2*n^2*\text{sgn}(F) - \pi^2*b*c^2*f^2*g^2*n^2 + 2*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))^2 - 4*b*d^2*f^2*g^2*n^2*\log(\text{abs}(F)) - 4*b*c*d*f^2*g^2*n^2*\log(\text{abs}(F)) + 4*b*d^2)*(3*\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)/((\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)^2) + 2*(\pi*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F)) + 2*\pi*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F)) + \pi*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F)) - \pi*b*d^2*f^2*g^2*n^2*x*\text{sgn}(F) + \pi*b*d^2*f^2*g^2*n^2*x - \pi*b*c*d*f^2*g^2*n^2*\text{sgn}(F) + \pi*b*c*d*f^2*g^2*n^2*(3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)$



$$\begin{aligned} & \log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)/((\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi* \\ & f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log( \\ & \text{abs}(F))^2)^2 + (3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3* \\ & \log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)^2))*\sin(-1/2*\pi*f*g*n*x*\text{sgn}(F) + \\ & 1/2*\pi*f*g*n*x - 1/2*\pi*g*n*e*\text{sgn}(F) + 1/2*\pi*g*n*e))*e^{(f*g*n*x*\log(\text{abs}(F) \\ & )) + g*n*e*\log(\text{abs}(F)))} + 1/2*I*((4*I*\pi^2*b*d^2*f^2*g^2*n^2*x^2*\text{sgn}(F) - 8 \\ & *\pi*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 4*I*\pi^2*b*d^2*f^2*g^2*n^2*x \\ & ^2 + 8*\pi*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F)) + 8*I*b*d^2*f^2*g^2*n^2*x^2*\log \\ & (\text{abs}(F))^2 + 8*I*\pi^2*b*c*d*f^2*g^2*n^2*x*\text{sgn}(F) - 16*\pi*b*c*d*f^2*g^2*n^2* \\ & x*\log(\text{abs}(F))*\text{sgn}(F) - 8*I*\pi^2*b*c*d*f^2*g^2*n^2*x + 16*\pi*b*c*d*f^2*g^2*n \\ & ^2*x*\log(\text{abs}(F)) + 16*I*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))^2 + 4*I*\pi^2*b*c^2* \\ & f^2*g^2*n^2*\text{sgn}(F) - 8*\pi*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) - 4*I*\pi^2*b \\ & *c^2*f^2*g^2*n^2 + 8*\pi*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F)) + 8*I*b*c^2*f^2*g^2*n \\ & ^2*\log(\text{abs}(F))^2 + 8*\pi*b*d^2*f*g*n*x*\text{sgn}(F) - 8*\pi*b*d^2*f*g*n*x - 16*I*b* \\ & d^2*f*g*n*x*\log(\text{abs}(F)) + 8*\pi*b*c*d*f*g*n*\text{sgn}(F) - 8*\pi*b*c*d*f*g*n - 16*I \\ & *b*c*d*f*g*n*\log(\text{abs}(F)) + 16*I*b*d^2)*e^{(1/2*I*\pi*f*g*n*x*\text{sgn}(F) - 1/2*I*\pi \\ & i*f*g*n*x + 1/2*I*\pi*g*n*e*\text{sgn}(F) - 1/2*I*\pi*g*n*e)/(-4*I*\pi^3*f^3*g^3*n^3* \\ & \text{sgn}(F) + 12*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) + 12*I*\pi*f^3*g^3*n^3*\log(a \\ & \text{bs}(F))^2*\text{sgn}(F) + 4*I*\pi^3*f^3*g^3*n^3 - 12*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) - \\ & 12*I*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2 + 8*f^3*g^3*n^3*\log(\text{abs}(F))^3) - (4*I*\pi^ \\ & 2*b*d^2*f^2*g^2*n^2*x^2*\text{sgn}(F) + 8*\pi*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))*\text{sgn} \\ & (F) - 4*I*\pi^2*b*d^2*f^2*g^2*n^2*x^2 - 8*\pi*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F) \\ & )) + 8*I*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))^2 + 8*I*\pi^2*b*c*d*f^2*g^2*n^2*x \\ & *\text{sgn}(F) + 16*\pi*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))*\text{sgn}(F) - 8*I*\pi^2*b*c*d*f^2 \\ & *g^2*n^2*x - 16*\pi*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F)) + 16*I*b*c*d*f^2*g^2*n^2 \\ & *x*\log(\text{abs}(F))^2 + 4*I*\pi^2*b*c^2*f^2*g^2*n^2*\text{sgn}(F) + 8*\pi*b*c^2*f^2*g^2*n \\ & ^2*\log(\text{abs}(F))*\text{sgn}(F) - 4*I*\pi^2*b*c^2*f^2*g^2*n^2 - 8*\pi*b*c^2*f^2*g^2*n^2 \\ & *\log(\text{abs}(F)) + 8*I*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))^2 - 8*\pi*b*d^2*f*g*n*x*\text{sgn} \\ & (F) + 8*\pi*b*d^2*f*g*n*x - 16*I*b*d^2*f*g*n*x*\log(\text{abs}(F)) - 8*\pi*b*c*d*f*g* \\ & n*\text{sgn}(F) + 8*\pi*b*c*d*f*g*n - 16*I*b*c*d*f*g*n*\log(\text{abs}(F)) + 16*I*b*d^2)*e^{ \\ & (-1/2*I*\pi*f*g*n*x*\text{sgn}(F) + 1/2*I*\pi*f*g*n*x - 1/2*I*\pi*g*n*e*\text{sgn}(F) + 1/2* \\ & I*\pi*g*n*e)/(4*I*\pi^3*f^3*g^3*n^3*\text{sgn}(F) + 12*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))* \\ & \text{sgn}(F) - 12*I*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 4*I*\pi^3*f^3*g^3*n^3 - \\ & 12*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 12*I*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2 + 8*f^3 \\ & *g^3*n^3*\log(\text{abs}(F))^3))*e^{(f*g*n*x*\log(\text{abs}(F)) + g*n*e*\log(\text{abs}(F)))} \end{aligned}$$

**maple [A]** time = 0.05, size = 199, normalized size = 1.73

$$\frac{a d^2 x^3}{3} + \frac{a c d x^2 + a c^2 x}{f g n \ln(F)} + \frac{b d^2 x^2 e^{n \ln(e^{(f x + e) g \ln(F)})}}{f g n \ln(F)} + \frac{b c^2 e^{n \ln(e^{(f x + e) g \ln(F)})}}{f g n \ln(F)} - \frac{2 b c d e^{n \ln(e^{(f x + e) g \ln(F)})}}{f^2 g^2 n^2 \ln(F)^2} + \frac{2 (c f g n \ln(F) - d) b d}{f^2 g^2 n^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F((f\*x+e)\*g))^n+a)\*(d\*x+c)^2,x)

[Out]  $a*c^2*x+a*c*d*x^2+b/\ln(F)/f/g/n*\exp(n*\ln(\exp(g*(f*x+e)*\ln(F))))*c^2-2*b/\ln(F)^2/f^2/g^2/n^2*\exp(n*\ln(\exp(g*(f*x+e)*\ln(F))))*c*d+2*b/\ln(F)^3/f^3/g^3/n^3*\exp(n*\ln(\exp(g*(f*x+e)*\ln(F))))*d^2+1/n/f/g/\ln(F)*b*d^2*x^2*\exp(n*\ln(\exp(g*(f*x+e)*\ln(F))))+1/3*a*d^2*x^3+2*b*d*(\ln(F)*c*f*g*n-d)/\ln(F)^2/f^2/g^2/n^2*x*\exp(n*\ln(\exp(g*(f*x+e)*\ln(F))))$

**maxima** [A] time = 1.26, size = 178, normalized size = 1.55

$$\frac{1}{3}ad^2x^3+acd^2x^2+ac^2x+\frac{(F^{fgx+eg})^n bc^2}{fgn \log(F)} + \frac{2((F^{eg})^n fgnx \log(F) - (F^{eg})^n)(F^{fgx})^n bcd}{f^2g^2n^2 \log(F)^2} + \frac{((F^{eg})^n f^2g^2n^2x^2 \log(F)^2 - 2(F^{eg})^n f^3g^3n^3 \log(F)^3)}{f^3g^3n^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F^(g*(f*x+e))))^n*(d*x+c)^2,x, algorithm="maxima")`

[Out]  $1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + (F^(f*g*x + e*g))^n*b*c^2/(f*g*n*\log(F)) + 2*((F^(e*g))^n*f*g*n*x*\log(F) - (F^(e*g))^n*(F^(f*g*x))^n*b*c*d/(f^2*g^2*n^2*\log(F)^2) + ((F^(e*g))^n*f^2*g^2*n^2*x^2*\log(F)^2 - 2*(F^(e*g))^n*f*g*n*x*\log(F) + 2*(F^(e*g))^n*(F^(f*g*x))^n*b*d^2/(f^3*g^3*n^3*\log(F)^3))$

**mupad** [B] time = 3.67, size = 135, normalized size = 1.17

$$(F^{fgx} F^{eg})^n \left( \frac{b(c^2 f^2 g^2 n^2 \ln(F)^2 - 2cd fgn \ln(F) + 2d^2)}{f^3 g^3 n^3 \ln(F)^3} + \frac{bd^2 x^2}{fgn \ln(F)} - \frac{2bdx(d - c fgn \ln(F))}{f^2 g^2 n^2 \ln(F)^2} \right) + \frac{ad^2 x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*(F^(g*(e + f*x))))^n*(c + d*x)^2,x)`

[Out]  $(F^(f*g*x)*F^(e*g))^n*((b*(2*d^2 + c^2*f^2*g^2*n^2*\log(F)^2 - 2*c*d*f*g*n*\log(F)))/(f^3*g^3*n^3*\log(F)^3) + (b*d^2*x^2)/(f*g*n*\log(F)) - (2*b*d*x*(d - c*f*g*n*\log(F)))/(f^2*g^2*n^2*\log(F)^2)) + (a*d^2*x^3)/3 + a*c^2*x + a*c*d*x^2$

**sympy** [A] time = 0.22, size = 196, normalized size = 1.70

$$ac^2x+acd^2x^2+\frac{ad^2x^3}{3} + \begin{cases} \frac{(bc^2f^2g^2n^2 \log(F)^2+2bcd^2f^2g^2n^2x \log(F)^2-2bcdfgn \log(F)+bd^2f^2g^2n^2x^2 \log(F)^2-2bd^2fgnx \log(F)+2bd^2)(F^{g(e+fx)})^n}{f^3g^3n^3 \log(F)^3} \\ bc^2x + bcdx^2 + \frac{bd^2x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F**(g*(f*x+e))))**n)*(d*x+c)**2,x)`

```
[Out] a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + Piecewise(((b*c**2*f**2*g**2*n**2*log(F)**2 + 2*b*c*d*f**2*g**2*n**2*x*log(F)**2 - 2*b*c*d*f*g*n*log(F) + b*d**2*f**2*g**2*n**2*x**2*log(F)**2 - 2*b*d**2*f*g*n*x*log(F) + 2*b*d**2)*(F**(g*(e + f*x)))**n/(f**3*g**3*n**3*log(F)**3), Ne(f**3*g**3*n**3*log(F)**3, 0)), (b*c**2*x + b*c*d*x**2 + b*d**2*x**3/3, True))
```

$$3.27 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx) dx$$

Optimal. Leaf size=77

$$\frac{a(c+dx)^2}{2d} + \frac{b(c+dx)(F^{eg+fgx})^n}{fgn \log(F)} - \frac{bd(F^{eg+fgx})^n}{f^2g^2n^2 \log^2(F)}$$

[Out]  $1/2*a*(d*x+c)^2/d-b*d*(F^{(f*g*x+e*g)})^n/f^2/g^2/n^2/\ln(F)^2+b*(F^{(f*g*x+e*g)})^n*(d*x+c)/f/g/n/\ln(F)$

**Rubi [A]** time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2183, 2176, 2194}

$$\frac{a(c+dx)^2}{2d} + \frac{b(c+dx)(F^{eg+fgx})^n}{fgn \log(F)} - \frac{bd(F^{eg+fgx})^n}{f^2g^2n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]\*(c + d\*x), x]

[Out]  $(a*(c + d*x)^2)/(2*d) - (b*d*(F^{(e*g + f*g*x)})^n)/(f^2*g^2*n^2*\text{Log}[F]^2) + (b*(F^{(e*g + f*g*x)})^n*(c + d*x))/(f*g*n*\text{Log}[F])$

#### Rule 2176

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2183

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

#### Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
\int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx) dx &= \int \left( a(c + dx) + b \left( F^{eg+fgx} \right)^n (c + dx) \right) dx \\
&= \frac{a(c + dx)^2}{2d} + b \int \left( F^{eg+fgx} \right)^n (c + dx) dx \\
&= \frac{a(c + dx)^2}{2d} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)}{fgn \log(F)} - \frac{(bd) \int \left( F^{eg+fgx} \right)^n dx}{fgn \log(F)} \\
&= \frac{a(c + dx)^2}{2d} - \frac{bd \left( F^{eg+fgx} \right)^n}{f^2 g^2 n^2 \log^2(F)} + \frac{b \left( F^{eg+fgx} \right)^n (c + dx)}{fgn \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 73, normalized size = 0.95

$$\frac{1}{2}ax(2c + dx) + \frac{b(c + dx) \left( F^{g(e+fx)} \right)^n}{fgn \log(F)} - \frac{bd \left( F^{g(e+fx)} \right)^n}{f^2 g^2 n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)\*(c + d\*x), x]

[Out] (a\*x\*(2\*c + d\*x))/2 - (b\*d\*(F^(g\*(e + f\*x)))^n)/(f^2\*g^2\*n^2\*Log[F]^2) + (b\*(F^(g\*(e + f\*x)))^n\*(c + d\*x))/(f\*g\*n\*Log[F])

**fricas [A]** time = 0.49, size = 87, normalized size = 1.13

$$\frac{(adf^2g^2n^2x^2 + 2acf^2g^2n^2x) \log(F)^2 - 2(bd - (bdfgnx + bcfgn) \log(F)) F^{fgnx+egn}}{2f^2g^2n^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)\*(d\*x+c), x, algorithm="fricas")

[Out] 1/2\*((a\*d\*f^2\*g^2\*n^2\*x^2 + 2\*a\*c\*f^2\*g^2\*n^2\*x)\*log(F)^2 - 2\*(b\*d - (b\*d\*f\*g\*n\*x + b\*c\*f\*g\*n)\*log(F))\*F^(f\*g\*n\*x + e\*g\*n))/(f^2\*g^2\*n^2\*log(F)^2)

**giac [C]** time = 0.37, size = 1117, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)\*(d\*x+c), x, algorithm="giac")

```
[Out] 1/2*a*d*x^2 + a*c*x + (2*((pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))*(pi*b*d*f*g*n*x*sgn(F) - pi*b*d*f*g*n*x + pi*b*c*f*g*n*sgn(F) - pi*b*c*f*g*n)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) + (pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)*(b*d*f*g*n*x*log(abs(F)) + b*c*f*g*n*log(abs(F)) - b*d)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2)))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*g*n*e*sgn(F) + 1/2*pi*g*n*e) + ((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)*(pi*b*d*f*g*n*x*sgn(F) - pi*b*d*f*g*n*x + pi*b*c*f*g*n*sgn(F) - pi*b*c*f*g*n)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) - 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))*(b*d*f*g*n*x*log(abs(F)) + b*c*f*g*n*log(abs(F)) - b*d)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*sin(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*g*n*e*sgn(F) + 1/2*pi*g*n*e))*e^(f*g*n*x*log(abs(F)) + g*n*e*log(abs(F))) - 1/2*I*((2*pi*b*d*f*g*n*x*sgn(F) - 2*pi*b*d*f*g*n*x - 4*I*b*d*f*g*n*x*log(abs(F)) + 2*pi*b*c*f*g*n*sgn(F) - 2*pi*b*c*f*g*n - 4*I*b*c*f*g*n*log(abs(F)) + 4*I*b*d)*e^(1/2*I*pi*f*g*n*x*sgn(F) - 1/2*I*pi*f*g*n*x + 1/2*I*pi*g*n*e*sgn(F) - 1/2*I*pi*g*n*e)/(2*pi^2*f^2*g^2*n^2*sgn(F) + 4*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - 2*pi^2*f^2*g^2*n^2 - 4*I*pi*f^2*g^2*n^2*log(abs(F)) + 4*f^2*g^2*n^2*log(abs(F))^2) + (2*pi*b*d*f*g*n*x*sgn(F) - 2*pi*b*d*f*g*n*x + 4*I*b*d*f*g*n*x*log(abs(F)) + 2*pi*b*c*f*g*n*sgn(F) - 2*pi*b*c*f*g*n + 4*I*b*c*f*g*n*log(abs(F)) - 4*I*b*d)*e^(-1/2*I*pi*f*g*n*x*sgn(F) + 1/2*I*pi*f*g*n*x - 1/2*I*pi*g*n*e*sgn(F) + 1/2*I*pi*g*n*e)/(2*pi^2*f^2*g^2*n^2*sgn(F) - 4*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - 2*pi^2*f^2*g^2*n^2 + 4*I*pi*f^2*g^2*n^2*log(abs(F)) + 4*f^2*g^2*n^2*log(abs(F))^2))*e^(f*g*n*x*log(abs(F)) + g*n*e*log(abs(F)))
```

**maple [A]** time = 0.04, size = 105, normalized size = 1.36

$$\frac{ad x^2}{2} + acx + \frac{bdx e^{n \ln(e^{(fx+e)g \ln(F)})}}{fgn \ln(F)} + \frac{bc e^{n \ln(e^{(fx+e)g \ln(F)})}}{fgn \ln(F)} - \frac{bd e^{n \ln(e^{(fx+e)g \ln(F)})}}{f^2 g^2 n^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(F^((f*x+e)*g))^n+a)*(d*x+c),x)
```

```
[Out] a*c*x+b/n/f/g/ln(F)*exp(n*ln(exp((f*x+e)*g*ln(F))))*c-b/n^2/f^2/g^2/ln(F)^2*exp(n*ln(exp((f*x+e)*g*ln(F))))*d+1/n/f/g/ln(F)*b*d*x*exp(n*ln(exp((f*x+e)*g*ln(F))))+1/2*a*d*x^2
```

**maxima** [A] time = 1.08, size = 88, normalized size = 1.14

$$\frac{1}{2} adx^2 + acx + \frac{(F^{fgx+eg})^n bc}{fgn \log(F)} + \frac{((F^{eg})^n fgnx \log(F) - (F^{eg})^n)(F^{fgx})^n bd}{f^2 g^2 n^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n\*(d\*x+c),x, algorithm="maxima")

[Out] 1/2\*a\*d\*x^2 + a\*c\*x + (F^(f\*g\*x + e\*g))^n\*b\*c/(f\*g\*n\*log(F)) + ((F^(e\*g))^n \*f\*g\*n\*x\*log(F) - (F^(e\*g))^n\*(F^(f\*g\*x))^n\*b\*d/(f^2\*g^2\*n^2\*log(F)^2)

**mupad** [B] time = 3.60, size = 72, normalized size = 0.94

$$acx - \left( \frac{b(d - c f g n \ln(F))}{f^2 g^2 n^2 \ln(F)^2} - \frac{bdx}{fgn \ln(F)} \right) (F^{fgx} F^{eg})^n + \frac{adx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n\*(c + d\*x),x)

[Out] a\*c\*x - ((b\*(d - c\*f\*g\*n\*log(F)))/(f^2\*g^2\*n^2\*log(F)^2) - (b\*d\*x)/(f\*g\*n\*log(F)))\*(F^(f\*g\*x)\*F^(e\*g))^n + (a\*d\*x^2)/2

**sympy** [A] time = 0.17, size = 94, normalized size = 1.22

$$acx + \frac{adx^2}{2} + \begin{cases} \frac{(bcfgn \log(F) + bdfgnx \log(F) - bd)(F^{g(e+fx)})^n}{f^2 g^2 n^2 \log(F)^2} & \text{for } f^2 g^2 n^2 \log(F)^2 \neq 0 \\ bcx + \frac{bdx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n\*(d\*x+c),x)

[Out] a\*c\*x + a\*d\*x\*\*2/2 + Piecewise(((b\*c\*f\*g\*n\*log(F) + b\*d\*f\*g\*n\*x\*log(F) - b\*d)\*(F\*\*(g\*(e + f\*x))))\*\*n/(f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2), Ne(f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2, 0)), (b\*c\*x + b\*d\*x\*\*2/2, True))

$$3.28 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) dx$$

Optimal. Leaf size=30

$$ax + \frac{b \left( F^{g(e+fx)} \right)^n}{fgn \log(F)}$$

[Out] a\*x+b\*(F^(g\*(f\*x+e)))^n/f/g/n/ln(F)

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2194}

$$ax + \frac{b \left( F^{g(e+fx)} \right)^n}{fgn \log(F)}$$

Antiderivative was successfully verified.

[In] Int[a + b\*(F^(g\*(e + f\*x)))^n,x]

[Out] a\*x + (b\*(F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F])

Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) dx &= ax + b \int \left( F^{g(e+fx)} \right)^n dx \\ &= ax + \frac{b \left( F^{g(e+fx)} \right)^n}{fgn \log(F)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 1.00

$$ax + \frac{b \left( F^{g(e+fx)} \right)^n}{fgn \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*(F^(g\*(e + f\*x)))^n,x]



[Out]  $a*x + (b*(F^{(g*(e + f*x))})^n)/(f*g*n*\text{Log}[F])$

**fricas** [A] time = 0.47, size = 37, normalized size = 1.23

$$\frac{afgnx \log(F) + Ffg^{nx+egn}b}{fgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="fricas")`

[Out]  $(a*f*g*n*x*\log(F) + F^{(f*g*n*x + e*g*n)}*b)/(f*g*n*\log(F))$

**giac** [A] time = 0.20, size = 32, normalized size = 1.07

$$ax + \frac{Ffg^{nx+gne}b}{fgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="giac")`

[Out]  $a*x + F^{(f*g*n*x + g*n*e)}*b/(f*g*n*\log(F))$

**maple** [A] time = 0.00, size = 31, normalized size = 1.03

$$ax + \frac{b \left( F^{(fx+e)g} \right)^n}{fgn \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(b*(F^((f*x+e)*g))^n+a,x)`

[Out]  $a*x+b*(F^{((f*x+e)*g)})^n/f/g/n/\ln(F)$

**maxima** [A] time = 0.93, size = 31, normalized size = 1.03

$$ax + \frac{(Ffg^{x+eg})^n b}{fgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="maxima")`

[Out]  $a*x + (F^{(f*g*x + e*g)})^n*b/(f*g*n*\log(F))$

mupad [B] time = 3.60, size = 31, normalized size = 1.03

$$ax + \frac{b(F^{eg+fgx})^n}{fgn \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a + b\*(F^(g\*(e + f\*x)))^n, x)

[Out] a\*x + (b\*(F^(e\*g + f\*g\*x))^n)/(f\*g\*n\*log(F))

sympy [A] time = 0.12, size = 32, normalized size = 1.07

$$ax + \begin{cases} \frac{b(F^{g(e+fx)})^n}{fgn \log(F)} & \text{for } fgn \log(F) \neq 0 \\ bx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*(F\*\*(g\*(f\*x+e)))\*\*n, x)

[Out] a\*x + Piecewise((b\*(F\*\*(g\*(e + f\*x)))\*\*n/(f\*g\*n\*log(F)), Ne(f\*g\*n\*log(F), 0)), (b\*x, True))

$$3.29 \quad \int \frac{a+b(F^{g(e+fx)})^n}{c+dx} dx$$

**Optimal.** Leaf size=68

$$\frac{a \log(c+dx)}{d} + \frac{b(F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \text{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}$$

[Out]  $bF^{((e-c*f/d)*g*n-g*n*(f*x+e))*(F^{(f*g*x+e*g)})^n*Ei(f*g*n*(d*x+c)*ln(F)/d)/d+a*ln(d*x+c)/d}$

**Rubi [A]** time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {2183, 2182, 2178}

$$\frac{a \log(c+dx)}{d} + \frac{b(F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \text{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)/(c + d\*x), x]

[Out]  $(bF^{((e-(c*f)/d)*g*n-g*n*(e+f*x))*(F^{(e*g+f*g*x)})^n*ExpIntegralEi[(f*g*n*(c+d*x)*Log[F])/d]}/d + (a*Log[c+d*x])/d)$

**Rule 2178**

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

**Rule 2182**

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Dist[(b\*F^(g\*(e + f\*x)))^n/F^(g\*n\*(e + f\*x)), Int[(c + d\*x)^m\*F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

**Rule 2183**

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n]^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a + b(Fg^{(e+fx)})^n}{c + dx} dx &= \int \left( \frac{a}{c + dx} + \frac{b(F^{eg+fgx})^n}{c + dx} \right) dx \\
&= \frac{a \log(c + dx)}{d} + b \int \frac{(F^{eg+fgx})^n}{c + dx} dx \\
&= \frac{a \log(c + dx)}{d} + \left( bF^{-n(eg+fgx)} (F^{eg+fgx})^n \right) \int \frac{F^{n(eg+fgx)}}{c + dx} dx \\
&= \frac{bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)} (F^{eg+fgx})^n \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{a \log(c + dx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 56, normalized size = 0.82

$$\frac{a \log(c + dx) + b(Fg^{(e+fx)})^n F^{-\frac{fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)/(c + d\*x), x]

[Out] ((b\*(F^(g\*(e + f\*x)))^n\*ExpIntegralEi[(f\*g\*n\*(c + d\*x)\*Log[F])/d])/F^((f\*g\*n\*(c + d\*x))/d) + a\*Log[c + d\*x])/d

**fricas [A]** time = 0.45, size = 50, normalized size = 0.74

$$\frac{F^{\frac{(de-cf)gn}{d}} b \operatorname{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right) + a \log(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)/(d\*x+c), x, algorithm="fricas")

[Out] (F^(((d\*e - c\*f)\*g\*n/d)\*b\*Ei((d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + a\*log(d\*x + c)))/d

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(F^{(fx+e)g})^n b + a}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c),x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)/(d\*x + c), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{b \left( F^{(f x + e) g} \right)^n + a}{d x + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)/(d\*x+c),x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)/(d\*x+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^n b \int \frac{(F^{fgx})^n}{dx + c} dx + \frac{a \log(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c),x, algorithm="maxima")

[Out] (F^(e\*g))^n\*b\*integrate((F^(f\*g\*x))^n/(d\*x + c), x) + a\*log(d\*x + c)/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \left( F^{g(e + f x)} \right)^n}{c + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)/(c + d\*x),x)

[Out] int((a + b\*(F^(g\*(e + f\*x))))^n)/(c + d\*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \left( F^{eg} F^{fgx} \right)^n}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)/(d\*x+c),x)

[Out] Integral((a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))\*\*n)/(c + d\*x), x)

$$3.30 \quad \int \frac{a+b(F^{g(e+fx)})^n}{(c+dx)^2} dx$$

**Optimal.** Leaf size=100

$$-\frac{a}{d(c+dx)} + \frac{bfgn \log(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx) \log(F)}{d}\right)}{d^2} - \frac{b(F^{eg+fgx})^n}{d(c+dx)}$$

[Out]  $-a/d/(d*x+c)-b*(F^{(f*g*x+e*g)})^n/d/(d*x+c)+b*f*F^{((e-c*f/d)*g*n-g*n*(f*x+e))}*(F^{(f*g*x+e*g)})^n*g*n*Ei(f*g*n*(d*x+c)*\ln(F)/d)*\ln(F)/d^2$

**Rubi [A]** time = 0.17, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {2183, 2177, 2182, 2178}

$$-\frac{a}{d(c+dx)} + \frac{bfgn \log(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx) \log(F)}{d}\right)}{d^2} - \frac{b(F^{eg+fgx})^n}{d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*(F^{(g*(e + f*x))})^n)/(c + d*x)^2, x]$

[Out]  $-(a/(d*(c + d*x))) - (b*(F^{(e*g + f*g*x)})^n)/(d*(c + d*x)) + (b*f*F^{((e - (c*f)/d)*g*n - g*n*(e + f*x))}*(F^{(e*g + f*g*x)})^n*g*n*\operatorname{ExpIntegralEi}[(f*g*n*(c + d*x)*\operatorname{Log}[F])/d]*\operatorname{Log}[F])/d^2$

**Rule 2177**

$\operatorname{Int}[(b*(F^{(g*(e + f*x))})^n)/(c + d*x)^2, x] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1}*(b*(F^{(g*(e + f*x))})^n)/(d*(m+1)), x] - \operatorname{Dist}[(f*g*n*\operatorname{Log}[F])/d, \operatorname{Int}[(c + d*x)^{m+1}*(b*(F^{(g*(e + f*x))})^n), x]] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

**Rule 2178**

$\operatorname{Int}[(F^{(g*(e - (c*f)/d))}*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

**Rule 2182**

$\operatorname{Int}[(b*(F^{(g*(e + f*x))})^n)/(c + d*x)^2, x] \rightarrow \operatorname{Dist}[(b*(F^{(g*(e + f*x))})^n)/F^{(g*n*(e + f*x))}, \operatorname{Int}[(c + d*x)^{m+1}*(b*(F^{(g*(e + f*x))})^n), x]] /;$

)^m \* F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

### Rule 2183

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b(F^{g(e+fx)})^n}{(c+dx)^2} dx &= \int \left( \frac{a}{(c+dx)^2} + \frac{b(F^{eg+fgx})^n}{(c+dx)^2} \right) dx \\
 &= -\frac{a}{d(c+dx)} + b \int \frac{(F^{eg+fgx})^n}{(c+dx)^2} dx \\
 &= -\frac{a}{d(c+dx)} - \frac{b(F^{eg+fgx})^n}{d(c+dx)} + \frac{(bfgn \log(F)) \int \frac{(F^{eg+fgx})^n}{c+dx} dx}{d} \\
 &= -\frac{a}{d(c+dx)} - \frac{b(F^{eg+fgx})^n}{d(c+dx)} + \frac{(bfF^{-n(eg+fgx)}(F^{eg+fgx})^n gn \log(F)) \int \frac{F^{n(eg+fgx)}}{c+dx} dx}{d} \\
 &= -\frac{a}{d(c+dx)} - \frac{b(F^{eg+fgx})^n}{d(c+dx)} + \frac{bfF^{(e-\frac{cf}{d})gn-gn(e+fx)}(F^{eg+fgx})^n gn \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right) \log(F)}{d^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.28, size = 78, normalized size = 0.78

$$\frac{bfgn \log(F) (F^{g(e+fx)})^n F^{-\frac{fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right) - \frac{d(a+b(F^{g(e+fx)})^n)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)/(c + d\*x)^2, x]

[Out] (-(d\*(a + b\*(F^(g\*(e + f\*x)))^n))/(c + d\*x)) + (b\*f\*(F^(g\*(e + f\*x)))^n\*g\*n\*ExpIntegralEi[(f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F])/F^((f\*g\*n\*(c + d\*x))/d)/d^2

**fricas** [A] time = 0.43, size = 87, normalized size = 0.87

$$\frac{(bdfgnx + bcfgn)F^{\frac{(de-cf)gn}{d}} \operatorname{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right) \log(F) - F^{fgnx+egnb}d - ad}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c)^2,x, algorithm="fricas")

[Out] ((b\*d\*f\*g\*n\*x + b\*c\*f\*g\*n)\*F^((d\*e - c\*f)\*g\*n/d)\*Ei((d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d)\*log(F) - F^(f\*g\*n\*x + e\*g\*n)\*b\*d - a\*d)/(d^3\*x + c\*d^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(F^{(fx+e)g})^n b + a}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)/(d\*x + c)^2, x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{b(F^{(fx+e)g})^n + a}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)/(d\*x+c)^2,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)/(d\*x+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^n b \int \frac{(F^{fgx})^n}{d^2x^2 + 2cdx + c^2} dx - \frac{a}{d^2x + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c)^2,x, algorithm="maxima")

[Out] (F^(e\*g))^n\*b\*integrate((F^(f\*g\*x))^n/(d^2\*x^2 + 2\*c\*d\*x + c^2), x) - a/(d^2\*x + c\*d)



mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \left( F^{g(e+fx)} \right)^n}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)/(c + d\*x)^2, x)

[Out] int((a + b\*(F^(g\*(e + f\*x))))^n)/(c + d\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \left( F^{egFfgx} \right)^n}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)/(d\*x+c)\*\*2, x)

[Out] Integral((a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))\*\*n)/(c + d\*x)\*\*2, x)

$$3.31 \quad \int \frac{a+b(F^{g(e+fx)})^n}{(c+dx)^3} dx$$

**Optimal.** Leaf size=147

$$-\frac{a}{2d(c+dx)^2} + \frac{bf^2g^2n^2 \log^2(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx) \log(F)}{d}\right)}{2d^3} - \frac{bfgn \log(F) (F^{eg+fgx})^n}{2d^2(c+dx)} - \frac{b(F^{eg+fgx})^n}{2d(c+dx)}$$

[Out]  $-1/2*a/d/(d*x+c)^2-1/2*b*(F^{(f*g*x+e*g)})^n/d/(d*x+c)^2-1/2*b*f*(F^{(f*g*x+e*g)})^n*g*n*\ln(F)/d^2/(d*x+c)+1/2*b*f^2*F^{((e-c*f/d)*g*n-g*n*(f*x+e))}*(F^{(f*g*x+e*g)})^n*g^2*n^2*\operatorname{Ei}(f*g*n*(d*x+c)*\ln(F)/d)*\ln(F)^2/d^3$

**Rubi [A]** time = 0.24, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {2183, 2177, 2182, 2178}

$$-\frac{a}{2d(c+dx)^2} + \frac{bf^2g^2n^2 \log^2(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx) \log(F)}{d}\right)}{2d^3} - \frac{bfgn \log(F) (F^{eg+fgx})^n}{2d^2(c+dx)} - \frac{b(F^{eg+fgx})^n}{2d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*(F^{(g*(e + f*x))})^n)/(c + d*x)^3, x]$

[Out]  $-a/(2*d*(c + d*x)^2) - (b*(F^{(e*g + f*g*x)})^n)/(2*d*(c + d*x)^2) - (b*f*(F^{(e*g + f*g*x)})^n*g*n*\operatorname{Log}[F])/(2*d^2*(c + d*x)) + (b*f^2*F^{((e - (c*f)/d)*g*n - g*n*(e + f*x))}*(F^{(e*g + f*g*x)})^n*g^2*n^2*\operatorname{ExpIntegralEi}[(f*g*n*(c + d*x)*\operatorname{Log}[F])/d]*\operatorname{Log}[F]^2)/(2*d^3)$

Rule 2177

$\operatorname{Int}[(b*(F^{(g*(e + f*x))})^n)/(c + d*x)^3, x] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1}*(b*(F^{(g*(e + f*x))})^n)/(d*(m+1)), x] - \operatorname{Dist}[(f*g*n*\operatorname{Log}[F])/(d*(m+1)), \operatorname{Int}[(c + d*x)^{m+1}*(b*(F^{(g*(e + f*x))})^n), x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

Rule 2178

$\operatorname{Int}[(F^{(g*(e - (c*f)/d))}*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2182

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] := Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

### Rule 2183

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^((p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b(F^{g(e+fx)})^n}{(c + dx)^3} dx &= \int \left( \frac{a}{(c + dx)^3} + \frac{b(F^{eg+fgx})^n}{(c + dx)^3} \right) dx \\
&= -\frac{a}{2d(c + dx)^2} + b \int \frac{(F^{eg+fgx})^n}{(c + dx)^3} dx \\
&= -\frac{a}{2d(c + dx)^2} - \frac{b(F^{eg+fgx})^n}{2d(c + dx)^2} + \frac{(bfgn \log(F)) \int \frac{(F^{eg+fgx})^n}{(c+dx)^2} dx}{2d} \\
&= -\frac{a}{2d(c + dx)^2} - \frac{b(F^{eg+fgx})^n}{2d(c + dx)^2} - \frac{bf(F^{eg+fgx})^n gn \log(F)}{2d^2(c + dx)} + \frac{(bf^2g^2n^2 \log^2(F)) \int \frac{(F^{eg+fgx})^n}{c+dx}}{2d^2} \\
&= -\frac{a}{2d(c + dx)^2} - \frac{b(F^{eg+fgx})^n}{2d(c + dx)^2} - \frac{bf(F^{eg+fgx})^n gn \log(F)}{2d^2(c + dx)} + \frac{(bf^2F^{-n(eg+fgx)}(F^{eg+fgx})^n g^2)}{2d^2} \\
&= -\frac{a}{2d(c + dx)^2} - \frac{b(F^{eg+fgx})^n}{2d(c + dx)^2} - \frac{bf(F^{eg+fgx})^n gn \log(F)}{2d^2(c + dx)} + \frac{bf^2F^{\left(-\frac{cf}{d}\right)gn-gn(e+fx)}(F^{eg+fgx})^n}{2d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.33, size = 111, normalized size = 0.76

$$\frac{ad^2 - bf^2g^2n^2 \log^2(F)(c + dx)^2 (F^{g(e+fx)})^n F^{-\frac{fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{fgn(c+dx) \log(F)}{d}\right) + bd (F^{g(e+fx)})^n (fgn \log(F)(c + dx) + \dots)}{2d^3(c + dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3,x]
```

[Out]  $-1/2*(a*d^2 - (b*f^2*(F^{(g*(e + f*x))})^n*g^2*n^2*(c + d*x)^2*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^{((f*g*n*(c + d*x))/d) + b*d*(F^{(g*(e + f*x))})^n*(d + f*g*n*(c + d*x)*Log[F]))/(d^3*(c + d*x)^2)$

**fricas** [A] time = 0.43, size = 160, normalized size = 1.09

$$\frac{(bd^2 f^2 g^2 n^2 x^2 + 2bcd f^2 g^2 n^2 x + bc^2 f^2 g^2 n^2) F^{\frac{(de-cf)gn}{d}} Ei\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right) \log(F)^2 - ad^2 - (bd^2 + (bd^2 fgnx + bc^2 d^2))}{2(d^5 x^2 + 2cd^4 x + c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F^(g*(f*x+e))))^n)/(d*x+c)^3,x, algorithm="fricas")`

[Out]  $1/2*((b*d^2*f^2*g^2*n^2*x^2 + 2*b*c*d*f^2*g^2*n^2*x + b*c^2*f^2*g^2*n^2)*F^{((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F)^2 - a*d^2 - (b*d^2 + (b*d^2*f*g*n*x + b*c*d*f*g*n)*log(F))*F^{(f*g*n*x + e*g*n)}}/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(F^{(f*x+e)g})^n b + a}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F^(g*(f*x+e))))^n)/(d*x+c)^3,x, algorithm="giac")`

[Out] `integrate(((F^((f*x + e)*g))^n*b + a)/(d*x + c)^3, x)`

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{b(F^{(f*x+e)g})^n + a}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*(F^((f*x+e)*g)))^n+a)/(d*x+c)^3,x)`

[Out] `int((b*(F^((f*x+e)*g)))^n+a)/(d*x+c)^3,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^n b \int \frac{(F^{fgx})^n}{d^3 x^3 + 3cd^2 x^2 + 3c^2 dx + c^3} dx - \frac{a}{2(d^3 x^2 + 2cd^2 x + c^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c)^3,x, algorithm="maxima")

[Out] (F^(e\*g))^n\*b\*integrate((F^(f\*g\*x))^n/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x) - 1/2\*a/(d^3\*x^2 + 2\*c\*d^2\*x + c^2\*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \left( F^{g(e+fx)} \right)^n}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)/(c + d\*x)^3,x)

[Out] int((a + b\*(F^(g\*(e + f\*x))))^n)/(c + d\*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \left( F^{egFfgx} \right)^n}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)/(d\*x+c)\*\*3,x)

[Out] Integral((a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))))\*\*n)/(c + d\*x)\*\*3, x)

$$3.32 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx)^3 dx$$

**Optimal.** Leaf size=322

$$\frac{a^2(c+dx)^4}{4d} + \frac{12abd^2(c+dx)(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{6abd(c+dx)^2(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{2ab(c+dx)^3(F^{eg+fgx})^n}{fgn\log(F)} - \frac{12abd^3(F^{eg+fgx})^n}{f^4g^4n^4\log^4(F)} +$$

[Out]  $\frac{1}{4}a^2(d*x+c)^4/d - 12*a*b*d^3*(F^(f*g*x+e*g))^n/f^4/g^4/n^4/\ln(F)^4 - 3/8*b^2*d^3*(F^(f*g*x+e*g))^(2*n)/f^4/g^4/n^4/\ln(F)^4 + 12*a*b*d^2*(F^(f*g*x+e*g))^n*(d*x+c)/f^3/g^3/n^3/\ln(F)^3 + 3/4*b^2*d^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^3/g^3/n^3/\ln(F)^3 - 6*a*b*d*(F^(f*g*x+e*g))^n*(d*x+c)^2/f^2/g^2/n^2/\ln(F)^2 - 3/4*b^2*d*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^2/f^2/g^2/n^2/\ln(F)^2 + 2*a*b*(F^(f*g*x+e*g))^n*(d*x+c)^3/f/g/n/\ln(F) + 1/2*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^3/f/g/n/\ln(F)$

**Rubi [A]** time = 0.49, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2183, 2176, 2194}

$$\frac{a^2(c+dx)^4}{4d} + \frac{12abd^2(c+dx)(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{6abd(c+dx)^2(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{2ab(c+dx)^3(F^{eg+fgx})^n}{fgn\log(F)} - \frac{12abd^3(F^{eg+fgx})^n}{f^4g^4n^4\log^4(F)} +$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]^2\*(c + d\*x)^3, x]

[Out]  $\frac{a^2*(c+d*x)^4}{(4*d)} - \frac{(12*a*b*d^3*(F^(e*g+f*g*x))^n)}{(f^4*g^4*n^4*\text{Log}[F]^4)} - \frac{(3*b^2*d^3*(F^(e*g+f*g*x))^(2*n))}{(8*f^4*g^4*n^4*\text{Log}[F]^4)} + \frac{(12*a*b*d^2*(F^(e*g+f*g*x))^n*(c+d*x))}{(f^3*g^3*n^3*\text{Log}[F]^3)} + \frac{(3*b^2*d^2*(F^(e*g+f*g*x))^(2*n)*(c+d*x))}{(4*f^3*g^3*n^3*\text{Log}[F]^3)} - \frac{(6*a*b*d*(F^(e*g+f*g*x))^n*(c+d*x)^2)}{(f^2*g^2*n^2*\text{Log}[F]^2)} - \frac{(3*b^2*d*(F^(e*g+f*g*x))^(2*n)*(c+d*x)^2)}{(4*f^2*g^2*n^2*\text{Log}[F]^2)} + \frac{(2*a*b*(F^(e*g+f*g*x))^n*(c+d*x)^3)}{(f*g*n*\text{Log}[F])} + \frac{(b^2*(F^(e*g+f*g*x))^(2*n)*(c+d*x)^3)}{(2*f*g*n*\text{Log}[F])}$

**Rule 2176**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.)+(f\_.)\*(x\_))))^(n\_.)\*((c\_.)+(d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c+d\*x)^m\*(b\*F^(g\*(e+f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c+d\*x)^(m-1)\*(b\*F^(g\*(e+f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

Rule 2183

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
 \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx)^3 dx &= \int \left( a^2(c + dx)^3 + 2ab \left( F^{eg+fgx} \right)^n (c + dx)^3 + b^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)^3 \right) dx \\
 &= \frac{a^2(c + dx)^4}{4d} + (2ab) \int \left( F^{eg+fgx} \right)^n (c + dx)^3 dx + b^2 \int \left( F^{eg+fgx} \right)^{2n} (c + dx)^3 dx \\
 &= \frac{a^2(c + dx)^4}{4d} + \frac{2ab \left( F^{eg+fgx} \right)^n (c + dx)^3}{fgn \log(F)} + \frac{b^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)^3}{2fgn \log(F)} - \frac{(6abd)}{fgn \log(F)} \\
 &= \frac{a^2(c + dx)^4}{4d} - \frac{6abd \left( F^{eg+fgx} \right)^n (c + dx)^2}{f^2 g^2 n^2 \log^2(F)} - \frac{3b^2 d \left( F^{eg+fgx} \right)^{2n} (c + dx)^2}{4f^2 g^2 n^2 \log^2(F)} + \frac{2a^2 b d}{fgn \log(F)} \\
 &= \frac{a^2(c + dx)^4}{4d} + \frac{12abd^2 \left( F^{eg+fgx} \right)^n (c + dx)}{f^3 g^3 n^3 \log^3(F)} + \frac{3b^2 d^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)}{4f^3 g^3 n^3 \log^3(F)} - \frac{6abd}{fgn \log(F)} \\
 &= \frac{a^2(c + dx)^4}{4d} - \frac{12abd^3 \left( F^{eg+fgx} \right)^n}{f^4 g^4 n^4 \log^4(F)} - \frac{3b^2 d^3 \left( F^{eg+fgx} \right)^{2n}}{8f^4 g^4 n^4 \log^4(F)} + \frac{12abd^2 \left( F^{eg+fgx} \right)^n}{f^3 g^3 n^3 \log^3(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.56, size = 239, normalized size = 0.74

$$a^2 c^3 x + \frac{3}{2} a^2 c^2 dx^2 + a^2 c d^2 x^3 + \frac{1}{4} a^2 d^3 x^4 + \frac{2ab \left( F^{g(e+fx)} \right)^n \left( 6d^2 fgn \log(F)(c + dx) + f^3 g^3 n^3 \log^3(F)(c + dx)^3 - 3df^2 g^3 \right)}{f^4 g^4 n^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^2\*(c + d\*x)^3,x]

[Out] a^2\*c^3\*x + (3\*a^2\*c^2\*d\*x^2)/2 + a^2\*c\*d^2\*x^3 + (a^2\*d^3\*x^4)/4 + (2\*a\*b\*(F^(g\*(e + f\*x)))^n\*(-6\*d^3 + 6\*d^2\*f\*g\*n\*(c + d\*x)\*Log[F] - 3\*d\*f^2\*g^2\*n^3

$$\frac{2*(c + d*x)^2*\text{Log}[F]^2 + f^3*g^3*n^3*(c + d*x)^3*\text{Log}[F]^3)}{(f^4*g^4*n^4*\text{Log}[F]^4) + (b^2*(F^(g*(e + f*x)))^(2*n))*(-3*d^3 + 6*d^2*f*g*n*(c + d*x)*\text{Log}[F] - 6*d*f^2*g^2*n^2*(c + d*x)^2*\text{Log}[F]^2 + 4*f^3*g^3*n^3*(c + d*x)^3*\text{Log}[F]^3)}{(8*f^4*g^4*n^4*\text{Log}[F]^4)}$$

**fricas** [A] time = 0.43, size = 482, normalized size = 1.50

$$\frac{2(a^2d^3f^4g^4n^4x^4 + 4a^2cd^2f^4g^4n^4x^3 + 6a^2c^2df^4g^4n^4x^2 + 4a^2c^3f^4g^4n^4x)\log(F)^4 - (3b^2d^3 - 4(b^2d^3f^3g^3n^3x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^3,x, algorithm="fricas")

[Out]  $\frac{1}{8}*(2*(a^2*d^3*f^4*g^4*n^4*x^4 + 4*a^2*c*d^2*f^4*g^4*n^4*x^3 + 6*a^2*c^2*d*f^4*g^4*n^4*x^2 + 4*a^2*c^3*f^4*g^4*n^4*x)*\log(F)^4 - (3*b^2*d^3 - 4*(b^2*d^3*f^3*g^3*n^3*x^3 + 3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*b^2*c^2*d*f^3*g^3*n^3*x + b^2*c^3*f^3*g^3*n^3)*\log(F)^3 + 6*(b^2*d^3*f^2*g^2*n^2*x^2 + 2*b^2*c*d^2*f^2*g^2*n^2*x + b^2*c^2*d*f^2*g^2*n^2)*\log(F)^2 - 6*(b^2*d^3*f*g*n*x + b^2*c*d^2*f*g*n)*\log(F))*F^(2*f*g*n*x + 2*e*g*n) - 16*(6*a*b*d^3 - (a*b*d^3*f^3*g^3*n^3*x^3 + 3*a*b*c*d^2*f^3*g^3*n^3*x^2 + 3*a*b*c^2*d*f^3*g^3*n^3*x + a*b*c^3*f^3*g^3*n^3)*\log(F)^3 + 3*(a*b*d^3*f^2*g^2*n^2*x^2 + 2*a*b*c*d^2*f^2*g^2*n^2*x + a*b*c^2*d*f^2*g^2*n^2)*\log(F)^2 - 6*(a*b*d^3*f*g*n*x + a*b*c*d^2*f*g*n)*\log(F))*F^(f*g*n*x + e*g*n))/(f^4*g^4*n^4*\log(F)^4)$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^3,x, algorithm="giac")

[Out] Timed out

**maple** [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(f*x+e)g} \right)^n + a \right)^2 (dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g)))^n+a)^2\*(d\*x+c)^3,x

[Out] int((b\*(F^((f\*x+e)\*g)))^n+a)^2\*(d\*x+c)^3,x



**maxima [A]** time = 1.89, size = 604, normalized size = 1.88

$$\frac{1}{4} a^2 d^3 x^4 + a^2 c d^2 x^3 + \frac{3}{2} a^2 c^2 d x^2 + a^2 c^3 x + \frac{2 (F^{fgx+eg})^n abc^3}{fgn \log(F)} + \frac{(F^{fgx+eg})^{2n} b^2 c^3}{2 fgn \log(F)} + \frac{6 ((F^{eg})^n fgnx \log(F) - (F^{eg})^n) (F^{fgx})}{f^2 g^2 n^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^3,x, algorithm="maxima")

[Out]  $\frac{1}{4} a^2 d^3 x^4 + a^2 c d^2 x^3 + \frac{3}{2} a^2 c^2 d x^2 + a^2 c^3 x + \frac{2 (F^{fgx+eg})^n abc^3}{fgn \log(F)} + \frac{(F^{fgx+eg})^{2n} b^2 c^3}{2 fgn \log(F)} + \frac{6 ((F^{eg})^n fgnx \log(F) - (F^{eg})^n) (F^{fgx})}{f^2 g^2 n^2 \log(F)^2}$

**mupad [B]** time = 3.97, size = 438, normalized size = 1.36

$$a^2 c^3 x - (F^{fgx} F^{eg})^n \left( \frac{2 a b (-c^3 f^3 g^3 n^3 \ln(F)^3 + 3 c^2 d f^2 g^2 n^2 \ln(F)^2 - 6 c d^2 f g n \ln(F) + 6 d^3)}{f^4 g^4 n^4 \ln(F)^4} - \frac{2 a b d^3 x^3}{f g n \ln(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x)^3,x)

[Out]  $a^2 c^3 x - (F^{fgx} F^{eg})^n \left( \frac{2 a b (-c^3 f^3 g^3 n^3 \ln(F)^3 + 3 c^2 d f^2 g^2 n^2 \ln(F)^2 - 6 c d^2 f g n \ln(F) + 6 d^3)}{f^4 g^4 n^4 \ln(F)^4} - \frac{2 a b d^3 x^3}{f g n \ln(F)} \right)$

sympy [A] time = 0.48, size = 709, normalized size = 2.20

$$a^2c^3x + \frac{3a^2c^2dx^2}{2} + a^2cd^2x^3 + \frac{a^2d^3x^4}{4} + \left\{ \frac{(4b^2c^3f^7g^7n^7 \log(F)^7 + 12b^2c^2df^7g^7n^7x \log(F)^7 - 6b^2c^2df^6g^6n^6 \log(F)^6 + 12b^2cd^2f^7g^7n^7x^2 \log(F)^7)}{x^4 \left( \frac{abd^3}{2} + \frac{b^2d^3}{4} \right) + x^3 (2abcd^2 + b^2cd^2) + x^2 \left( 3abc^2d + \frac{3b^2c^2d}{2} \right) + x (2abc^3 + \dots)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*2\*(d\*x+c)\*\*3,x)

[Out] a\*\*2\*c\*\*3\*x + 3\*a\*\*2\*c\*\*2\*d\*x\*\*2/2 + a\*\*2\*c\*d\*\*2\*x\*\*3 + a\*\*2\*d\*\*3\*x\*\*4/4 + Piecewise((((4\*b\*\*2\*c\*\*3\*f\*\*7\*g\*\*7\*n\*\*7\*log(F)\*\*7 + 12\*b\*\*2\*c\*\*2\*d\*f\*\*7\*g\*\*7\*n\*\*7\*x\*log(F)\*\*7 - 6\*b\*\*2\*c\*\*2\*d\*f\*\*6\*g\*\*6\*n\*\*6\*log(F)\*\*6 + 12\*b\*\*2\*c\*d\*\*2\*f\*\*7\*g\*\*7\*n\*\*7\*x\*\*2\*log(F)\*\*7 - 12\*b\*\*2\*c\*d\*\*2\*f\*\*6\*g\*\*6\*n\*\*6\*x\*log(F)\*\*6 + 6\*b\*\*2\*c\*d\*\*2\*f\*\*5\*g\*\*5\*n\*\*5\*log(F)\*\*5 + 4\*b\*\*2\*d\*\*3\*f\*\*7\*g\*\*7\*n\*\*7\*x\*\*3\*log(F)\*\*7 - 6\*b\*\*2\*d\*\*3\*f\*\*6\*g\*\*6\*n\*\*6\*x\*\*2\*log(F)\*\*6 + 6\*b\*\*2\*d\*\*3\*f\*\*5\*g\*\*5\*n\*\*5\*x\*log(F)\*\*5 - 3\*b\*\*2\*d\*\*3\*f\*\*4\*g\*\*4\*n\*\*4\*log(F)\*\*4)\*(F\*\*(g\*(e + f\*x)))\*\*2\*n) + (16\*a\*b\*c\*\*3\*f\*\*7\*g\*\*7\*n\*\*7\*log(F)\*\*7 + 48\*a\*b\*c\*\*2\*d\*f\*\*7\*g\*\*7\*n\*\*7\*x\*log(F)\*\*7 - 48\*a\*b\*c\*\*2\*d\*f\*\*6\*g\*\*6\*n\*\*6\*log(F)\*\*6 + 48\*a\*b\*c\*d\*\*2\*f\*\*7\*g\*\*7\*n\*\*7\*x\*\*2\*log(F)\*\*7 - 96\*a\*b\*c\*d\*\*2\*f\*\*6\*g\*\*6\*n\*\*6\*x\*log(F)\*\*6 + 96\*a\*b\*c\*d\*\*2\*f\*\*5\*g\*\*5\*n\*\*5\*log(F)\*\*5 + 16\*a\*b\*d\*\*3\*f\*\*7\*g\*\*7\*n\*\*7\*x\*\*3\*log(F)\*\*7 - 48\*a\*b\*d\*\*3\*f\*\*6\*g\*\*6\*n\*\*6\*x\*\*2\*log(F)\*\*6 + 96\*a\*b\*d\*\*3\*f\*\*5\*g\*\*5\*n\*\*5\*x\*log(F)\*\*5 - 96\*a\*b\*d\*\*3\*f\*\*4\*g\*\*4\*n\*\*4\*log(F)\*\*4)\*(F\*\*(g\*(e + f\*x)))\*\*n)/(8\*f\*\*8\*g\*\*8\*n\*\*8\*log(F)\*\*8), Ne(8\*f\*\*8\*g\*\*8\*n\*\*8\*log(F)\*\*8, 0)), (x\*\*4\*(a\*b\*d\*\*3/2 + b\*\*2\*d\*\*3/4) + x\*\*3\*(2\*a\*b\*c\*d\*\*2 + b\*\*2\*c\*d\*\*2) + x\*\*2\*(3\*a\*b\*c\*\*2\*d + 3\*b\*\*2\*c\*\*2\*d/2) + x\*(2\*a\*b\*c\*\*3 + b\*\*2\*c\*\*3), True))

$$3.33 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx)^2 dx$$

**Optimal.** Leaf size=239

$$\frac{a^2(c+dx)^3}{3d} - \frac{4abd(c+dx)(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{2ab(c+dx)^2(F^{eg+fgx})^n}{fgn\log(F)} + \frac{4abd^2(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{b^2d(c+dx)(F^{eg+fgx})^{2n}}{2f^2g^2n^2\log^2(F)} + \frac{b^2(c+dx)^2(F^{eg+fgx})^{2n}}{2f^2g^2n^2\log^2(F)}$$

[Out]  $\frac{1}{3}a^2(d*x+c)^3/d+4*a*b*d^2*(F^{(f*g*x+e*g)})^n/f^3/g^3/n^3/\ln(F)^3+1/4*b^2*d^2*(F^{(f*g*x+e*g)})^{(2*n)}/f^3/g^3/n^3/\ln(F)^3-4*a*b*d*(F^{(f*g*x+e*g)})^n*(d*x+c)/f^2/g^2/n^2/\ln(F)^2-1/2*b^2*d*(F^{(f*g*x+e*g)})^{(2*n)}*(d*x+c)/f^2/g^2/n^2/\ln(F)^2+2*a*b*(F^{(f*g*x+e*g)})^n*(d*x+c)^2/f/g/n/\ln(F)+1/2*b^2*(F^{(f*g*x+e*g)})^{(2*n)}*(d*x+c)^2/f/g/n/\ln(F)$

**Rubi [A]** time = 0.32, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2183, 2176, 2194}

$$\frac{a^2(c+dx)^3}{3d} - \frac{4abd(c+dx)(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{2ab(c+dx)^2(F^{eg+fgx})^n}{fgn\log(F)} + \frac{4abd^2(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{b^2d(c+dx)(F^{eg+fgx})^{2n}}{2f^2g^2n^2\log^2(F)} + \frac{b^2(c+dx)^2(F^{eg+fgx})^{2n}}{2f^2g^2n^2\log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]^2\*(c + d\*x)^2,x]

[Out]  $(a^2*(c+d*x)^3)/(3*d) + (4*a*b*d^2*(F^{(e*g+f*g*x)})^n)/(f^3*g^3*n^3*\text{Log}[F]^3) + (b^2*d^2*(F^{(e*g+f*g*x)})^{(2*n)})/(4*f^3*g^3*n^3*\text{Log}[F]^3) - (4*a*b*d*(F^{(e*g+f*g*x)})^n*(c+d*x))/(f^2*g^2*n^2*\text{Log}[F]^2) - (b^2*d*(F^{(e*g+f*g*x)})^{(2*n)}*(c+d*x))/(2*f^2*g^2*n^2*\text{Log}[F]^2) + (2*a*b*(F^{(e*g+f*g*x)})^n*(c+d*x)^2)/(f*g*n*\text{Log}[F]) + (b^2*(F^{(e*g+f*g*x)})^{(2*n)}*(c+d*x)^2)/(2*f*g*n*\text{Log}[F])$

**Rule 2176**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.)+(f\_.)\*(x\_))))^(n\_.)\*((c\_.)+(d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c+d\*x)^m\*(b\*F^(g\*(e+f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c+d\*x)^(m-1)\*(b\*F^(g\*(e+f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

**Rule 2183**

Int[((a\_.)+(b\_.)\*((F\_)^((g\_.)\*((e\_.)+(f\_.)\*(x\_))))^(n\_.))^p\_.\*(c\_.)+(d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c+d\*x)^m, (a+b\*(F

$\int (g*(e + f*x))^n)^p, x] /; \text{FreeQ}[\{F, a, b, c, d, e, f, g, m, n\}, x] \&\&$   
 $\text{IGtQ}[p, 0]$

### Rule 2194

$\text{Int}[(F_*)^{((c_*)*(a_*) + (b_*)*(x_*))}^{(n_*)}, x\_Symbol] := \text{Simp}[(F^{(c*(a + b*x))})^n / (b*c*n*\text{Log}[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

### Rubi steps

$$\begin{aligned} \int (a + b(F^{g(e+fx)})^n)^2 (c + dx)^2 dx &= \int (a^2(c + dx)^2 + 2ab(F^{eg+fgx})^n (c + dx)^2 + b^2(F^{eg+fgx})^{2n} (c + dx)^2) dx \\ &= \frac{a^2(c + dx)^3}{3d} + (2ab) \int (F^{eg+fgx})^n (c + dx)^2 dx + b^2 \int (F^{eg+fgx})^{2n} (c + dx)^2 dx \\ &= \frac{a^2(c + dx)^3}{3d} + \frac{2ab(F^{eg+fgx})^n (c + dx)^2}{fgn \log(F)} + \frac{b^2(F^{eg+fgx})^{2n} (c + dx)^2}{2fgn \log(F)} - \frac{(4abd)}{fgn \log(F)} \\ &= \frac{a^2(c + dx)^3}{3d} - \frac{4abd(F^{eg+fgx})^n (c + dx)}{f^2g^2n^2 \log^2(F)} - \frac{b^2d(F^{eg+fgx})^{2n} (c + dx)}{2f^2g^2n^2 \log^2(F)} + \frac{2ab(F^{eg+fgx})^n (c + dx)}{fgn \log(F)} \\ &= \frac{a^2(c + dx)^3}{3d} + \frac{4abd^2(F^{eg+fgx})^n}{f^3g^3n^3 \log^3(F)} + \frac{b^2d^2(F^{eg+fgx})^{2n}}{4f^3g^3n^3 \log^3(F)} - \frac{4abd(F^{eg+fgx})^n (c + dx)}{f^2g^2n^2 \log^2(F)} \end{aligned}$$

**Mathematica [A]** time = 0.37, size = 171, normalized size = 0.72

$$a^2c^2x + a^2cdx^2 + \frac{1}{3}a^2d^2x^3 + \frac{2ab(F^{g(e+fx)})^n (f^2g^2n^2 \log^2(F)(c + dx)^2 - 2dfgn \log(F)(c + dx) + 2d^2)}{f^3g^3n^3 \log^3(F)} + \frac{b^2(F^{g(e+fx)})^{2n} (f^2g^2n^2 \log^2(F)(c + dx)^2 - 2dfgn \log(F)(c + dx) + 2d^2)}{4f^3g^3n^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n]^2\*(c + d\*x)^2,x]

[Out]  $a^2c^2x + a^2c*d*x^2 + (a^2*d^2*x^3)/3 + (2*a*b*(F^{g*(e + f*x)})^n*(2*d^2 - 2*d*f*g*n*(c + d*x)*\text{Log}[F] + f^2*g^2*n^2*(c + d*x)^2*\text{Log}[F]^2))/(f^3*g^3*n^3*\text{Log}[F]^3) + (b^2*(F^{g*(e + f*x)})^{2*n}*(d^2 - 2*d*f*g*n*(c + d*x)*\text{Log}[F] + 2*f^2*g^2*n^2*(c + d*x)^2*\text{Log}[F]^2))/(4*f^3*g^3*n^3*\text{Log}[F]^3)$

**fricas [A]** time = 0.43, size = 287, normalized size = 1.20

$$4(a^2d^2f^3g^3n^3x^3 + 3a^2cdf^3g^3n^3x^2 + 3a^2c^2f^3g^3n^3x) \log(F)^3 + 3(b^2d^2 + 2(b^2d^2f^2g^2n^2x^2 + 2b^2cdf^2g^2n^2x + b^2c^2)) \log(F)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^2,x, algorithm="fricas")

[Out]  $\frac{1}{12} \cdot (4 \cdot (a^2 \cdot d^2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot x^3 + 3 \cdot a^2 \cdot c \cdot d \cdot f^3 \cdot g^3 \cdot n^3 \cdot x^2 + 3 \cdot a^2 \cdot c^2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot x) \cdot \log(F)^3 + 3 \cdot (b^2 \cdot d^2 + 2 \cdot (b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 + 2 \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x + b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2)) \cdot \log(F)^2 - 2 \cdot (b^2 \cdot d^2 \cdot f \cdot g \cdot n \cdot x + b^2 \cdot c \cdot d \cdot f \cdot g \cdot n) \cdot \log(F)) \cdot F^{(2 \cdot f \cdot g \cdot n \cdot x + 2 \cdot e \cdot g \cdot n)} + 24 \cdot (2 \cdot a \cdot b \cdot d^2 + (a \cdot b \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 + 2 \cdot a \cdot b \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x + a \cdot b \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2)) \cdot \log(F)^2 - 2 \cdot (a \cdot b \cdot d^2 \cdot f \cdot g \cdot n \cdot x + a \cdot b \cdot c \cdot d \cdot f \cdot g \cdot n) \cdot \log(F)) \cdot F^{(f \cdot g \cdot n \cdot x + e \cdot g \cdot n)} / (f^3 \cdot g^3 \cdot n^3 \cdot \log(F)^3)$

**giac** [C] time = 1.23, size = 5699, normalized size = 23.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^2,x, algorithm="giac")

[Out]  $\frac{1}{3} \cdot a^2 \cdot d^2 \cdot x^3 + a^2 \cdot c \cdot d \cdot x^2 + a^2 \cdot c^2 \cdot x - \frac{1}{2} \cdot (((2 \cdot \pi \cdot b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - 2 \cdot \pi \cdot b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 \cdot \log(\text{abs}(F)) + 4 \cdot \pi \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - 4 \cdot \pi \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x \cdot \log(\text{abs}(F)) + 2 \cdot \pi \cdot b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - 2 \cdot \pi \cdot b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) - \pi \cdot b^2 \cdot d^2 \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) + \pi \cdot b^2 \cdot d^2 \cdot f \cdot g \cdot n \cdot x - \pi \cdot b^2 \cdot c \cdot d \cdot f \cdot g \cdot n \cdot \text{sgn}(F) + \pi \cdot b^2 \cdot c \cdot d \cdot f \cdot g \cdot n) \cdot (\pi^3 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \text{sgn}(F) - 3 \cdot \pi \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)))^2 \cdot \text{sgn}(F) - \pi^3 \cdot f^3 \cdot g^3 \cdot n^3 + 3 \cdot \pi \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)))^2) / ((\pi^3 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \text{sgn}(F) - 3 \cdot \pi \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)))^2 \cdot \text{sgn}(F) - \pi^3 \cdot f^3 \cdot g^3 \cdot n^3 + 3 \cdot \pi \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)))^2 + (3 \cdot \pi^2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - 3 \cdot \pi^2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)) + 2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)))^3)^2 - (\pi^2 \cdot b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 \cdot \text{sgn}(F) - \pi^2 \cdot b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 + 2 \cdot b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 \cdot \log(\text{abs}(F)))^2 + 2 \cdot \pi^2 \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x \cdot \text{sgn}(F) - 2 \cdot \pi^2 \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x + 4 \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x \cdot \log(\text{abs}(F)))^2 + \pi^2 \cdot b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - \pi^2 \cdot b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2 + 2 \cdot b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2 - 2 \cdot b^2 \cdot d^2 \cdot f \cdot g \cdot n \cdot x \cdot \log(\text{abs}(F)) - 2 \cdot b^2 \cdot c \cdot d \cdot f \cdot g \cdot n \cdot \log(\text{abs}(F)) + b^2 \cdot d^2) \cdot (3 \cdot \pi^2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - 3 \cdot \pi^2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)) + 2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F))) / ((\pi^3 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \text{sgn}(F) - 3 \cdot \pi \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)))^2 \cdot \text{sgn}(F) - \pi^3 \cdot f^3 \cdot g^3 \cdot n^3 + 3 \cdot \pi \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)))^2 + (3 \cdot \pi^2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - 3 \cdot \pi^2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)) + 2 \cdot f^3 \cdot g^3 \cdot n^3 \cdot \log(\text{abs}(F)))^3)^2) \cdot \cos(-\pi \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) + \pi \cdot f \cdot g \cdot n \cdot x - \pi \cdot g \cdot n \cdot e \cdot \text{sgn}(F) + \pi \cdot g \cdot n \cdot e) - ((\pi^2 \cdot b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 \cdot \text{sgn}(F) - \pi^2 \cdot b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 + 2 \cdot b^2 \cdot d^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 \cdot \log(\text{abs}(F)))^2 + 2 \cdot \pi^2 \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x \cdot \text{sgn}(F) - 2 \cdot \pi^2 \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x + 4 \cdot b^2 \cdot c \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x \cdot \log(\text{abs}(F)))^2 + \pi^2 \cdot b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - \pi^2 \cdot b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2 + 2 \cdot b^2 \cdot c^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2 - 2 \cdot b^2 \cdot d^2 \cdot f \cdot g \cdot n \cdot x \cdot \log(\text{abs}(F)) - 2 \cdot b^2 \cdot c \cdot d \cdot f \cdot g \cdot n \cdot \log(\text{abs}(F)) + b^2 \cdot d^2) \cdot (\pi^3 \cdot$

$$\begin{aligned}
& f^3 g^3 n^3 \operatorname{sgn}(F) - 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 f^3 g^3 n^3 \\
& \quad + 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 / ((\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2) \\
& \quad + (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^2)^2 + (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^2)^2 \\
& \quad + (2\pi b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2\pi b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + 4\pi b^2 c d f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 4\pi b^2 c d f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + \\
& \quad 2\pi b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2\pi b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) - \pi b^2 d^2 f g n x \operatorname{sgn}(F) + \pi b^2 d^2 f g n x - \pi b^2 c d f g n \\
& \quad n \operatorname{sgn}(F) + \pi b^2 c d f g n) * (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 + (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^2) \\
& \quad + (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^2) * \sin(-\pi f g n x \operatorname{sgn}(F) + \pi f g n x - \pi g n e \operatorname{sgn}(F) + \pi g n e)) * e^{(2 f g n x \log(\operatorname{abs}(F)) + 2 g n e \log(\operatorname{abs}(F)))} \\
& \quad + 1/2 I * ((2 I \pi^2 b^2 d^2 f^2 g^2 n^2 x^2 \operatorname{sgn}(F) - 4\pi b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 I \pi^2 b^2 d^2 f^2 g^2 n^2 x^2 + 4\pi b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + 4 I b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F))^2 + 4 I \pi^2 b^2 c d f^2 g^2 n^2 x \operatorname{sgn}(F) - 8\pi b^2 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 4 I \pi^2 b^2 c d f^2 g^2 n^2 x + 8\pi b^2 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F)) + 8 I b^2 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F))^2 + 2 I \pi^2 b^2 c^2 f^2 g^2 n^2 \operatorname{sgn}(F) - 4\pi b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 I \pi^2 b^2 c^2 f^2 g^2 n^2 + 4\pi b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) + 4 I b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F))^2 + 2\pi b^2 d^2 f g n x \operatorname{sgn}(F) - 2\pi b^2 d^2 f g n x - 4 I b^2 d^2 f g n x \log(\operatorname{abs}(F)) + 2\pi b^2 c d f g n \operatorname{sgn}(F) - 2\pi b^2 c d f g n - 4 I b^2 c d f g n \log(\operatorname{abs}(F)) + 2 I b^2 d^2) * e^{(I \pi f g n x \operatorname{sgn}(F) - I \pi f g n x + I \pi g n e \operatorname{sgn}(F) - I \pi g n e)} / (-4 I \pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) + 12 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) + 12 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) + 4 I \pi^3 f^3 g^3 n^3 - 12 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) - 12 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 + 8 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3 - (2 I \pi^2 b^2 d^2 f^2 g^2 n^2 x^2 \operatorname{sgn}(F) + 4\pi b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 I \pi^2 b^2 d^2 f^2 g^2 n^2 x^2 - 4\pi b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + 4 I b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)))^2 + 4 I \pi^2 b^2 c d f^2 g^2 n^2 x \operatorname{sgn}(F) + 8\pi b^2 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 4 I \pi^2 b^2 c d f^2 g^2 n^2 x - 8\pi b^2 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F)) + 8 I b^2 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F))^2 + 2 I \pi^2 b^2 c^2 f^2 g^2 n^2 \operatorname{sgn}(F) + 4\pi b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 I \pi^2 b^2 c^2 f^2 g^2 n^2 - 4\pi b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) + 4 I b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F))^2 - 2\pi b^2 d^2 f g n x \operatorname{sgn}(F) + 2\pi b^2 d^2 f g n x - 4 I b^2 d^2 f g n x \log(\operatorname{abs}(F)) - 2\pi b^2 c d f g n \operatorname{sgn}(F) + 2\pi b^2 c d f g n - 4 I b^2 c d f g n \log(\operatorname{abs}(F)) + 2 I b^2 d^2) * e^{(-I \pi f g n x \operatorname{sgn}(F) + I \pi f g n x - I \pi g n e \operatorname{sgn}(F) + I \pi g n e)} / (4 I \pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) + 12 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 12 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - 4 I \pi^3 f^3 g^3 n^3 - 12 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 12 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 + 8 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3) * e^{(
\end{aligned}$$

$$\begin{aligned}
& 2*f*g*n*x*\log(\text{abs}(F)) + 2*g*n*e*\log(\text{abs}(F))) - 2*((2*(\pi*a*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*a*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F)) + 2*\pi*a*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi*a*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F)) + \pi*a*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*a*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F)) - \pi*a*b*d^2*f*g*n*x*\text{sgn}(F) + \pi*a*b*d^2*f*g*n*x - \pi*a*b*c*d*f*g*n*\text{sgn}(F) + \pi*a*b*c*d*f*g*n)*( \pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)/((\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)^2) - (\pi^2*a*b*d^2*f^2*g^2*n^2*x^2*\text{sgn}(F) - \pi^2*a*b*d^2*f^2*g^2*n^2*x^2 + 2*a*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))^2 + 2*\pi^2*a*b*c*d*f^2*g^2*n^2*x*\text{sgn}(F) - 2*\pi^2*a*b*c*d*f^2*g^2*n^2*x + 4*a*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))^2 + \pi^2*a*b*c^2*f^2*g^2*n^2*\text{sgn}(F) - \pi^2*a*b*c^2*f^2*g^2*n^2 + 2*a*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))^2 - 4*a*b*d^2*f*g*n*x*\log(\text{abs}(F)) - 4*a*b*c*d*f*g*n*\log(\text{abs}(F)) + 4*a*b*d^2)*(3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)/((\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)^2))*\cos(-1/2*\pi*f*g*n*x*\text{sgn}(F) + 1/2*\pi*f*g*n*x - 1/2*\pi*g*n*e*\text{sgn}(F) + 1/2*\pi*g*n*e) - ((\pi^2*a*b*d^2*f^2*g^2*n^2*x^2*\text{sgn}(F) - \pi^2*a*b*d^2*f^2*g^2*n^2*x^2 + 2*a*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))^2 + 2*\pi^2*a*b*c*d*f^2*g^2*n^2*x*\text{sgn}(F) - 2*\pi^2*a*b*c*d*f^2*g^2*n^2*x + 4*a*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))^2 + \pi^2*a*b*c^2*f^2*g^2*n^2*\text{sgn}(F) - \pi^2*a*b*c^2*f^2*g^2*n^2 + 2*a*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))^2 - 4*a*b*d^2*f*g*n*x*\log(\text{abs}(F)) - 4*a*b*c*d*f*g*n*\log(\text{abs}(F)) + 4*a*b*d^2)*(\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)/((\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)^2) + 2*(\pi*a*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*a*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F)) + 2*\pi*a*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F))*\text{sgn}(F) - 2*\pi*a*b*c*d*f^2*g^2*n^2*x*\log(\text{abs}(F)) + \pi*a*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*a*b*c^2*f^2*g^2*n^2*\log(\text{abs}(F)) - \pi*a*b*d^2*f*g*n*x*\text{sgn}(F) + \pi*a*b*d^2*f*g*n*x - \pi*a*b*c*d*f*g*n*\text{sgn}(F) + \pi*a*b*c*d*f*g*n)*(3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)/((\pi^3*f^3*g^3*n^3*\text{sgn}(F) - 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*f^3*g^3*n^3 + 3*\pi*f^3*g^3*n^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*f^3*g^3*n^3*\log(\text{abs}(F)) + 2*f^3*g^3*n^3*\log(\text{abs}(F))^3)^2))*\sin(-1/2*\pi*f*g*n*x*\text{sgn}(F) + 1/2*\pi*f*g*n*x - 1/2*\pi*g*n*e*\text{sgn}(F) + 1/2*\pi*g*n*e))e^{(f*g*n*x*\log(\text{abs}(F)) + g*n*e*\log(\text{abs}(F)))} + 1/2*I*((8*I*\pi^2*a*b*d^2*f^2*g^2*n^2*x^2*\text{sgn}(F) - 16*\pi*a*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 8*I*\pi^2*a*b*d^2*f^2*g^2*n^2*x^2 + 16*\pi*a*b*d^2*f^2*g^2*n^2*x^2*\log(\text{abs}(F))) + 16*I*\pi^2*a*b*c*d*f^2*g^2
\end{aligned}$$

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*n^2*x*sgn(F) - 32*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 16*I*pi^2*
a*b*c*d*f^2*g^2*n^2*x + 32*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(F)) + 32*I*a*b*
c*d*f^2*g^2*n^2*x*log(abs(F))^2 + 8*I*pi^2*a*b*c^2*f^2*g^2*n^2*sgn(F) - 16*
pi*a*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 8*I*pi^2*a*b*c^2*f^2*g^2*n^2 +
16*pi*a*b*c^2*f^2*g^2*n^2*log(abs(F)) + 16*I*a*b*c^2*f^2*g^2*n^2*log(abs(F)
)^2 + 16*pi*a*b*d^2*f*g*n*x*sgn(F) - 16*pi*a*b*d^2*f*g*n*x - 32*I*a*b*d^2*f
*g*n*x*log(abs(F)) + 16*pi*a*b*c*d*f*g*n*sgn(F) - 16*pi*a*b*c*d*f*g*n - 32*
I*a*b*c*d*f*g*n*log(abs(F)) + 32*I*a*b*d^2)*e^(1/2*I*pi*f*g*n*x*sgn(F) - 1/
2*I*pi*f*g*n*x + 1/2*I*pi*g*n*e*sgn(F) - 1/2*I*pi*g*n*e)/(-4*I*pi^3*f^3*g^3
*n^3*sgn(F) + 12*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) + 12*I*pi*f^3*g^3*n^3*
log(abs(F))^2*sgn(F) + 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(F)
)) - 12*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F))^3) - (8*
I*pi^2*a*b*d^2*f^2*g^2*n^2*x^2*sgn(F) + 16*pi*a*b*d^2*f^2*g^2*n^2*x^2*log(a
bs(F))*sgn(F) - 8*I*pi^2*a*b*d^2*f^2*g^2*n^2*x^2 - 16*pi*a*b*d^2*f^2*g^2*n^
2*x^2*log(abs(F)) + 16*I*a*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 16*I*pi^2*
a*b*c*d*f^2*g^2*n^2*x*sgn(F) + 32*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(
F) - 16*I*pi^2*a*b*c*d*f^2*g^2*n^2*x - 32*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(
F)) + 32*I*a*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 + 8*I*pi^2*a*b*c^2*f^2*g^2*n
^2*sgn(F) + 16*pi*a*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 8*I*pi^2*a*b*c^2
*f^2*g^2*n^2 - 16*pi*a*b*c^2*f^2*g^2*n^2*log(abs(F)) + 16*I*a*b*c^2*f^2*g^2
*n^2*log(abs(F))^2 - 16*pi*a*b*d^2*f*g*n*x*sgn(F) + 16*pi*a*b*d^2*f*g*n*x -
32*I*a*b*d^2*f*g*n*x*log(abs(F)) - 16*pi*a*b*c*d*f*g*n*sgn(F) + 16*pi*a*b*
c*d*f*g*n - 32*I*a*b*c*d*f*g*n*log(abs(F)) + 32*I*a*b*d^2)*e^(-1/2*I*pi*f*g
*n*x*sgn(F) + 1/2*I*pi*f*g*n*x - 1/2*I*pi*g*n*e*sgn(F) + 1/2*I*pi*g*n*e)/(4
*I*pi^3*f^3*g^3*n^3*sgn(F) + 12*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 12*I*
pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^
3*n^3*log(abs(F)) + 12*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(a
bs(F))^3))*e^(f*g*n*x*log(abs(F)) + g*n*e*log(abs(F)))

```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(fx+e)g} \right)^n + a \right)^2 (dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^2\*(d\*x+c)^2,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^2\*(d\*x+c)^2,x)

**maxima [A]** time = 1.61, size = 368, normalized size = 1.54

$$\frac{1}{3} a^2 d^2 x^3 + a^2 c d x^2 + a^2 c^2 x + \frac{2 (F^{fgx+eg})^n abc^2}{fgn \log(F)} + \frac{(F^{fgx+eg})^{2n} b^2 c^2}{2 fgn \log(F)} + \frac{4 ((F^{eg})^n fgn x \log(F) - (F^{eg})^n) (F^{fgx})^n abcd}{f^2 g^2 n^2 \log(F)^2} + \frac{(2 (F^{eg})^n fgn x \log(F) - (F^{eg})^n) (F^{fgx})^n abcd}{f^2 g^2 n^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}a^2d^2x^3 + a^2cdx^2 + a^2c^2x + 2(F^{fgx+eg})^{n+1}abc^2/(fgn \log(F)) + \frac{1}{2}(F^{fgx+eg})^{2n}b^2c^2/(fgn \log(F)) + 4((F^{eg})^n f g n x \log(F) - (F^{eg})^n (F^{fgx})^n abc d / (f^2 g^2 n^2 \log(F)^2) + \frac{1}{2}(2(F^{eg})^{2n} f g n x \log(F) - (F^{eg})^{2n}) (F^{fgx})^{2n} b^2 c d / (f^2 g^2 n^2 \log(F)^2) + 2((F^{eg})^n f^2 g^2 n^2 x^2 \log(F)^2 - 2(F^{eg})^n f g n x \log(F) + 2(F^{eg})^n (F^{fgx})^n abc d^2 / (f^3 g^3 n^3 \log(F)^3) + \frac{1}{4}(2(F^{eg})^{2n} f^2 g^2 n^2 x^2 \log(F)^2 - 2(F^{eg})^{2n} f g n x \log(F) + (F^{eg})^{2n}) (F^{fgx})^{2n} b^2 d^2 / (f^3 g^3 n^3 \log(F)^3)$

**mupad [B]** time = 3.82, size = 267, normalized size = 1.12

$$(F^{fgx} F^{eg})^{2n} \left( \frac{b^2 (2c^2 f^2 g^2 n^2 \ln(F)^2 - 2cdfgn \ln(F) + d^2)}{4f^3 g^3 n^3 \ln(F)^3} + \frac{b^2 d^2 x^2}{2fgn \ln(F)} - \frac{b^2 dx (d - 2cdfgn \ln(F))}{2f^2 g^2 n^2 \ln(F)^2} \right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x)^2,x)

[Out]  $(F^{fgx} F^{eg})^{2n} ((b^2(d^2 + 2c^2 f^2 g^2 n^2 \log(F)^2 - 2c d f g n \log(F)) / (4f^3 g^3 n^3 \log(F)^3) + (b^2 d^2 x^2) / (2f g n \log(F)) - (b^2 d x (d - 2c f g n \log(F)) / (2f^2 g^2 n^2 \log(F)^2)) + (F^{fgx} F^{eg})^n ((2a b (2d^2 + c^2 f^2 g^2 n^2 \log(F)^2 - 2c d f g n \log(F)) / (f^3 g^3 n^3 \log(F)^3) + (2a b d^2 x^2) / (f g n \log(F)) - (4a b d x (d - c f g n \log(F)) / (f^2 g^2 n^2 \log(F)^2)) + a^2 c^2 x + (a^2 d^2 x^3) / 3 + a^2 c d x^2$

**sympy [A]** time = 0.37, size = 439, normalized size = 1.84

$$a^2 c^2 x + a^2 c d x^2 + \frac{a^2 d^2 x^3}{3} + \left\{ \frac{(2b^2 c^2 f^5 g^5 n^5 \log(F)^5 + 4b^2 c d f^5 g^5 n^5 x \log(F)^5 - 2b^2 c d f^4 g^4 n^4 \log(F)^4 + 2b^2 d^2 f^5 g^5 n^5 x^2 \log(F)^5 - 2b^2 d^2 f^4 g^4 n^4 x \log(F)^4)}{x^3 \left( \frac{2abd^2}{3} + \frac{b^2 d^2}{3} \right) + x^2 (2abcd + b^2 cd) + x (2abc^2 + b^2 c^2)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*2\*(d\*x+c)\*\*2,x)

[Out]  $a^{**2}c^{**2}x + a^{**2}c*d*x^{**2} + a^{**2}d^{**2}x^{**3}/3 + \text{Piecewise}(((2*b^{**2}c^{**2}f^{**5}g^{**5}n^{**5} \log(F)^{**5} + 4*b^{**2}c*d*f^{**5}g^{**5}n^{**5}x \log(F)^{**5} - 2*b^{**2}c*d*f^{**4}g^{**4}n^{**4} \log(F)^{**4} + 2*b^{**2}d^{**2}f^{**5}g^{**5}n^{**5}x^2 \log(F)^{**5} - 2*b^{**2}d^{**2}f^{**4}g^{**4}n^{**4}x \log(F)^{**4} + b^{**2}d^{**2}f^{**3}g^{**3}n^{**3} \log(F)^{**3}) * (F^{**}(g*(e + f*x)))^{**}(2*n) + (8*a*b*c^{**2}f^{**5}g^{**5}n^{**5} \log(F)^{**5} + 16*a*b*c$

```

*d*f**5*g**5*n**5*x*log(F)**5 - 16*a*b*c*d*f**4*g**4*n**4*log(F)**4 + 8*a*b
*d**2*f**5*g**5*n**5*x**2*log(F)**5 - 16*a*b*d**2*f**4*g**4*n**4*x*log(F)**
4 + 16*a*b*d**2*f**3*g**3*n**3*log(F)**3*(F**(g*(e + f*x)))**n)/(4*f**6*g*
*6*n**6*log(F)**6), Ne(4*f**6*g**6*n**6*log(F)**6, 0)), (x**3*(2*a*b*d**2/3
+ b**2*d**2/3) + x**2*(2*a*b*c*d + b**2*c*d) + x*(2*a*b*c**2 + b**2*c**2),
True))

```

$$3.34 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx) dx$$

**Optimal.** Leaf size=156

$$\frac{a^2(c+dx)^2}{2d} + \frac{2ab(c+dx)(F^{eg+fgx})^n}{fgn \log(F)} - \frac{2abd(F^{eg+fgx})^n}{f^2g^2n^2 \log^2(F)} + \frac{b^2(c+dx)(F^{eg+fgx})^{2n}}{2fgn \log(F)} - \frac{b^2d(F^{eg+fgx})^{2n}}{4f^2g^2n^2 \log^2(F)}$$

[Out]  $1/2*a^2*(d*x+c)^2/d-2*a*b*d*(F^(f*g*x+e*g))^n/f^2/g^2/n^2/\ln(F)^2-1/4*b^2*d*(F^(f*g*x+e*g))^(2*n)/f^2/g^2/n^2/\ln(F)^2+2*a*b*(F^(f*g*x+e*g))^n*(d*x+c)/f/g/n/\ln(F)+1/2*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f/g/n/\ln(F)$

**Rubi [A]** time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {2183, 2176, 2194}

$$\frac{a^2(c+dx)^2}{2d} + \frac{2ab(c+dx)(F^{eg+fgx})^n}{fgn \log(F)} - \frac{2abd(F^{eg+fgx})^n}{f^2g^2n^2 \log^2(F)} + \frac{b^2(c+dx)(F^{eg+fgx})^{2n}}{2fgn \log(F)} - \frac{b^2d(F^{eg+fgx})^{2n}}{4f^2g^2n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^2\*(c + d\*x), x]

[Out]  $(a^2*(c+d*x)^2)/(2*d) - (2*a*b*d*(F^(e*g+f*g*x))^n)/(f^2*g^2*n^2*\text{Log}[F]^2) - (b^2*d*(F^(e*g+f*g*x))^(2*n))/(4*f^2*g^2*n^2*\text{Log}[F]^2) + (2*a*b*(F^(e*g+f*g*x))^n*(c+d*x))/(f*g*n*\text{Log}[F]) + (b^2*(F^(e*g+f*g*x))^(2*n)*(c+d*x))/(2*f*g*n*\text{Log}[F])$

**Rule 2176**

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

**Rule 2183**

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

**Rule 2194**

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

### Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx) dx &= \int \left( a^2(c + dx) + 2ab \left( F^{eg+fgx} \right)^n (c + dx) + b^2 \left( F^{eg+fgx} \right)^{2n} (c + dx) \right) dx \\ &= \frac{a^2(c + dx)^2}{2d} + (2ab) \int \left( F^{eg+fgx} \right)^n (c + dx) dx + b^2 \int \left( F^{eg+fgx} \right)^{2n} (c + dx) dx \\ &= \frac{a^2(c + dx)^2}{2d} + \frac{2ab \left( F^{eg+fgx} \right)^n (c + dx)}{fgn \log(F)} + \frac{b^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)}{2fgn \log(F)} - \frac{(2abd) \int \left( F^{eg+fgx} \right)^n (c + dx) dx}{fgn} \\ &= \frac{a^2(c + dx)^2}{2d} - \frac{2abd \left( F^{eg+fgx} \right)^n}{f^2g^2n^2 \log^2(F)} - \frac{b^2d \left( F^{eg+fgx} \right)^{2n}}{4f^2g^2n^2 \log^2(F)} + \frac{2ab \left( F^{eg+fgx} \right)^n (c + dx)}{fgn \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 117, normalized size = 0.75

$$\frac{2a^2f^2g^2n^2x \log^2(F)(2c + dx) + 2bfgn \log(F)(c + dx) \left( F^{g(e+fx)} \right)^n \left( 4a + b \left( F^{g(e+fx)} \right)^n \right) - bd \left( F^{g(e+fx)} \right)^n \left( 8a + b \left( F^{g(e+fx)} \right)^n \right)}{4f^2g^2n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^2\*(c + d\*x), x]

[Out]  $(- (b*d*(F^{g*(e + f*x)}))^n*(8*a + b*(F^{g*(e + f*x)}))^n) + 2*b*f*(F^{g*(e + f*x)})^n*(4*a + b*(F^{g*(e + f*x)}))^n*g*n*(c + d*x)*Log[F] + 2*a^2*f^2*g^2*n^2*x*(2*c + d*x)*Log[F]^2)/(4*f^2*g^2*n^2*Log[F]^2)$

**fricas [A]** time = 0.44, size = 139, normalized size = 0.89

$$\frac{2 \left( a^2 d f^2 g^2 n^2 x^2 + 2 a^2 c f^2 g^2 n^2 x \right) \log(F)^2 - \left( b^2 d - 2 \left( b^2 d f g n x + b^2 c f g n \right) \log(F) \right) F^{2 f g n x + 2 e g n} - 8 \left( a b d - \left( a b d f g n \right) \log(F) \right) F^{g n x + e g n}}{4 f^2 g^2 n^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^2\*(d\*x+c), x, algorithm="fricas")

[Out]  $1/4*(2*(a^2*d*f^2*g^2*n^2*x^2 + 2*a^2*c*f^2*g^2*n^2*x)*\log(F)^2 - (b^2*d - 2*(b^2*d*f*g*n*x + b^2*c*f*g*n)*\log(F))*F^{(2*f*g*n*x + 2*e*g*n)} - 8*(a*b*d - (a*b*d*f*g*n*x + a*b*c*f*g*n)*\log(F))*F^{(f*g*n*x + e*g*n)})/(f^2*g^2*n^2*\log(F)^2)$

giac [C] time = 0.98, size = 2312, normalized size = 14.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)^2\*(d\*x+c),x, algorithm="giac")

[Out] 
$$\frac{1}{2}a^2dx^2 + a^2cx + \frac{1}{2} \left( (2(\pi b^2 d f g^n x \operatorname{sgn}(F) - \pi b^2 d f g^n x + \pi b^2 c f g^n \operatorname{sgn}(F) - \pi b^2 c f g^n) (\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F))) / ((\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2)^2 + 4(\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F)))^2 + (\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2) (2 b^2 d f g^n x \log(\operatorname{abs}(F)) + 2 b^2 c f g^n \operatorname{sgn}(F) - b^2 d) / ((\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2)^2 + 4(\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F)))^2) \right) \cos(-\pi f g^n x \operatorname{sgn}(F) + \pi f g^n x - \pi g^n e \operatorname{sgn}(F) + \pi g^n e) + ((\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2) (\pi b^2 d f g^n x \operatorname{sgn}(F) - \pi b^2 d f g^n x + \pi b^2 c f g^n \operatorname{sgn}(F) - \pi b^2 c f g^n) / ((\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2)^2 + 4(\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F)))^2) - 2(\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F))) (2 b^2 d f g^n x \log(\operatorname{abs}(F)) + 2 b^2 c f g^n \log(\operatorname{abs}(F)) - b^2 d) / ((\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2)^2 + 4(\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F)))^2) \right) \sin(-\pi f g^n x \operatorname{sgn}(F) + \pi f g^n x - \pi g^n e \operatorname{sgn}(F) + \pi g^n e) \right) e^{(2 f g^n x \log(\operatorname{abs}(F)) + 2 g^n e \log(\operatorname{abs}(F)))} - \frac{1}{2} I \left( (\pi b^2 d f g^n x \operatorname{sgn}(F) - \pi b^2 d f g^n x - 2 I b^2 d f g^n x \log(\operatorname{abs}(F)) + \pi b^2 c f g^n \operatorname{sgn}(F) - \pi b^2 c f g^n - 2 I b^2 c f g^n \log(\operatorname{abs}(F)) + I b^2 d) e^{(I \pi f g^n x \operatorname{sgn}(F) - I \pi f g^n x + I \pi g^n e \operatorname{sgn}(F) - I \pi g^n e)} / (2 \pi^2 f^2 g^{2n} \operatorname{sgn}(F) + 4 I \pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 \pi^2 f^2 g^{2n} - 4 I \pi f^2 g^{2n} \log(\operatorname{abs}(F)) + 4 f^2 g^{2n} \log(\operatorname{abs}(F))^2) + (\pi b^2 d f g^n x \operatorname{sgn}(F) - \pi b^2 d f g^n x + 2 I b^2 d f g^n x \log(\operatorname{abs}(F)) + \pi b^2 c f g^n \operatorname{sgn}(F) - \pi b^2 c f g^n + 2 I b^2 c f g^n \log(\operatorname{abs}(F)) - I b^2 d) e^{(-I \pi f g^n x \operatorname{sgn}(F) + I \pi f g^n x - I \pi g^n e \operatorname{sgn}(F) + I \pi g^n e)} / (2 \pi^2 f^2 g^{2n} \operatorname{sgn}(F) - 4 I \pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 \pi^2 f^2 g^{2n} + 4 I \pi f^2 g^{2n} \log(\operatorname{abs}(F)) + 4 f^2 g^{2n} \log(\operatorname{abs}(F))^2) \right) e^{(2 f g^n x \log(\operatorname{abs}(F)) + 2 g^n e \log(\operatorname{abs}(F)))} + 2 \left( (2(\pi a b d f g^n x \operatorname{sgn}(F) - \pi a b d f g^n x + \pi a b c f g^n \operatorname{sgn}(F) - \pi a b c f g^n) (\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F))) / ((\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2)^2 + 4(\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F)))^2 + (\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2) (a b d f g^n x \log(\operatorname{abs}(F)) + a b c f g^n \log(\operatorname{abs}(F)) - a b d) / ((\pi^2 f^2 g^{2n} \operatorname{sgn}(F) - \pi^2 f^2 g^{2n} + 2 f^2 g^{2n} \log(\operatorname{abs}(F))^2)^2 + 4(\pi f^2 g^{2n} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi f^2 g^{2n} \log(\operatorname{abs}(F)))^2) \right)$$

$$2n^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi f^2 g^2 n^2 \log(\text{abs}(F))^2) \cos(-1/2 \pi f g n x \text{sgn}(F) + 1/2 \pi f g n x - 1/2 \pi g n e \text{sgn}(F) + 1/2 \pi g n e) + ((\pi^2 f^2 g^2 n^2 \text{sgn}(F) - \pi^2 f^2 g^2 n^2 + 2 f^2 g^2 n^2 \log(\text{abs}(F))^2) (\pi a b d f g n x \text{sgn}(F) - \pi a b d f g n x + \pi a b c f g n \text{sgn}(F) - \pi a b c f g n) / ((\pi^2 f^2 g^2 n^2 \text{sgn}(F) - \pi^2 f^2 g^2 n^2 + 2 f^2 g^2 n^2 \log(\text{abs}(F))^2)^2 + 4 (\pi f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi f^2 g^2 n^2 \log(\text{abs}(F)))^2) - 4 (\pi f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi f^2 g^2 n^2 \log(\text{abs}(F))) (a b d f g n x \log(\text{abs}(F)) + a b c f g n \log(\text{abs}(F)) - a b d) / ((\pi^2 f^2 g^2 n^2 \text{sgn}(F) - \pi^2 f^2 g^2 n^2 + 2 f^2 g^2 n^2 \log(\text{abs}(F))^2)^2 + 4 (\pi f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi f^2 g^2 n^2 \log(\text{abs}(F)))^2) \sin(-1/2 \pi f g n x \text{sgn}(F) + 1/2 \pi f g n x - 1/2 \pi g n e \text{sgn}(F) + 1/2 \pi g n e) e^{(f g n x \log(\text{abs}(F)) + g n e \log(\text{abs}(F)))} - 1/2 I * ((4 \pi a b d f g n x \text{sgn}(F) - 4 \pi a b d f g n x - 8 I a b d f g n x \log(\text{abs}(F)) + 4 \pi a b c f g n \text{sgn}(F) - 4 \pi a b c f g n - 8 I a b c f g n \log(\text{abs}(F)) + 8 I a b d) e^{(1/2 I \pi f g n x \text{sgn}(F) - 1/2 I \pi f g n x + 1/2 I \pi g n e \text{sgn}(F) - 1/2 I \pi g n e) / (2 \pi^2 f^2 g^2 n^2 \text{sgn}(F) + 4 I \pi f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - 2 \pi^2 f^2 g^2 n^2 - 4 I \pi f^2 g^2 n^2 \log(\text{abs}(F)) + 4 f^2 g^2 n^2 \log(\text{abs}(F)))^2} + (4 \pi a b d f g n x \text{sgn}(F) - 4 \pi a b d f g n x + 8 I a b d f g n x \log(\text{abs}(F)) + 4 \pi a b c f g n \text{sgn}(F) - 4 \pi a b c f g n + 8 I a b c f g n \log(\text{abs}(F)) - 8 I a b d) e^{(-1/2 I \pi f g n x \text{sgn}(F) + 1/2 I \pi f g n x - 1/2 I \pi g n e \text{sgn}(F) + 1/2 I \pi g n e) / (2 \pi^2 f^2 g^2 n^2 \text{sgn}(F) - 4 I \pi f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - 2 \pi^2 f^2 g^2 n^2 + 4 I \pi f^2 g^2 n^2 \log(\text{abs}(F)) + 4 f^2 g^2 n^2 \log(\text{abs}(F))^2)} e^{(f g n x \log(\text{abs}(F)) + g n e \log(\text{abs}(F)))}$$

**maple** [A] time = 0.06, size = 220, normalized size = 1.41

$$\frac{a^2 dx^2}{2} + a^2 cx + \frac{2abdx e^{n \ln(e^{(fx+e)g \ln(F)})}}{fgn \ln(F)} + \frac{b^2 dx e^{2n \ln(e^{(fx+e)g \ln(F)})}}{2fgn \ln(F)} + \frac{2abc e^{n \ln(e^{(fx+e)g \ln(F)})}}{fgn \ln(F)} + \frac{b^2 c e^{2n \ln(e^{(fx+e)g \ln(F)})}}{2fgn \ln(F)} - \frac{2abd e^{n \ln(e^{(fx+e)g \ln(F)})}}{f^2 g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^2\*(d\*x+c),x)

[Out]  $a^2 c x + 1/2 a^2 d x^2 + 1/2 b^2/n/f/g/\ln(F) \exp(n \ln(\exp((f*x+e)*g*\ln(F))))^2 * c - 1/4 b^2/n^2/f^2/g^2/\ln(F)^2 \exp(n \ln(\exp((f*x+e)*g*\ln(F))))^2 d + 2 a b/n/f/g/\ln(F) \exp(n \ln(\exp((f*x+e)*g*\ln(F)))) * c - 2 a b/n^2/f^2/g^2/\ln(F)^2 \exp(n \ln(\exp((f*x+e)*g*\ln(F)))) * d + 1/2/n/f/g/\ln(F) * b^2 * d * x * \exp(n \ln(\exp((f*x+e)*g*\ln(F))))^2 + 2/n/f/g/\ln(F) * a * b * d * x * \exp(n \ln(\exp((f*x+e)*g*\ln(F))))$

**maxima** [A] time = 1.25, size = 187, normalized size = 1.20

$$\frac{1}{2} a^2 dx^2 + a^2 cx + \frac{2 (F^{fgx+eg})^n abc}{fgn \log(F)} + \frac{(F^{fgx+eg})^{2n} b^2 c}{2 fgn \log(F)} + \frac{2 ((F^{eg})^n fgnx \log(F) - (F^{eg})^n) (F^{fgx})^n abd}{f^2 g^2 n^2 \log(F)^2} + \frac{(2 (F^{eg})^{2n} fgnx \log(F) - (F^{eg})^{2n})}{4 f^2 g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c),x, algorithm="maxima")

[Out]  $\frac{1}{2}a^2d^2x^2 + a^2c^2x + 2*(F^{(f*gx + e*g)})^n*a*b*c/(f*g*n*\log(F)) + \frac{1}{2}*(F^{(f*gx + e*g)})^{(2*n)}*b^2*c/(f*g*n*\log(F)) + 2*((F^{(e*g)})^n*f*g*n*x*\log(F) - (F^{(e*g)})^n*(F^{(f*gx)}))^{(2*n)}*a*b*d/(f^2*g^2*n^2*\log(F)^2) + \frac{1}{4}*(2*(F^{(e*g)})^{(2*n)}*f*g*n*x*\log(F) - (F^{(e*g)})^{(2*n)}*(F^{(f*gx)}))^{(2*n)}*b^2*d/(f^2*g^2*n^2*\log(F)^2)$

**mupad [B]** time = 3.67, size = 146, normalized size = 0.94

$$\frac{a^2 d x^2}{2} - (F^{f g x} F^{e g})^{2n} \left( \frac{b^2 (d - 2 c f g n \ln(F))}{4 f^2 g^2 n^2 \ln(F)^2} - \frac{b^2 d x}{2 f g n \ln(F)} \right) - (F^{f g x} F^{e g})^n \left( \frac{2 a b (d - c f g n \ln(F))}{f^2 g^2 n^2 \ln(F)^2} - \frac{2 a b}{f g n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x),x

[Out]  $(a^2*d*x^2)/2 - (F^{(f*gx)}*F^{(e*g)})^{(2*n)}*((b^2*(d - 2*c*f*g*n*\log(F)))/(4*f^2*g^2*n^2*\log(F)^2) - (b^2*d*x)/(2*f*g*n*\log(F))) - (F^{(f*gx)}*F^{(e*g)})^n*((2*a*b*(d - c*f*g*n*\log(F)))/(f^2*g^2*n^2*\log(F)^2) - (2*a*b*d*x)/(f*g*n*\log(F))) + a^2*c*x$

**sympy [A]** time = 0.27, size = 233, normalized size = 1.49

$$a^2cx + \frac{a^2dx^2}{2} + \left\{ \begin{array}{l} \frac{(2b^2cf^3g^3n^3 \log(F)^3 + 2b^2df^3g^3n^3x \log(F)^3 - b^2df^2g^2n^2 \log(F)^2)(F^{g(e+fx)})^{2n} + (8abcf^3g^3n^3 \log(F)^3 + 8abdf^3g^3n^3x \log(F)^3 - 8a^2b^2c^2f^2g^2n^2 \log(F)^2)}{4f^4g^4n^4 \log(F)^4} \\ x^2 \left( abd + \frac{b^2d}{2} \right) + x(2abc + b^2c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*2\*(d\*x+c),x

[Out]  $a**2*c*x + a**2*d*x**2/2 + \text{Piecewise}(\left( \left( (2*b**2*c*f**3*g**3*n**3*\log(F)**3 + 2*b**2*d*f**3*g**3*n**3*x*\log(F)**3 - b**2*d*f**2*g**2*n**2*\log(F)**2) \right) * (F**(g*(e + f*x)))^{(2*n)} + (8*a*b*c*f**3*g**3*n**3*\log(F)**3 + 8*a*b*d*f**3*g**3*n**3*x*\log(F)**3 - 8*a*b*d*f**2*g**2*n**2*\log(F)**2) * (F**(g*(e + f*x)))^{(2*n)} \right) / (4*f**4*g**4*n**4*\log(F)**4), \text{Ne}(4*f**4*g**4*n**4*\log(F)**4, 0)), (x**2*(a*b*d + b**2*d/2) + x*(2*a*b*c + b**2*c), \text{True}))$

$$3.35 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 dx$$

Optimal. Leaf size=67

$$a^2x + \frac{2ab \left( F^{g(e+fx)} \right)^n}{fgn \log(F)} + \frac{b^2 \left( F^{g(e+fx)} \right)^{2n}}{2fgn \log(F)}$$

[Out]  $a^2x + 2ab(F^{g(f*x+e)})^n/f/g/n/\ln(F) + 1/2*b^2*(F^{g(f*x+e)})^{(2*n)}/f/g/n/\ln(F)$

**Rubi [A]** time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2282, 266, 43}

$$a^2x + \frac{2ab \left( F^{g(e+fx)} \right)^n}{fgn \log(F)} + \frac{b^2 \left( F^{g(e+fx)} \right)^{2n}}{2fgn \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]^2,x]

[Out]  $a^2x + (2*a*b*(F^{g*(e + f*x)})^n)/(f*g*n*\text{Log}[F]) + (b^2*(F^{g*(e + f*x)})^{(2*n)})/(2*f*g*n*\text{Log}[F])$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*



(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 dx &= \frac{\text{Subst} \left( \int \frac{(a+bx)^2}{x} dx, x, F^{g(e+fx)} \right)}{fg \log(F)} \\ &= \frac{\text{Subst} \left( \int \frac{(a+bx)^2}{x} dx, x, \left( F^{g(e+fx)} \right)^n \right)}{fgn \log(F)} \\ &= \frac{\text{Subst} \left( \int \left( 2ab + \frac{a^2}{x} + b^2x \right) dx, x, \left( F^{g(e+fx)} \right)^n \right)}{fgn \log(F)} \\ &= a^2x + \frac{2ab \left( F^{g(e+fx)} \right)^n}{fgn \log(F)} + \frac{b^2 \left( F^{g(e+fx)} \right)^{2n}}{2fgn \log(F)} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 52, normalized size = 0.78

$$a^2x + \frac{b \left( F^{g(e+fx)} \right)^n \left( 4a + b \left( F^{g(e+fx)} \right)^n \right)}{2fgn \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^2, x]

[Out] a^2\*x + (b\*(F^(g\*(e + f\*x)))^n\*(4\*a + b\*(F^(g\*(e + f\*x)))^n))/(2\*f\*g\*n\*Log[F])

**fricas** [A] time = 0.43, size = 61, normalized size = 0.91

$$\frac{2a^2fgnx \log(F) + 4Ffg^{nx+egn}ab + F^2fg^{nx+2egn}b^2}{2fgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)^2,x, algorithm="fricas")

[Out] 1/2\*(2\*a^2\*f\*g\*n\*x\*log(F) + 4\*F^(f\*g\*n\*x + e\*g\*n)\*a\*b + F^(2\*f\*g\*n\*x + 2\*e\*g\*n)\*b^2)/(f\*g\*n\*log(F))

**giac** [A] time = 0.49, size = 77, normalized size = 1.15

$$\frac{4 F f g^{n x} F g^{n e} a b + F^2 f g^{n x} F^2 g^{n e} b^2 + 2 a^2 \log(|F|^{f g^{n x}} |F|^{g^{n e}})}{2 f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="giac")

[Out] 1/2\*(4\*F^(f\*g\*n\*x)\*F^(g\*n\*e)\*a\*b + F^(2\*f\*g\*n\*x)\*F^(2\*g\*n\*e)\*b^2 + 2\*a^2\*log(abs(F)^(f\*g\*n\*x)\*abs(F)^(g\*n\*e)))/(f\*g\*n\*log(F))

**maple** [A] time = 0.02, size = 90, normalized size = 1.34

$$\frac{a^2 \ln\left(\left(F^{(f x+e) g}\right)^n\right)}{f g n \ln(F)} + \frac{2 a b \left(F^{(f x+e) g}\right)^n}{f g n \ln(F)} + \frac{b^2 \left(F^{(f x+e) g}\right)^{2 n}}{2 f g n \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g)))^n+a)^2,x)

[Out] 1/2/g/f/ln(F)/n\*b^2\*((F^((f\*x+e)\*g))^n)^2+2\*a\*b\*(F^((f\*x+e)\*g))^n/f/g/n/ln(F)+1/g/f/ln(F)/n\*a^2\*ln((F^((f\*x+e)\*g))^n)

**maxima** [A] time = 1.19, size = 75, normalized size = 1.12

$$a^2 x + \frac{2 (F f g^x)^n (F e g)^n a b}{f g n \log(F)} + \frac{(F f g^x)^{2 n} (F e g)^{2 n} b^2}{2 f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="maxima")

[Out] a^2\*x + 2\*(F^(f\*g\*x))^n\*(F^(e\*g))^n\*a\*b/(f\*g\*n\*log(F)) + 1/2\*(F^(f\*g\*x))^(2\*n)\*(F^(e\*g))^(2\*n)\*b^2/(f\*g\*n\*log(F))

**mupad** [B] time = 3.72, size = 75, normalized size = 1.12

$$\frac{b^2 (F^{e+f g x})^{2 n}}{2} + \frac{2 a b (F^{e+f g x})^n}{f g n \ln(F)} + \frac{a^2 \ln\left(F^{g(e+f x)}\right)}{f g \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^2,x)

```
[Out] ((b^2*(F^(e*g + f*g*x))^(2*n))/2 + 2*a*b*(F^(e*g + f*g*x))^n)/(f*g*n*log(F)
) + (a^2*log(F^(g*(e + f*x))))/(f*g*log(F))
```

sympy [A] time = 0.18, size = 94, normalized size = 1.40

$$a^2x + \begin{cases} \frac{4abfgn\left(F^{g(e+fx)}\right)^n \log(F) + b^2fgn\left(F^{g(e+fx)}\right)^{2n} \log(F)}{2f^2g^2n^2 \log(F)^2} & \text{for } 2f^2g^2n^2 \log(F)^2 \neq 0 \\ x(2ab + b^2) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(F**(g*(f*x+e))))**n)**2,x)
```

```
[Out] a**2*x + Piecewise(((4*a*b*f*g*n*(F**(g*(e + f*x))))**n*log(F) + b**2*f*g*n*
(F**(g*(e + f*x)))**2*n*log(F))/(2*f**2*g**2*n**2*log(F)**2), Ne(2*f**2*g
**2*n**2*log(F)**2, 0)), (x*(2*a*b + b**2), True))
```

$$3.36 \quad \int \frac{\left(a + b(F^{g(e+fx)})^n\right)^2}{c+dx} dx$$

**Optimal.** Leaf size=134

$$\frac{a^2 \log(c+dx)}{d} + \frac{2ab(F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{b^2(F^{eg+fgx})^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$

[Out]  $2*a*b*F^{((e-c*f/d)*g*n-g*n*(f*x+e))*(F^{(f*g*x+e*g)})^n*Ei(f*g*n*(d*x+c)*\ln(F)/d)/d+b^2*F^{(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^{(f*g*x+e*g)})^{(2*n)*Ei(2*f*g*n*(d*x+c)*\ln(F)/d)/d+a^2*\ln(d*x+c)/d}$

**Rubi [A]** time = 0.25, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2183, 2182, 2178}

$$\frac{a^2 \log(c+dx)}{d} + \frac{2ab(F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{b^2(F^{eg+fgx})^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^2/(c + d\*x), x]

[Out]  $(2*a*b*F^{((e-(c*f)/d)*g*n-g*n*(e+f*x))*(F^{(e*g+f*g*x)})^n*ExpIntegralEi[(f*g*n*(c+d*x)*Log[F])/d])/d + (b^2*F^{(2*(e-(c*f)/d)*g*n-2*g*n*(e+f*x))*(F^{(e*g+f*g*x)})^{(2*n)*ExpIntegralEi[(2*f*g*n*(c+d*x)*Log[F])/d]})/d + (a^2*Log[c+d*x])/d$

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2182

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(b\*F^(g\*(e + f\*x)))^n/F^(g\*n\*(e + f\*x)), Int[(c + d\*x)^m\*F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

#### Rule 2183

Int[((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p], x]

$\int (g*(e + f*x))^n dx$ ,  $x$  /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&  
IGtQ[p, 0]

### Rubi steps

$$\begin{aligned} \int \frac{(a + b(F^{g(e+fx)})^n)^2}{c + dx} dx &= \int \left( \frac{a^2}{c + dx} + \frac{2ab(F^{eg+fgx})^n}{c + dx} + \frac{b^2(F^{eg+fgx})^{2n}}{c + dx} \right) dx \\ &= \frac{a^2 \log(c + dx)}{d} + (2ab) \int \frac{(F^{eg+fgx})^n}{c + dx} dx + b^2 \int \frac{(F^{eg+fgx})^{2n}}{c + dx} dx \\ &= \frac{a^2 \log(c + dx)}{d} + \left( 2abF^{-n(eg+fgx)} (F^{eg+fgx})^n \right) \int \frac{F^{n(eg+fgx)}}{c + dx} dx + \left( b^2F^{-2n(eg+fgx)} (F^{eg+fgx})^{2n} \right) \int \frac{F^{2n(eg+fgx)}}{c + dx} dx \\ &= \frac{2abF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)} (F^{eg+fgx})^n \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + b^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)} (F^{eg+fgx})^{2n} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 108, normalized size = 0.81

$$\frac{a^2 \log(c + dx) + 2ab (F^{g(e+fx)})^n F^{-\frac{fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + b^2 (F^{g(e+fx)})^{2n} F^{-\frac{2fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^2/(c + d\*x), x]

[Out] ((2\*a\*b\*(F^(g\*(e + f\*x)))^n\*ExpIntegralEi[(f\*g\*n\*(c + d\*x)\*Log[F])/d])/F^((f\*g\*n\*(c + d\*x))/d) + (b^2\*(F^(g\*(e + f\*x)))^(2\*n)\*ExpIntegralEi[(2\*f\*g\*n\*(c + d\*x)\*Log[F])/d])/F^((2\*f\*g\*n\*(c + d\*x))/d) + a^2\*Log[c + d\*x])/d

**fricas [A]** time = 0.42, size = 95, normalized size = 0.71

$$\frac{F^{\frac{2(de-cf)gn}{d}} b^2 \operatorname{Ei}\left(\frac{2(dfgnx+cfgn)\log(F)}{d}\right) + 2F^{\frac{(de-cf)gn}{d}} ab \operatorname{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right) + a^2 \log(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)^2/(d\*x+c), x, algorithm="fricas")

[Out] (F^(2\*(d\*e - c\*f)\*g\*n/d)\*b^2\*Ei(2\*(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + 2\*F^((d\*e - c\*f)\*g\*n/d)\*a\*b\*Ei((d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + a^2\*log(d\*x + c))/d

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left( \left( F^{(f^{x+e})g} \right)^n b + a \right)^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c),x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^2/(d\*x + c), x)

**maple** [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left( b \left( F^{(f^{x+e})g} \right)^n + a \right)^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c),x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^{2n} b^2 \int \frac{(F^{fgx})^{2n}}{dx + c} dx + 2 (F^{eg})^n ab \int \frac{(F^{fgx})^n}{dx + c} dx + \frac{a^2 \log(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c),x, algorithm="maxima")

[Out] (F^(e\*g))^(2\*n)\*b^2\*integrate((F^(f\*g\*x))^(2\*n)/(d\*x + c), x) + 2\*(F^(e\*g))  
^n\*a\*b\*integrate((F^(f\*g\*x))^n/(d\*x + c), x) + a^2\*log(d\*x + c)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left( a + b \left( F^{g(e+fx)} \right)^n \right)^2}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^2/(c + d\*x),x)

[Out] `int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \left(F^{eg} F^{fgx}\right)^n\right)^2}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c), x)`

[Out] `Integral((a + b*(F**(e*g)*F**(f*g*x)))**n)**2/(c + d*x), x)`

$$3.37 \quad \int \frac{\left(a + b(F^{g(e+fx)})^n\right)^2}{(c+dx)^2} dx$$

**Optimal.** Leaf size=202

$$-\frac{a^2}{d(c+dx)} + \frac{2abfgn \log(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{2ab(F^{eg+fgx})^n}{d(c+dx)} + \frac{2b^2fgn \log(F) (F^{eg+fgx})^{2n}}{d(c+dx)}$$

[Out]  $-a^2/d/(d*x+c) - 2*a*b*(F^(f*g*x+e*g))^n/d/(d*x+c) - b^2*(F^(f*g*x+e*g))^(2*n)/d/(d*x+c) + 2*a*b*f*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g*n*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2 + 2*b^2*f*g*n*(f*x+e)*(F^(f*g*x+e*g))^(2*n)*g*n*Ei(2*f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2$

**Rubi [A]** time = 0.34, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {2183, 2177, 2182, 2178}

$$-\frac{a^2}{d(c+dx)} + \frac{2abfgn \log(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{2ab(F^{eg+fgx})^n}{d(c+dx)} + \frac{2b^2fgn \log(F) (F^{eg+fgx})^{2n}}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^2/(c + d\*x)^2, x]

[Out]  $-(a^2/(d*(c + d*x))) - (2*a*b*(F^(e*g + f*g*x))^n/(d*(c + d*x)) - (b^2*(F^(e*g + f*g*x))^(2*n))/(d*(c + d*x)) + (2*a*b*f*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*g*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2 + (2*b^2*f*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*g*n*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2$

**Rule 2177**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n)/(d\*(m + 1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

**Rule 2178**

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; F



```
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2182

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

### Rule 2183

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b(F^{g(e+fx)})^n)^2}{(c + dx)^2} dx &= \int \left( \frac{a^2}{(c + dx)^2} + \frac{2ab(F^{eg+fgx})^n}{(c + dx)^2} + \frac{b^2(F^{eg+fgx})^{2n}}{(c + dx)^2} \right) dx \\
 &= -\frac{a^2}{d(c + dx)} + (2ab) \int \frac{(F^{eg+fgx})^n}{(c + dx)^2} dx + b^2 \int \frac{(F^{eg+fgx})^{2n}}{(c + dx)^2} dx \\
 &= -\frac{a^2}{d(c + dx)} - \frac{2ab(F^{eg+fgx})^n}{d(c + dx)} - \frac{b^2(F^{eg+fgx})^{2n}}{d(c + dx)} + \frac{(2abfgn \log(F)) \int \frac{(F^{eg+fgx})^n}{c+dx} dx}{d} + \dots \\
 &= -\frac{a^2}{d(c + dx)} - \frac{2ab(F^{eg+fgx})^n}{d(c + dx)} - \frac{b^2(F^{eg+fgx})^{2n}}{d(c + dx)} + \frac{(2abfF^{-n(eg+fgx)}(F^{eg+fgx})^n g n \log(F))}{d} \\
 &= -\frac{a^2}{d(c + dx)} - \frac{2ab(F^{eg+fgx})^n}{d(c + dx)} - \frac{b^2(F^{eg+fgx})^{2n}}{d(c + dx)} + \frac{2abfF^{\left(\frac{e-cf}{d}\right)gn-gn(e+fx)}(F^{eg+fgx})^n g}{d^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.68, size = 136, normalized size = 0.67

$$\frac{2abfgn \log(F) (F^{g(e+fx)})^n F^{-\frac{fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{fgn(c+dx) \log(F)}{d}\right) - \frac{d(a+b(F^{g(e+fx)})^n)^2}{c+dx} + 2b^2fgn \log(F) (F^{g(e+fx)})^{2n} F^{-\frac{2fgn(c+dx)}{d}}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n]^2/(c + d\*x)^2,x]

[Out] -((d\*(a + b\*(F^(g\*(e + f\*x))))^n)^2/(c + d\*x)) + (2\*a\*b\*f\*(F^(g\*(e + f\*x)))^n\*g\*n\*ExpIntegralEi[(f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F])/F^((f\*g\*n\*(c + d\*x))/d) + (2\*b^2\*f\*(F^(g\*(e + f\*x)))^(2\*n)\*g\*n\*ExpIntegralEi[(2\*f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F])/F^((2\*f\*g\*n\*(c + d\*x))/d)/d^2

**fricas** [A] time = 0.42, size = 171, normalized size = 0.85

$$\frac{2 F f g n x + e g n a b d + F^2 f g n x + 2 e g n b^2 d - 2 (b^2 d f g n x + b^2 c f g n) F^{\frac{2(d e - c f) g n}{d}} \operatorname{Ei}\left(\frac{2(d f g n x + c f g n) \log(F)}{d}\right) \log(F) - 2(a b d f g n x}{d^3 x + c d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c)^2,x, algorithm="fricas")

[Out] -(2\*F^(f\*g\*n\*x + e\*g\*n)\*a\*b\*d + F^(2\*f\*g\*n\*x + 2\*e\*g\*n)\*b^2\*d - 2\*(b^2\*d\*f\*g\*n\*x + b^2\*c\*f\*g\*n)\*F^(2\*(d\*e - c\*f)\*g\*n/d)\*Ei(2\*(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d)\*log(F) - 2\*(a\*b\*d\*f\*g\*n\*x + a\*b\*c\*f\*g\*n)\*F^((d\*e - c\*f)\*g\*n/d)\*Ei((d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d)\*log(F) + a^2\*d)/(d^3\*x + c\*d^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(F^{(f x + e) g}\right)^n b + a\right)^2}{(d x + c)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^2/(d\*x + c)^2, x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\left(F^{(f x + e) g}\right)^n + a\right)^2}{(d x + c)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c)^2,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^{2n} b^2 \int \frac{(F^{fgx})^{2n}}{d^2 x^2 + 2cdx + c^2} dx + 2(F^{eg})^n ab \int \frac{(F^{fgx})^n}{d^2 x^2 + 2cdx + c^2} dx - \frac{a^2}{d^2 x + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c)^2,x, algorithm="maxima")

[Out] (F^(e\*g))^(2\*n)\*b^2\*integrate((F^(f\*g\*x))^(2\*n)/(d^2\*x^2 + 2\*c\*d\*x + c^2), x) + 2\*(F^(e\*g))^n\*a\*b\*integrate((F^(f\*g\*x))^n/(d^2\*x^2 + 2\*c\*d\*x + c^2), x) - a^2/(d^2\*x + c\*d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^2/(c + d\*x)^2,x)

[Out] int((a + b\*(F^(g\*(e + f\*x))))^n)^2/(c + d\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \left(F^{eg} F^{fgx}\right)^n\right)^2}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*2/(d\*x+c)\*\*2,x)

[Out] Integral((a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))))\*\*n)\*\*2/(c + d\*x)\*\*2, x)

$$3.38 \quad \int \frac{\left(a + b(F^g(e+fx))^n\right)^2}{(c+dx)^3} dx$$

Optimal. Leaf size=286

$$-\frac{a^2}{2d(c+dx)^2} + \frac{abf^2g^2n^2 \log^2(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^3} - \frac{abfgn \log(F) (F^{eg+fgx})^n}{d^2(c+dx)} - \frac{ab(F^{eg+fgx})^n}{d(c+dx)}$$

[Out]  $-1/2*a^2/d/(d*x+c)^2-a*b*(F^(f*g*x+e*g))^n/d/(d*x+c)^2-1/2*b^2*(F^(f*g*x+e*g))^(2*n)/d/(d*x+c)^2-a*b*f*(F^(f*g*x+e*g))^n*g*n*\ln(F)/d^2/(d*x+c)-b^2*f*(F^(f*g*x+e*g))^(2*n)*g*n*\ln(F)/d^2/(d*x+c)+a*b*f^2*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g^2*n^2*Ei(f*g*n*(d*x+c)*\ln(F)/d)*\ln(F)^2/d^3+2*b^2*f^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g^2*n^2*Ei(2*f*g*n*(d*x+c)*\ln(F)/d)*\ln(F)^2/d^3$

**Rubi [A]** time = 0.47, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {2183, 2177, 2182, 2178}

$$-\frac{a^2}{2d(c+dx)^2} + \frac{abf^2g^2n^2 \log^2(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^3} - \frac{abfgn \log(F) (F^{eg+fgx})^n}{d^2(c+dx)} - \frac{ab(F^{eg+fgx})^n}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]^2/(c + d\*x)^3, x]

[Out]  $-a^2/(2*d*(c+d*x)^2) - (a*b*(F^(e*g+f*g*x))^n)/(d*(c+d*x)^2) - (b^2*(F^(e*g+f*g*x))^(2*n))/(2*d*(c+d*x)^2) - (a*b*f*(F^(e*g+f*g*x))^n*g*n*\operatorname{Log}[F])/(d^2*(c+d*x)) - (b^2*f*(F^(e*g+f*g*x))^(2*n)*g*n*\operatorname{Log}[F])/(d^2*(c+d*x)) + (a*b*f^2*F^((e-(c*f)/d)*g*n-g*n*(e+f*x))*(F^(e*g+f*g*x))^n*g^2*n^2*\operatorname{ExpIntegralEi}[(f*g*n*(c+d*x)*\operatorname{Log}[F])/d]*\operatorname{Log}[F]^2)/d^3 + (2*b^2*f^2*F^(2*(e-(c*f)/d)*g*n-2*g*n*(e+f*x))*(F^(e*g+f*g*x))^(2*n)*g^2*n^2*\operatorname{ExpIntegralEi}[(2*f*g*n*(c+d*x)*\operatorname{Log}[F])/d]*\operatorname{Log}[F]^2)/d^3$

Rule 2177

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.)+(f\_.)\*(x\_))))^(n\_.)\*((c\_.)+(d\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((c+d\*x)^(m+1)\*(b\*F^(g\*(e+f\*x)))^n)/(d\*(m+1)), x] - Dist[(f\*g\*n\*Log[F])/(d\*(m+1)), Int[(c+d\*x)^(m+1)\*(b\*F^(g\*(e+f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2182

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] := Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2183

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3} dx &= \int \left( \frac{a^2}{(c+dx)^3} + \frac{2ab \left(F^{eg+fgx}\right)^n}{(c+dx)^3} + \frac{b^2 \left(F^{eg+fgx}\right)^{2n}}{(c+dx)^3} \right) dx \\
&= -\frac{a^2}{2d(c+dx)^2} + (2ab) \int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^3} dx + b^2 \int \frac{\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^3} dx \\
&= -\frac{a^2}{2d(c+dx)^2} - \frac{ab \left(F^{eg+fgx}\right)^n}{d(c+dx)^2} - \frac{b^2 \left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} + \frac{(abfgn \log(F)) \int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^2} dx}{d} + \dots \\
&= -\frac{a^2}{2d(c+dx)^2} - \frac{ab \left(F^{eg+fgx}\right)^n}{d(c+dx)^2} - \frac{b^2 \left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{abf \left(F^{eg+fgx}\right)^n gn \log(F)}{d^2(c+dx)} - \frac{b^2 f \left(F^{eg+fgx}\right)^{2n} gn \log(F)}{d^2(c+dx)} \\
&= -\frac{a^2}{2d(c+dx)^2} - \frac{ab \left(F^{eg+fgx}\right)^n}{d(c+dx)^2} - \frac{b^2 \left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{abf \left(F^{eg+fgx}\right)^n gn \log(F)}{d^2(c+dx)} - \frac{b^2 f \left(F^{eg+fgx}\right)^{2n} gn \log(F)}{d^2(c+dx)} \\
&= -\frac{a^2}{2d(c+dx)^2} - \frac{ab \left(F^{eg+fgx}\right)^n}{d(c+dx)^2} - \frac{b^2 \left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{abf \left(F^{eg+fgx}\right)^n gn \log(F)}{d^2(c+dx)} - \frac{b^2 f \left(F^{eg+fgx}\right)^{2n} gn \log(F)}{d^2(c+dx)}
\end{aligned}$$

**Mathematica [A]** time = 0.73, size = 217, normalized size = 0.76

$$a^2d^2 - 2abf^2g^2n^2 \log^2(F)(c + dx)^2 \left(F^{g(e+fx)}\right)^n F^{-\frac{fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + 2abd \left(F^{g(e+fx)}\right)^n (fgn \log(F)(c + d$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n/(c + d\*x)^3,x]

[Out]  $-\frac{1}{2}(a^2d^2 - (2abf^2g^2n^2(F^{g(e+fx)})^n)^2(c+dx)^2 \operatorname{ExpIntegralEi}[\frac{(f*g*n*(c+dx)*\log[F])}{d}*\log[F]^2]/F^{((f*g*n*(c+dx))/d)} - (4b^2f^2(F^{g(e+fx)})^{2n})^2(c+dx)^2 \operatorname{ExpIntegralEi}[\frac{(2f*g*n*(c+dx)*\log[F])}{d}*\log[F]^2]/F^{((2f*g*n*(c+dx))/d)} + 2a*b*d*(F^{g(e+fx)})^n(d+f*g*n*(c+dx)*\log[F]) + b^2*d*(F^{g(e+fx)})^{2n}(d+2f*g*n*(c+dx)*\log[F]))/(d^3*(c+dx)^2)$

**fricas [A]** time = 0.46, size = 314, normalized size = 1.10

$$4(b^2d^2f^2g^2n^2x^2 + 2b^2cdf^2g^2n^2x + b^2c^2f^2g^2n^2)F^{\frac{2(de-cf)gn}{d}} \operatorname{Ei}\left(\frac{2(dfgnx+cfgn)\log(F)}{d}\right) \log(F)^2 + 2(abd^2f^2g^2n^2x^2 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n/(d\*x+c)^3,x, algorithm="fricas")

[Out]  $\frac{1}{2}(4(b^2d^2f^2g^2n^2x^2 + 2b^2c*d*f^2g^2n^2x + b^2c^2f^2g^2n^2)*F^{(2*(d*e - c*f)*g*n/d)}*Ei(2*(d*f*g*n*x + c*f*g*n)*\log(F)/d)*\log(F)^2 + 2*(a*b*d^2f^2g^2n^2x^2 + 2*a*b*c*d*f^2g^2n^2x + a*b*c^2f^2g^2n^2)*F^{((d*e - c*f)*g*n/d)}*Ei((d*f*g*n*x + c*f*g*n)*\log(F)/d)*\log(F)^2 - a^2*d^2 - (b^2*d^2 + 2*(b^2*d^2*f*g*n*x + b^2*c*d*f*g*n)*\log(F))*F^{(2*f*g*n*x + 2*e*g*n)} - 2*(a*b*d^2 + (a*b*d^2*f*g*n*x + a*b*c*d*f*g*n)*\log(F))*F^{(f*g*n*x + e*g*n)})/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(F^{(fx+e)g}\right)^n b + a\right)^2}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n/(d\*x+c)^3,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^2/(d\*x + c)^3, x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b \left(F^{(f x+e) g}\right)^n + a\right)^2}{(d x+c)^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c)^3,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^{2n} b^2 \int \frac{(F^{fgx})^{2n}}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3} dx + 2 (F^{eg})^n a b \int \frac{(F^{fgx})^n}{d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3} dx - \frac{a^2}{2 (d^3 x^2 + 2 c d^2 x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c)^3,x, algorithm="maxima")

[Out] (F^(e\*g))^(2\*n)\*b^2\*integrate((F^(f\*g\*x))^(2\*n)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x) + 2\*(F^(e\*g))^n\*a\*b\*integrate((F^(f\*g\*x))^n/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x) - 1/2\*a^2/(d^3\*x^2 + 2\*c\*d^2\*x + c^2\*d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \left(F^{g(e+f x)}\right)^n\right)^2}{(c+d x)^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^2/(c + d\*x)^3,x)

[Out] int((a + b\*(F^(g\*(e + f\*x))))^n)^2/(c + d\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \left(F^{eg} F^{fgx}\right)^n\right)^2}{(c+d x)^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*2/(d\*x+c)\*\*3,x)

[Out] Integral((a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))))\*\*n)\*\*2/(c + d\*x)\*\*3, x)

$$3.39 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^3 dx$$

**Optimal.** Leaf size=496

$$\frac{a^3(c+dx)^4}{4d} + \frac{18a^2bd^2(c+dx)(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{9a^2bd(c+dx)^2(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{3a^2b(c+dx)^3(F^{eg+fgx})^n}{fgn\log(F)} - \frac{18a^2bd^3(F^{eg+fgx})^n}{f^4g^4n^4\log^4(F)}$$

[Out]  $\frac{1}{4}a^3(d*x+c)^4/d - 18a^2b*d^3*(F^{(f*g*x+e*g)})^n/f^4/g^4/n^4/\ln(F)^4 - 9/8*a*b^2*d^3*(F^{(f*g*x+e*g)})^{(2*n)}/f^4/g^4/n^4/\ln(F)^4 - 2/27*b^3*d^3*(F^{(f*g*x+e*g)})^{(3*n)}/f^4/g^4/n^4/\ln(F)^4 + 18a^2*b*d^2*(F^{(f*g*x+e*g)})^n*(d*x+c)/f^3/g^3/n^3/\ln(F)^3 + 2/9*b^3*d^2*(F^{(f*g*x+e*g)})^{(3*n)}*(d*x+c)/f^3/g^3/n^3/\ln(F)^3 - 9a^2*b*d*(F^{(f*g*x+e*g)})^n*(d*x+c)^2/f^2/g^2/n^2/\ln(F)^2 - 9/4*a*b^2*d*(F^{(f*g*x+e*g)})^{(2*n)}*(d*x+c)^2/f^2/g^2/n^2/\ln(F)^2 - 1/3*b^3*d*(F^{(f*g*x+e*g)})^{(3*n)}*(d*x+c)^2/f^2/g^2/n^2/\ln(F)^2 + 3a^2*b*(F^{(f*g*x+e*g)})^n*(d*x+c)^3/f/g/n/\ln(F) + 3/2*a*b^2*(F^{(f*g*x+e*g)})^{(2*n)}*(d*x+c)^3/f/g/n/\ln(F) + 1/3*b^3*(F^{(f*g*x+e*g)})^{(3*n)}*(d*x+c)^3/f/g/n/\ln(F)$

**Rubi [A]** time = 0.71, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2183, 2176, 2194}

$$\frac{18a^2bd^2(c+dx)(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{9a^2bd(c+dx)^2(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{3a^2b(c+dx)^3(F^{eg+fgx})^n}{fgn\log(F)} - \frac{18a^2bd^3(F^{eg+fgx})^n}{f^4g^4n^4\log^4(F)} + \frac{a^3(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^3\*(c + d\*x)^3,x]

[Out]  $(a^3*(c + d*x)^4)/(4*d) - (18*a^2*b*d^3*(F^{(e*g + f*g*x)})^n)/(f^4*g^4*n^4*\text{Log}[F]^4) - (9*a*b^2*d^3*(F^{(e*g + f*g*x)})^{(2*n)})/(8*f^4*g^4*n^4*\text{Log}[F]^4) - (2*b^3*d^3*(F^{(e*g + f*g*x)})^{(3*n)})/(27*f^4*g^4*n^4*\text{Log}[F]^4) + (18*a^2*b*d^2*(F^{(e*g + f*g*x)})^n*(c + d*x))/(f^3*g^3*n^3*\text{Log}[F]^3) + (9*a*b^2*d^2*(F^{(e*g + f*g*x)})^{(2*n)}*(c + d*x))/(4*f^3*g^3*n^3*\text{Log}[F]^3) + (2*b^3*d^2*(F^{(e*g + f*g*x)})^{(3*n)}*(c + d*x))/(9*f^3*g^3*n^3*\text{Log}[F]^3) - (9*a^2*b*d*(F^{(e*g + f*g*x)})^n*(c + d*x)^2)/(f^2*g^2*n^2*\text{Log}[F]^2) - (9*a*b^2*d*(F^{(e*g + f*g*x)})^{(2*n)}*(c + d*x)^2)/(4*f^2*g^2*n^2*\text{Log}[F]^2) - (b^3*d*(F^{(e*g + f*g*x)})^{(3*n)}*(c + d*x)^2)/(3*f^2*g^2*n^2*\text{Log}[F]^2) + (3*a^2*b*(F^{(e*g + f*g*x)})^n*(c + d*x)^3)/(f*g*n*\text{Log}[F]) + (3*a*b^2*(F^{(e*g + f*g*x)})^{(2*n)}*(c + d*x)^3)/(2*f*g*n*\text{Log}[F]) + (b^3*(F^{(e*g + f*g*x)})^{(3*n)}*(c + d*x)^3)/(3*f*g*n*\text{Log}[F])$

Rule 2176



```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma === True
```

### Rule 2183

```
Int[((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^((p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

### Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^3 dx &= \int \left( a^3(c + dx)^3 + 3a^2b \left( F^{eg+fgx} \right)^n (c + dx)^3 + 3ab^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)^3 + b^3 \left( F^{eg+fgx} \right)^{3n} (c + dx)^3 \right) dx \\
 &= \frac{a^3(c + dx)^4}{4d} + (3a^2b) \int \left( F^{eg+fgx} \right)^n (c + dx)^3 dx + (3ab^2) \int \left( F^{eg+fgx} \right)^{2n} (c + dx)^3 dx + b^3 \int \left( F^{eg+fgx} \right)^{3n} (c + dx)^3 dx \\
 &= \frac{a^3(c + dx)^4}{4d} + \frac{3a^2b \left( F^{eg+fgx} \right)^n (c + dx)^3}{fgn \log(F)} + \frac{3ab^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)^3}{2fgn \log(F)} + \frac{b^3 \left( F^{eg+fgx} \right)^{3n} (c + dx)^3}{3fgn \log(F)} \\
 &= \frac{a^3(c + dx)^4}{4d} - \frac{9a^2bd \left( F^{eg+fgx} \right)^n (c + dx)^2}{f^2g^2n^2 \log^2(F)} - \frac{9ab^2d \left( F^{eg+fgx} \right)^{2n} (c + dx)^2}{4f^2g^2n^2 \log^2(F)} - \frac{b^3d \left( F^{eg+fgx} \right)^{3n} (c + dx)^2}{12f^2g^2n^2 \log^2(F)} \\
 &= \frac{a^3(c + dx)^4}{4d} + \frac{18a^2bd^2 \left( F^{eg+fgx} \right)^n (c + dx)}{f^3g^3n^3 \log^3(F)} + \frac{9ab^2d^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)}{4f^3g^3n^3 \log^3(F)} + \frac{b^3d^2 \left( F^{eg+fgx} \right)^{3n} (c + dx)}{12f^3g^3n^3 \log^3(F)} \\
 &= \frac{a^3(c + dx)^4}{4d} - \frac{18a^2bd^3 \left( F^{eg+fgx} \right)^n}{f^4g^4n^4 \log^4(F)} - \frac{9ab^2d^3 \left( F^{eg+fgx} \right)^{2n}}{8f^4g^4n^4 \log^4(F)} - \frac{2b^3d^3 \left( F^{eg+fgx} \right)^{3n}}{27f^4g^4n^4 \log^4(F)}
 \end{aligned}$$

**Mathematica** [A] time = 0.74, size = 341, normalized size = 0.69

$$a^3c^3x + \frac{3}{2}a^3c^2dx^2 + a^3cd^2x^3 + \frac{1}{4}a^3d^3x^4 + \frac{3a^2b \left( F^{g(e+fx)} \right)^n \left( 6d^2fgn \log(F)(c + dx) + f^3g^3n^3 \log^3(F)(c + dx)^3 - 3df^2g^2n^2 \log^2(F)(c + dx)^2 \right)}{f^4g^4n^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n]^3\*(c + d\*x)^3,x]

[Out]  $a^3c^3x + (3a^3c^2dx^2)/2 + a^3cd^2x^3 + (a^3d^3x^4)/4 + (3a^2b(F^{g(e+fx)})^n(-6d^3 + 6d^2f*gn*(c+dx)*\text{Log}[F] - 3d^2f^2g^2n^2(c+dx)^2\text{Log}[F]^2 + f^3g^3n^3(c+dx)^3\text{Log}[F]^3))/(f^4g^4n^4\text{Log}[F]^4) + (3ab^2(F^{g(e+fx)})^{2n}(-3d^3 + 6d^2f*gn*(c+dx)*\text{Log}[F] - 6d^2f^2g^2n^2(c+dx)^2\text{Log}[F]^2 + 4f^3g^3n^3(c+dx)^3\text{Log}[F]^3))/(8f^4g^4n^4\text{Log}[F]^4) + (b^3(F^{g(e+fx)})^{3n}(-2d^3 + 6d^2f*gn*(c+dx)*\text{Log}[F] - 9d^2f^2g^2n^2(c+dx)^2\text{Log}[F]^2 + 9f^3g^3n^3(c+dx)^3\text{Log}[F]^3))/(27f^4g^4n^4\text{Log}[F]^4)$

**fricas** [A] time = 0.43, size = 708, normalized size = 1.43

$$54(a^3d^3f^4g^4n^4x^4 + 4a^3cd^2f^4g^4n^4x^3 + 6a^3c^2df^4g^4n^4x^2 + 4a^3c^3f^4g^4n^4x)\log(F)^4 - 8(2b^3d^3 - 9(b^3d^3f^3g^3n^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^3,x, algorithm="fricas")

[Out]  $1/216*(54*(a^3d^3f^4g^4n^4x^4 + 4a^3cd^2f^4g^4n^4x^3 + 6a^3c^2df^4g^4n^4x^2 + 4a^3c^3f^4g^4n^4x)*\log(F)^4 - 8*(2b^3d^3 - 9(b^3d^3f^3g^3n^3x^3 + 3b^3cd^2f^3g^3n^3x^2 + 3b^3c^2d^2f^3g^3n^3x + b^3c^3d^2f^3g^3n^3)*\log(F)^3 + 9*(b^3d^3f^2g^2n^2x^2 + 2b^3cd^2f^2g^2n^2x + b^3c^2d^2f^2g^2n^2)*\log(F)^2 - 6*(b^3d^3f*gn*x + b^3cd^2f*gn)*\log(F))*F^{(3f*gn*x + 3e*gn)} - 81*(3a^2b^2d^3 - 4(a^2b^2d^3f^3g^3n^3x^3 + 3a^2b^2cd^2f^3g^3n^3x^2 + 3a^2b^2c^2d^2f^3g^3n^3x + a^2b^2c^3d^2f^3g^3n^3)*\log(F)^3 + 6*(a^2b^2d^3f^2g^2n^2x^2 + 2a^2b^2cd^2f^2g^2n^2x + a^2b^2c^2d^2f^2g^2n^2)*\log(F)^2 - 6*(a^2b^2d^3f*gn*x + a^2b^2cd^2f*gn)*\log(F))*F^{(2f*gn*x + 2e*gn)} - 64*8*(6a^2b^2d^3 - (a^2b^2d^3f^3g^3n^3x^3 + 3a^2b^2cd^2f^3g^3n^3x^2 + 3a^2b^2c^2d^2f^3g^3n^3x + a^2b^2c^3d^2f^3g^3n^3)*\log(F)^3 + 3*(a^2b^2d^3f^2g^2n^2x^2 + 2a^2b^2cd^2f^2g^2n^2x + a^2b^2c^2d^2f^2g^2n^2)*\log(F)^2 - 6*(a^2b^2d^3f*gn*x + a^2b^2cd^2f*gn)*\log(F))*F^{(f*gn*x + e*gn)})/(f^4g^4n^4\log(F)^4)$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^3,x, algorithm="giac")

[Out] Timed out

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(fx+e)g} \right)^n + a \right)^3 (dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^3\*(d\*x+c)^3,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^3\*(d\*x+c)^3,x)

**maxima [A]** time = 2.37, size = 914, normalized size = 1.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}a^3d^3x^4 + a^3cd^2x^3 + \frac{3}{2}a^3c^2dx^2 + a^3c^3x + 3(F^{(fgx+e)g})^n a^2 b^2 c^3 / (fgn \log(F)) + \frac{3}{2}(F^{(fgx+e)g})^{2n} a^2 b^2 c^3 / (fgn \log(F)) + \frac{1}{3}(F^{(fgx+e)g})^{3n} b^3 c^3 / (fgn \log(F)) + 9((F^{(eg)})^n f g n x \log(F) - (F^{(eg)})^n (F^{(fgx)g})^n a^2 b^2 c^2 d / (f^2 g^2 n^2 \log(F)^2) + \frac{9}{4}(2(F^{(eg)})^{2n} f g n x \log(F) - (F^{(eg)})^{2n}) (F^{(fgx)g})^{2n} a^2 b^2 c^2 d / (f^2 g^2 n^2 \log(F)^2) + \frac{1}{3}(3(F^{(eg)})^{3n} f g n x \log(F) - (F^{(eg)})^{3n}) (F^{(fgx)g})^{3n} b^3 c^2 d / (f^2 g^2 n^2 \log(F)^2) + 9((F^{(eg)})^n f^2 g^2 n^2 x^2 \log(F)^2 - 2(F^{(eg)})^n f g n x \log(F) + 2(F^{(eg)})^n (F^{(fgx)g})^n a^2 b^2 c^2 d / (f^3 g^3 n^3 \log(F)^3) + \frac{9}{4}(2(F^{(eg)})^{2n} f^2 g^2 n^2 x^2 \log(F)^2 - 2(F^{(eg)})^{2n} f g n x \log(F) + (F^{(eg)})^{2n}) (F^{(fgx)g})^{2n} a^2 b^2 c^2 d / (f^3 g^3 n^3 \log(F)^3) + \frac{1}{9}(9(F^{(eg)})^{3n} f^2 g^2 n^2 x^2 \log(F)^2 - 6(F^{(eg)})^{3n} f g n x \log(F) + 2(F^{(eg)})^{3n}) (F^{(fgx)g})^{3n} b^3 c^2 d / (f^3 g^3 n^3 \log(F)^3) + 3((F^{(eg)})^n f^3 g^3 n^3 x^3 \log(F)^3 - 3(F^{(eg)})^n f^2 g^2 n^2 x^2 \log(F)^2 + 6(F^{(eg)})^n f g n x \log(F) - 6(F^{(eg)})^n (F^{(fgx)g})^n a^2 b^2 d^3 / (f^4 g^4 n^4 \log(F)^4) + \frac{3}{8}(4(F^{(eg)})^{2n} f^3 g^3 n^3 x^3 \log(F)^3 - 6(F^{(eg)})^{2n} f^2 g^2 n^2 x^2 \log(F)^2 + 6(F^{(eg)})^{2n} f g n x \log(F) - 3(F^{(eg)})^{2n}) (F^{(fgx)g})^{2n} a^2 b^2 d^3 / (f^4 g^4 n^4 \log(F)^4) + \frac{1}{27}(9(F^{(eg)})^{3n} f^3 g^3 n^3 x^3 \log(F)^3 - 9(F^{(eg)})^{3n} f^2 g^2 n^2 x^2 \log(F)^2 + 6(F^{(eg)})^{3n} f g n x \log(F) - 2(F^{(eg)})^{3n}) (F^{(fgx)g})^{3n} b^3 d^3 / (f^4 g^4 n^4 \log(F)^4)$

**mupad [B]** time = 4.37, size = 652, normalized size = 1.31

$$a^3 c^3 x - (F^{fgx} F^{eg})^n \left( \frac{3 a^2 b (-c^3 f^3 g^3 n^3 \ln(F)^3 + 3 c^2 d f^2 g^2 n^2 \ln(F)^2 - 6 c d^2 f g n \ln(F) + 6 d^3)}{f^4 g^4 n^4 \ln(F)^4} - \frac{3 a^2 b d^3 x^3}{f g n \ln(F)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*(F^(g*(e + f*x))))^n)^3*(c + d*x)^3,x`

[Out]  $a^3c^3x - (F^{(f*gx)}F^{(eg)})^n \left( \frac{(3a^2b(6d^3 - c^3f^3g^3n^3 \log(F)^3 - 6cd^2f*gn \log(F) + 3c^2d^2f^2g^2n^2 \log(F)^2))}{(f^4g^4n^4 \log(F)^4) - (3a^2bd^3x^3)/(f*gn \log(F)) - (9a^2bd^2x(2d^2 + c^2f^2g^2n^2 \log(F)^2 - 2cd^2f*gn \log(F)))/(f^3g^3n^3 \log(F)^3) + (9a^2bd^2x^2(d - cf*gn \log(F)))/(f^2g^2n^2 \log(F)^2)} - (F^{(f*gx)}F^{(eg)})^{(2n)} \left( \frac{(3ab^2(3d^3 - 4c^3f^3g^3n^3 \log(F)^3 - 6cd^2f*gn \log(F) + 6c^2d^2f^2g^2n^2 \log(F)^2))}{(8f^4g^4n^4 \log(F)^4) - (3ab^2d^3x^3)/(2f*gn \log(F)) - (9ab^2d^2x(d^2 + 2c^2f^2g^2n^2 \log(F)^2 - 2cd^2f*gn \log(F)))/(4f^3g^3n^3 \log(F)^3) + (9ab^2d^2x^2(d - 2cf*gn \log(F)))/(4f^2g^2n^2 \log(F)^2)} - (F^{(f*gx)}F^{(eg)})^{(3n)} \left( \frac{(b^3(2d^3 - 9c^3f^3g^3n^3 \log(F)^3 - 6cd^2f*gn \log(F) + 9c^2d^2f^2g^2n^2 \log(F)^2))}{(27f^4g^4n^4 \log(F)^4) - (b^3d^3x^3)/(3f*gn \log(F)) - (b^3d^2x(2d^2 + 9c^2f^2g^2n^2 \log(F)^2 - 6cd^2f*gn \log(F)))/(9f^3g^3n^3 \log(F)^3) + (b^3d^2x^2(d - 3cf*gn \log(F)))/(3f^2g^2n^2 \log(F)^2)} + (a^3d^3x^4)/4 + (3a^3c^2d^2x^2)/2 + a^3cd^2x^3 \right. \right.$

**sympy** [A] time = 0.65, size = 1074, normalized size = 2.17

$$a^3c^3x + \frac{3a^3c^2dx^2}{2} + a^3cd^2x^3 + \frac{a^3d^3x^4}{4} + \left\{ \frac{(72b^3c^3f^{11}g^{11}n^{11} \log(F)^{11} + 216b^3c^2df^{11}g^{11}n^{11}x \log(F)^{11} - 72b^3c^2df^{10}g^{10}n^{10} \log(F)^{10} + 216b^3cd^2f^{11}g^{11}n^{11}x \log(F)^{11} - 72b^3c^2d^2f^{10}g^{10}n^{10} \log(F)^{10} + 216b^3cd^2f^{10}g^{10}n^{10}x \log(F)^{10} - 144b^3cd^2f^{10}g^{10}n^{10}x^2 \log(F)^{10} + 48b^3cd^2f^9g^9n^9x \log(F)^9 + 72b^3d^3f^{11}g^{11}n^{11}x^3 \log(F)^{11} - 72b^3d^3f^{10}g^{10}n^{10}x^2 \log(F)^{10} + 48b^3d^3f^9g^9n^9x^2 \log(F)^9 - 16b^3d^3f^8g^8n^8 \log(F)^8) (F^{(g*(e + f*x))})^{(3n)} + (324ab^2c^3f^{11}g^{11}n^{11} \log(F)^{11} + 972ab^2c^2d^2f^{11}g^{11}n^{11}x \log(F)^{11} - 486ab^2c^2d^2f^{10}g^{10}n^{10} \log(F)^{10} + 972ab^2c^2d^2f^{11}g^{11}n^{11}x^2 \log(F)^{11} - 972ab^2c^2d^2f^{10}g^{10}n^{10}x \log(F)^{10} + 486ab^2c^2d^2f^9g^9n^9 \log(F)^9 + 324ab^2d^3f^{11}g^{11}n^{11}x^3 \log(F)^{11} - 486ab^2d^3f^{10}g^{10}n^{10}x^2 \log(F)^{10} + 486ab^2d^3f^9g^9n^9x \log(F)^9 - 243ab^2d^3f^8g^8n^8 \log(F)^8) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F**(g*(f*x+e))))**n)**3*(d*x+c)**3,x`

[Out]  $a**3*c**3*x + 3*a**3*c**2*d*x**2/2 + a**3*c*d**2*x**3 + a**3*d**3*x**4/4 + \text{Piecewise}(\left( \frac{(72*b**3*c**3*f**11*g**11*n**11*\log(F)**11 + 216*b**3*c**2*d*f**11*g**11*n**11*x*\log(F)**11 - 72*b**3*c**2*d^2*f**10*g**10*n**10*\log(F)**10 + 216*b**3*c*d**2*f**11*g**11*n**11*x**2*\log(F)**11 - 144*b**3*c*d**2*f**10*g**10*n**10*x*\log(F)**10 + 48*b**3*c*d**2*f**9*g**9*n**9*\log(F)**9 + 72*b**3*d**3*f**11*g**11*n**11*x**3*\log(F)**11 - 72*b**3*d**3*f**10*g**10*n**10*x**2*\log(F)**10 + 48*b**3*d**3*f**9*g**9*n**9*x*\log(F)**9 - 16*b**3*d**3*f**8*g**8*n**8*\log(F)**8) (F**(g*(e + f*x)))**(3*n) + (324*a*b**2*c**3*f**11*g**11*n**11*\log(F)**11 + 972*a*b**2*c**2*d^2*f**11*g**11*n**11*x*\log(F)**11 - 486*a*b**2*c**2*d^2*f**10*g**10*n**10*\log(F)**10 + 972*a*b**2*c*d**2*f**11*g**11*n**11*x**2*\log(F)**11 - 972*a*b**2*c*d**2*f**10*g**10*n**10*x*\log(F)**10 + 486*a*b**2*c*d**2*f**9*g**9*n**9*\log(F)**9 + 324*a*b**2*d**3*f**11*g**11*n**11*x**3*\log(F)**11 - 486*a*b**2*d**3*f**10*g**10*n**10*x**2*\log(F)**10 + 486*a*b**2*d**3*f**9*g**9*n**9*x*\log(F)**9 - 243*a*b**2*d**3*f**8*g**8n**8*\log(F)**8) \right.$

```

n**8*log(F)**8*(F**(g*(e + f*x)))**(2*n) + (648*a**2*b*c**3*f**11*g**11*n*
*11*log(F)**11 + 1944*a**2*b*c**2*d*f**11*g**11*n**11*x*log(F)**11 - 1944*a
**2*b*c**2*d*f**10*g**10*n**10*log(F)**10 + 1944*a**2*b*c*d**2*f**11*g**11*
n**11*x**2*log(F)**11 - 3888*a**2*b*c*d**2*f**10*g**10*n**10*x*log(F)**10 +
 3888*a**2*b*c*d**2*f**9*g**9*n**9*log(F)**9 + 648*a**2*b*d**3*f**11*g**11*
n**11*x**3*log(F)**11 - 1944*a**2*b*d**3*f**10*g**10*n**10*x**2*log(F)**10
+ 3888*a**2*b*d**3*f**9*g**9*n**9*x*log(F)**9 - 3888*a**2*b*d**3*f**8*g**8*
n**8*log(F)**8*(F**(g*(e + f*x)))**n)/(216*f**12*g**12*n**12*log(F)**12),
Ne(216*f**12*g**12*n**12*log(F)**12, 0)), (x**4*(3*a**2*b*d**3/4 + 3*a*b**2
*d**3/4 + b**3*d**3/4) + x**3*(3*a**2*b*c*d**2 + 3*a*b**2*c*d**2 + b**3*c*d
**2) + x**2*(9*a**2*b*c**2*d/2 + 9*a*b**2*c**2*d/2 + 3*b**3*c**2*d/2) + x*(
3*a**2*b*c**3 + 3*a*b**2*c**3 + b**3*c**3), True))

```

$$3.40 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^2 dx$$

**Optimal.** Leaf size=366

$$\frac{a^3(c+dx)^3}{3d} - \frac{6a^2bd(c+dx)(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{3a^2b(c+dx)^2(F^{eg+fgx})^n}{fgn\log(F)} + \frac{6a^2bd^2(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} - \frac{3ab^2d(c+dx)(F^{eg+fgx})^{2n}}{2f^2g^2n^2\log^2(F)}$$

[Out]  $\frac{1}{3}a^3(d*x+c)^3/d + 6a^2*b*d^2*(F^(f*g*x+e*g))^n/f^3/g^3/n^3/\ln(F)^3 + 3/4*a*b^2*d^2*(F^(f*g*x+e*g))^(2*n)/f^3/g^3/n^3/\ln(F)^3 + 2/27*b^3*d^2*(F^(f*g*x+e*g))^(3*n)/f^3/g^3/n^3/\ln(F)^3 - 6a^2*b*d*(F^(f*g*x+e*g))^n*(d*x+c)/f^2/g^2/n^2/\ln(F)^2 - 3/2*a*b^2*d*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^2/g^2/n^2/\ln(F)^2 - 2/9*b^3*d*(F^(f*g*x+e*g))^(3*n)*(d*x+c)/f^2/g^2/n^2/\ln(F)^2 + 3*a^2*b*(F^(f*g*x+e*g))^n*(d*x+c)^2/f/g/n/\ln(F) + 3/2*a*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^2/f/g/n/\ln(F) + 1/3*b^3*(F^(f*g*x+e*g))^(3*n)*(d*x+c)^2/f/g/n/\ln(F)$

**Rubi [A]** time = 0.46, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2183, 2176, 2194}

$$-\frac{6a^2bd(c+dx)(F^{eg+fgx})^n}{f^2g^2n^2\log^2(F)} + \frac{3a^2b(c+dx)^2(F^{eg+fgx})^n}{fgn\log(F)} + \frac{6a^2bd^2(F^{eg+fgx})^n}{f^3g^3n^3\log^3(F)} + \frac{a^3(c+dx)^3}{3d} - \frac{3ab^2d(c+dx)(F^{eg+fgx})^{2n}}{2f^2g^2n^2\log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]^3\*(c + d\*x)^2, x]

[Out]  $\frac{a^3(c+dx)^3}{(3d)} + \frac{(6a^2*b*d^2*(F^(e*g+f*g*x))^n)}{(f^3*g^3*n^3*Log[F]^3)} + \frac{(3*a*b^2*d^2*(F^(e*g+f*g*x))^(2*n))}{(4*f^3*g^3*n^3*Log[F]^3)} + \frac{(2*b^3*d^2*(F^(e*g+f*g*x))^(3*n))}{(27*f^3*g^3*n^3*Log[F]^3)} - \frac{(6a^2*b*d*(F^(e*g+f*g*x))^n*(c+dx))}{(f^2*g^2*n^2*Log[F]^2)} - \frac{(3*a*b^2*d*(F^(e*g+f*g*x))^(2*n)*(c+dx))}{(2*f^2*g^2*n^2*Log[F]^2)} - \frac{(2*b^3*d*(F^(e*g+f*g*x))^(3*n)*(c+dx))}{(9*f^2*g^2*n^2*Log[F]^2)} + \frac{(3*a^2*b*(F^(e*g+f*g*x))^n*(c+dx)^2)}{(f*g*n*Log[F])} + \frac{(3*a*b^2*(F^(e*g+f*g*x))^(2*n)*(c+dx)^2)}{(2*f*g*n*Log[F])} + \frac{(b^3*(F^(e*g+f*g*x))^(3*n)*(c+dx)^2)}{(3*f*g*n*Log[F])}$

**Rule 2176**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.)+(f\_.)\*(x\_))))^(n\_.)\*((c\_.)+(d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c+dx)^m\*(b\*F^(g\*(e+f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c+dx)^(m-1)\*(b\*F^(g\*(e+f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma === True

Rule 2183

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

Rule 2194

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
 \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^2 dx &= \int \left( a^3(c + dx)^2 + 3a^2b \left( F^{eg+fgx} \right)^n (c + dx)^2 + 3ab^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)^2 + \right. \\
 &= \frac{a^3(c + dx)^3}{3d} + (3a^2b) \int \left( F^{eg+fgx} \right)^n (c + dx)^2 dx + (3ab^2) \int \left( F^{eg+fgx} \right)^{2n} (c + dx)^2 dx \\
 &= \frac{a^3(c + dx)^3}{3d} + \frac{3a^2b \left( F^{eg+fgx} \right)^n (c + dx)^2}{fgn \log(F)} + \frac{3ab^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)^2}{2fgn \log(F)} + \frac{b^3 \left( F^{eg+fgx} \right)^{3n} (c + dx)^2}{3fgn \log(F)} \\
 &= \frac{a^3(c + dx)^3}{3d} - \frac{6a^2bd \left( F^{eg+fgx} \right)^n (c + dx)}{f^2g^2n^2 \log^2(F)} - \frac{3ab^2d \left( F^{eg+fgx} \right)^{2n} (c + dx)}{2f^2g^2n^2 \log^2(F)} - \frac{2b^3d \left( F^{eg+fgx} \right)^{3n} (c + dx)}{3f^2g^2n^2 \log^2(F)} \\
 &= \frac{a^3(c + dx)^3}{3d} + \frac{6a^2bd^2 \left( F^{eg+fgx} \right)^n}{f^3g^3n^3 \log^3(F)} + \frac{3ab^2d^2 \left( F^{eg+fgx} \right)^{2n}}{4f^3g^3n^3 \log^3(F)} + \frac{2b^3d^2 \left( F^{eg+fgx} \right)^3}{27f^3g^3n^3 \log^3(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.53, size = 248, normalized size = 0.68

$$a^3c^2x + a^3cdx^2 + \frac{1}{3}a^3d^2x^3 + \frac{3a^2b \left( F^{g(e+fx)} \right)^n \left( f^2g^2n^2 \log^2(F)(c + dx)^2 - 2dfgn \log(F)(c + dx) + 2d^2 \right)}{f^3g^3n^3 \log^3(F)} + \frac{3ab^2 \left( F^{g(e+fx)} \right)^{2n} \left( f^2g^2n^2 \log^2(F)(c + dx)^2 - 2dfgn \log(F)(c + dx) + 2d^2 \right)}{4f^3g^3n^3 \log^3(F)} + \frac{2b^3d^2 \left( F^{g(e+fx)} \right)^3}{27f^3g^3n^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^3\*(c + d\*x)^2,x]

[Out] a^3\*c^2\*x + a^3\*c\*d\*x^2 + (a^3\*d^2\*x^3)/3 + (3\*a^2\*b\*(F^(g\*(e + f\*x)))^n\*(2\*d^2 - 2\*d\*f\*g\*n\*(c + d\*x)\*Log[F] + f^2\*g^2\*n^2\*(c + d\*x)^2\*Log[F]^2))/(f^3\*g^3\*n^3\*Log[F]^3) + (3\*a\*b^2\*(F^(g\*(e + f\*x)))^(2\*n)\*(d^2 - 2\*d\*f\*g\*n\*(c + d\*x)\*Log[F] + 2\*f^2\*g^2\*n^2\*(c + d\*x)^2\*Log[F]^2))/(4\*f^3\*g^3\*n^3\*Log[F]^3)

) + (b^3\*(F^(g\*(e + f\*x)))^(3\*n)\*(2\*d^2 - 6\*d\*f\*g\*n\*(c + d\*x)\*Log[F] + 9\*f^2\*g^2\*n^2\*(c + d\*x)^2\*Log[F]^2))/(27\*f^3\*g^3\*n^3\*Log[F]^3)

**fricas** [A] time = 0.44, size = 414, normalized size = 1.13

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$$36(a^3d^2f^3g^3n^3x^3 + 3a^3cdf^3g^3n^3x^2 + 3a^3c^2f^3g^3n^3x)\log(F)^3 + 4(2b^3d^2 + 9(b^3d^2f^2g^2n^2x^2 + 2b^3cdf^2g^2n^2x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^2,x, algorithm="fricas")

[Out] 1/108\*(36\*(a^3\*d^2\*f^3\*g^3\*n^3\*x^3 + 3\*a^3\*c\*d\*f^3\*g^3\*n^3\*x^2 + 3\*a^3\*c^2\*f^3\*g^3\*n^3\*x)\*log(F)^3 + 4\*(2\*b^3\*d^2 + 9\*(b^3\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*b^3\*c\*d\*f^2\*g^2\*n^2\*x + b^3\*c^2\*f^2\*g^2\*n^2)\*log(F)^2 - 6\*(b^3\*d^2\*f\*g\*n\*x + b^3\*c\*d\*f\*g\*n)\*log(F))\*F^(3\*f\*g\*n\*x + 3\*e\*g\*n) + 81\*(a\*b^2\*d^2 + 2\*(a\*b^2\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*a\*b^2\*c\*d\*f^2\*g^2\*n^2\*x + a\*b^2\*c^2\*f^2\*g^2\*n^2)\*log(F)^2 - 2\*(a\*b^2\*d^2\*f\*g\*n\*x + a\*b^2\*c\*d\*f\*g\*n)\*log(F))\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n) + 324\*(2\*a^2\*b\*d^2 + (a^2\*b\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*a^2\*b\*c\*d\*f^2\*g^2\*n^2\*x + a^2\*b\*c^2\*f^2\*g^2\*n^2)\*log(F)^2 - 2\*(a^2\*b\*d^2\*f\*g\*n\*x + a^2\*b\*c\*d\*f\*g\*n)\*log(F))\*F^(f\*g\*n\*x + e\*g\*n))/(f^3\*g^3\*n^3\*log(F)^3)

**giac** [C] time = 1.66, size = 8854, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^2,x, algorithm="giac")

[Out] 1/3\*a^3\*d^2\*x^3 + a^3\*c\*d\*x^2 + a^3\*c^2\*x - 1/27\*((6\*(3\*pi\*b^3\*d^2\*f^2\*g^2\*n^2\*x^2\*log(abs(F))\*sgn(F) - 3\*pi\*b^3\*d^2\*f^2\*g^2\*n^2\*x^2\*log(abs(F)) + 6\*pi\*b^3\*c\*d\*f^2\*g^2\*n^2\*x\*log(abs(F))\*sgn(F) - 6\*pi\*b^3\*c\*d\*f^2\*g^2\*n^2\*x\*log(abs(F)) + 3\*pi\*b^3\*c^2\*f^2\*g^2\*n^2\*log(abs(F))\*sgn(F) - 3\*pi\*b^3\*c^2\*f^2\*g^2\*n^2\*log(abs(F)) - pi\*b^3\*d^2\*f\*g\*n\*x\*sgn(F) + pi\*b^3\*d^2\*f\*g\*n\*x - pi\*b^3\*c\*d\*f\*g\*n\*sgn(F) + pi\*b^3\*c\*d\*f\*g\*n)\*(pi^3\*f^3\*g^3\*n^3\*sgn(F) - 3\*pi\*f^3\*g^3\*n^3\*log(abs(F))^2\*sgn(F) - pi^3\*f^3\*g^3\*n^3 + 3\*pi\*f^3\*g^3\*n^3\*log(abs(F))^2)/(pi^3\*f^3\*g^3\*n^3\*sgn(F) - 3\*pi\*f^3\*g^3\*n^3\*log(abs(F))^2\*sgn(F) - pi^3\*f^3\*g^3\*n^3 + 3\*pi\*f^3\*g^3\*n^3\*log(abs(F))^2)^2 + (3\*pi^2\*f^3\*g^3\*n^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*f^3\*g^3\*n^3\*log(abs(F)) + 2\*f^3\*g^3\*n^3\*log(abs(F))^3)^2 - (9\*pi^2\*b^3\*d^2\*f^2\*g^2\*n^2\*x^2\*sgn(F) - 9\*pi^2\*b^3\*d^2\*f^2\*g^2\*n^2\*x^2 + 18\*b^3\*d^2\*f^2\*g^2\*n^2\*x^2\*log(abs(F))^2 + 18\*pi^2\*b^3\*c\*d\*f^2\*g^2\*n^2\*x\*sgn(F) - 18\*pi^2\*b^3\*c\*d\*f^2\*g^2\*n^2\*x + 36\*b^3\*c\*d\*f^2\*g^2\*n^2\*x\*log(abs(F))^2 + 9\*pi^2\*b^3\*c^2\*f^2\*g^2\*n^2\*sgn(F) - 9\*pi^2\*b^3\*c^2\*f^2\*g^2\*n^2 + 18\*b^3\*c^2\*f^2\*g^2\*n^2\*log(abs(F))^2 - 12\*b^3\*d^2\*f\*g\*n\*x\*log(abs(F)) - 12\*b^3\*c\*d\*f\*g\*n\*log(abs(F)) + 4\*b^3\*d^2)\*(3\*pi^2\*f^3\*g^3\*n^3\*log(abs(F))



$$\begin{aligned}
& )) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2f^3 g^3 n^3 \log(\operatorname{abs}(F))^3 / ( \\
& (\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 f^3 g^3 n^3 \\
& * g^3 n^3 + 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2)^2 + (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}( \\
& F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2f^3 g^3 n^3 \log(\operatorname{abs}(F))^3)^2 \\
& )) * \cos(-3/2 \pi f g n x \operatorname{sgn}(F) + 3/2 \pi f g n x - 3/2 \pi g n e \operatorname{sgn}(F) + 3/2 \\
& * \pi g n e) - ((9\pi^2 b^3 d^2 f^2 g^2 n^2 x^2 \operatorname{sgn}(F) - 9\pi^2 b^3 d^2 f^2 g^2 n^2 x^2 \\
& ^2 n^2 x^2 + 18b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F))^2 + 18\pi^2 b^3 c d f^2 \\
& * g^2 n^2 x \operatorname{sgn}(F) - 18\pi^2 b^3 c d f^2 g^2 n^2 x + 36b^3 c d f^2 g^2 n^2 x \\
& * \log(\operatorname{abs}(F))^2 + 9\pi^2 b^3 c^2 f^2 g^2 n^2 \operatorname{sgn}(F) - 9\pi^2 b^3 c^2 f^2 g^2 n^2 \\
& + 18b^3 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F))^2 - 12b^3 d^2 f g n x \log(\operatorname{abs}(F) \\
& )) - 12b^3 c d f g n \log(\operatorname{abs}(F)) + 4b^3 d^2) * (\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3 \\
& * \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 f^3 g^3 n^3 + 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2) / \\
& ((\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 f^3 g^3 n^3 + 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2)^2 \\
& + (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2f^3 g^3 n^3 \\
& * \log(\operatorname{abs}(F))^3)^2) + 6 * (3\pi b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \\
& * \pi b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + 6\pi b^3 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 6\pi b^3 c d f^2 g^2 n^2 x \\
& * \log(\operatorname{abs}(F)) + 3\pi b^3 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi b^3 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) - \pi b^3 d^2 \\
& * f g n x \operatorname{sgn}(F) + \pi b^3 d^2 f g n x - \pi b^3 c d f g n \operatorname{sgn}(F) + \pi b^3 c d f g n) * \\
& (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2f^3 g^3 n^3 \log(\operatorname{abs}(F))^3) / \\
& ((\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 f^3 g^3 n^3 + 3\pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2)^2 \\
& + (3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2f^3 g^3 n^3 \log(\operatorname{abs}(F))^3)^2) * \sin(-3/2 \pi f g n x \operatorname{sgn}(F) + 3/2 \\
& \pi f g n x - 3/2 \pi g n e \operatorname{sgn}(F) + 3/2 \pi g n e) * e^{(3f g n x \log(\operatorname{abs}(F)) \\
& + 3g n e \log(\operatorname{abs}(F)))} + 1/2 * I * ((36 * I * \pi^2 b^3 d^2 f^2 g^2 n^2 x^2 \operatorname{sgn}(F) - \\
& 72 * \pi b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 36 * I * \pi^2 b^3 d^2 f^2 g^2 n^2 x^2 \\
& ^2 n^2 x^2 + 72 * \pi b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + 72 * I * b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F))^2 \\
& + 72 * I * \pi^2 b^3 c d f^2 g^2 n^2 x \operatorname{sgn}(F) - 144 * \pi b^3 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 72 * I * \pi^2 b^3 c d f^2 g^2 n^2 x \\
& + 144 * \pi b^3 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F)) + 144 * I * b^3 c d f^2 g^2 n^2 x \log(\operatorname{abs}(F))^2 + 36 * I * \pi^2 b^3 c^2 f^2 g^2 n^2 \operatorname{sgn}(F) - 72 * \pi b^3 c^2 f^2 g^2 n^2 \\
& * \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 36 * I * \pi^2 b^3 c^2 f^2 g^2 n^2 + 72 * \pi b^3 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) + 72 * I * b^3 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F))^2 + 24 * \pi b^3 d^2 \\
& * f g n x \operatorname{sgn}(F) - 24 * \pi b^3 d^2 f g n x - 48 * I * b^3 d^2 f g n x \log(\operatorname{abs}(F)) \\
& + 24 * \pi b^3 c d f g n \operatorname{sgn}(F) - 24 * \pi b^3 c d f g n - 48 * I * b^3 c d f g n \log(\operatorname{abs}(F)) + 16 * I * b^3 d^2) * e^{(3/2 * I * \pi f g n x \operatorname{sgn}(F) - 3/2 * I * \pi f g n x + 3 \\
& /2 * I * \pi g n e \operatorname{sgn}(F) - 3/2 * I * \pi g n e) / (-108 * I * \pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) + 32 \\
& 4 * \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) + 324 * I * \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) + 108 * I * \pi^3 f^3 g^3 n^3 - 324 * \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) - 324 * I * \\
& \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 + 216 * f^3 g^3 n^3 \log(\operatorname{abs}(F))^3) - (36 * I * \pi^2 b^3 d^2 f^2 g^2 n^2 x^2 \operatorname{sgn}(F) + 72 * \pi b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) * \\
& \operatorname{sgn}(F) - 36 * I * \pi^2 b^3 d^2 f^2 g^2 n^2 x^2 - 72 * \pi b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + 72 * I * b^3 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F))^2 + 72 * I * \pi^2 b^3 c}
\end{aligned}$$

$$\begin{aligned}
& d^2 f^2 g^{2n^2} x \operatorname{sgn}(F) + 144 \pi b^3 c d^2 f^2 g^{2n^2} x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \\
& 72 I \pi^2 b^3 c d^2 f^2 g^{2n^2} x - 144 \pi b^3 c d^2 f^2 g^{2n^2} x \log(\operatorname{abs}(F)) \\
& + 144 I b^3 c d^2 f^2 g^{2n^2} x \log(\operatorname{abs}(F))^2 + 36 I \pi^2 b^3 c^2 f^2 g^{2n^2} \\
& \operatorname{sgn}(F) + 72 \pi b^3 c^2 f^2 g^{2n^2} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 36 I \pi^2 b^3 c^2 f^2 \\
& g^{2n^2} - 72 \pi b^3 c^2 f^2 g^{2n^2} \log(\operatorname{abs}(F)) + 72 I b^3 c^2 f^2 g^{2n^2} \\
& n^2 \log(\operatorname{abs}(F))^2 - 24 \pi b^3 d^2 f g n x \operatorname{sgn}(F) + 24 \pi b^3 d^2 f g n x - \\
& 48 I b^3 d^2 f g n x \log(\operatorname{abs}(F)) - 24 \pi b^3 c d^2 f g n \operatorname{sgn}(F) + 24 \pi b^3 c \\
& d^2 f g n - 48 I b^3 c d^2 f g n \log(\operatorname{abs}(F)) + 16 I b^3 d^2 e^{(-3/2 I \pi f g n x \operatorname{sgn}(F) + 3/2 I \pi f g n x - 3/2 I \pi i g n e \operatorname{sgn}(F) + 3/2 I \pi i g n e)} / (10 \\
& 8 I \pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) + 324 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 324 \\
& I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - 108 I \pi^3 f^3 g^3 n^3 - 324 \pi^2 f^3 g^3 n^3 \\
& \log(\operatorname{abs}(F)) + 324 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 + 216 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3) e^{(3 f g n x \log(\operatorname{abs}(F)) + 3 g n e \log(\operatorname{abs}(F)))} - 3/2 ( \\
& ((2 \pi a b^2 d^2 f^2 g^{2n^2} x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 \pi a b^2 d^2 f^2 g^{2n^2} x^2 \log(\operatorname{abs}(F)) + 4 \pi a b^2 c d^2 f^2 g^{2n^2} x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 4 \\
& \pi a b^2 c d^2 f^2 g^{2n^2} x \log(\operatorname{abs}(F)) + 2 \pi a b^2 c^2 f^2 g^{2n^2} \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 \pi a b^2 c^2 f^2 g^{2n^2} \log(\operatorname{abs}(F)) - \pi a b^2 d^2 f g n x \operatorname{sgn}(F) + \pi a b^2 d^2 f g n x - \pi a b^2 c d^2 f g n \operatorname{sgn}(F) + \pi a b^2 c d^2 f g n) (\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi i^3 f^3 g^3 n^3 + 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2) / ((\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi i^3 f^3 g^3 n^3 + 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2)^2 + (3 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3)^2) - (\pi^2 a b^2 d^2 f^2 g^{2n^2} x^2 \operatorname{sgn}(F) - \pi^2 a b^2 d^2 f^2 g^{2n^2} x^2 + 2 a b^2 d^2 f^2 g^{2n^2} x^2 \log(\operatorname{abs}(F))^2 + 2 \pi^2 a b^2 c d^2 f^2 g^{2n^2} x \operatorname{sgn}(F) - 2 \pi^2 a b^2 c d^2 f^2 g^{2n^2} x + 4 a b^2 c d^2 f^2 g^{2n^2} x \log(\operatorname{abs}(F))^2 + \pi^2 a b^2 c^2 f^2 g^{2n^2} \operatorname{sgn}(F) - \pi^2 a b^2 c^2 f^2 g^{2n^2} + 2 a b^2 c^2 f^2 g^{2n^2} \log(\operatorname{abs}(F))^2 - 2 a b^2 d^2 f g n x \log(\operatorname{abs}(F)) - 2 a b^2 c d^2 f g n \log(\operatorname{abs}(F)) + a b^2 d^2 (3 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3) / ((\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi i^3 f^3 g^3 n^3 + 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2)^2 + (3 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3)^2) \cos(-\pi f g n x \operatorname{sgn}(F) + \pi f g n x - \pi g n e \operatorname{sgn}(F) + \pi g n e) - ((\pi^2 a b^2 d^2 f^2 g^{2n^2} x^2 \operatorname{sgn}(F) - \pi^2 a b^2 d^2 f^2 g^{2n^2} x^2 + 2 a b^2 d^2 f^2 g^{2n^2} x^2 \log(\operatorname{abs}(F))^2 + 2 \pi^2 a b^2 c d^2 f^2 g^{2n^2} x \operatorname{sgn}(F) - 2 \pi^2 a b^2 c d^2 f^2 g^{2n^2} x + 4 a b^2 c d^2 f^2 g^{2n^2} x \log(\operatorname{abs}(F))^2 + \pi^2 a b^2 c^2 f^2 g^{2n^2} \operatorname{sgn}(F) - \pi^2 a b^2 c^2 f^2 g^{2n^2} + 2 a b^2 c^2 f^2 g^{2n^2} \log(\operatorname{abs}(F))^2 - 2 a b^2 d^2 f g n x \log(\operatorname{abs}(F)) - 2 a b^2 c d^2 f g n \log(\operatorname{abs}(F)) + a b^2 d^2 (\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi i^3 f^3 g^3 n^3 + 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2) / ((\pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) - 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi i^3 f^3 g^3 n^3 + 3 \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2)^2 + (3 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 2 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3)^2) + (2 \pi a b^2 d^2 f^2 g^{2n^2} x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 2 \pi a b^2 d^2 f^2 g^{2n^2} x^2 \log(\operatorname{abs}(F))
\end{aligned}$$

$$\begin{aligned} & \text{abs}(F)) + 4\pi a b^2 c d f^2 g^2 n^2 x \log(\text{abs}(F)) \text{sgn}(F) - 4\pi a a b^2 c d f^2 g^2 n^2 x \log(\text{abs}(F)) + 2\pi a a b^2 c^2 f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - \\ & 2\pi a a b^2 c^2 f^2 g^2 n^2 \log(\text{abs}(F)) - \pi a a b^2 d^2 f g n x \text{sgn}(F) + \pi a a b^2 d^2 f g n x - \pi a a b^2 c d f g n \text{sgn}(F) + \pi a a b^2 c d f g n (3\pi^2 \\ & f^3 g^3 n^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\text{abs}(F)) + 2f^3 g^3 n^3 \log(\text{abs}(F))^3) / ((\pi^3 f^3 g^3 n^3 \text{sgn}(F) - 3\pi f^3 g^3 n^3 \log(\text{abs}(F)) \\ & )^2 \text{sgn}(F) - \pi^3 f^3 g^3 n^3 + 3\pi f^3 g^3 n^3 \log(\text{abs}(F))^2)^2 + (3\pi^2 f^3 g^3 n^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 f^3 g^3 n^3 \log(\text{abs}(F)) + 2f^3 g^3 n^3 \log(\text{abs}(F))^3)^2 \\ & )^2 \sin(-\pi f g n x \text{sgn}(F) + \pi f g n x - \pi g n e \text{sgn}(F) + \pi g n e) e^{(2f g n x \log(\text{abs}(F)) + 2g n e \log(\text{abs}(F)))} + 1/2 I * ( \\ & (6 I \pi^2 a b^2 d^2 f^2 g^2 n^2 x^2 \text{sgn}(F) - 12 \pi a a b^2 d^2 f^2 g^2 n^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 6 I \pi^2 a a b^2 d^2 f^2 g^2 n^2 x^2 + 12 \pi a a b^2 d^2 \\ & f^2 g^2 n^2 x^2 \log(\text{abs}(F)) + 12 I a a b^2 d^2 f^2 g^2 n^2 x^2 \log(\text{abs}(F))^2 + 12 I \pi^2 a a b^2 c d f^2 g^2 n^2 x \text{sgn}(F) - 24 \pi a a b^2 c d f^2 g^2 n^2 x \\ & \log(\text{abs}(F)) \text{sgn}(F) - 12 I \pi^2 a a b^2 c d f^2 g^2 n^2 x + 24 \pi a a b^2 c d f^2 g^2 n^2 x \log(\text{abs}(F)) + 24 I a a b^2 c d f^2 g^2 n^2 x \log(\text{abs}(F))^2 + 6 I \\ & \pi^2 a a b^2 c^2 f^2 g^2 n^2 \text{sgn}(F) - 12 \pi a a b^2 c^2 f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - 6 I \pi^2 a a b^2 c^2 f^2 g^2 n^2 + 12 \pi a a b^2 c^2 f^2 g^2 n^2 \log \\ & (\text{abs}(F)) + 12 I a a b^2 c^2 f^2 g^2 n^2 \log(\text{abs}(F))^2 + 6 \pi a a b^2 d^2 f g n x \text{sgn}(F) - 6 \pi a a b^2 d^2 f g n x - 12 I a a b^2 d^2 f g n x \log(\text{abs}(F)) + 6 \\ & \pi a a b^2 c d f g n \text{sgn}(F) - 6 \pi a a b^2 c d f g n - 12 I a a b^2 c d f g n \log(\text{abs}(F)) + 6 I a a b^2 d^2 e^{(I \pi f g n x \text{sgn}(F) - I \pi f g n x + I \pi g n e \\ & \text{sgn}(F) - I \pi g n e) / (-4 I \pi^3 f^3 g^3 n^3 \text{sgn}(F) + 12 \pi^2 f^3 g^3 n^3 \log(\text{abs}(F)) \text{sgn}(F) + 12 I \pi^2 f^3 g^3 n^3 \log(\text{abs}(F))^2 \text{sgn}(F) + 4 I \pi^3 f^3 \\ & g^3 n^3 - 12 \pi^2 f^3 g^3 n^3 \log(\text{abs}(F)) - 12 I \pi^2 f^3 g^3 n^3 \log(\text{abs}(F))^2 + 8 f^3 g^3 n^3 \log(\text{abs}(F))^3) - (6 I \pi^2 a a b^2 d^2 f^2 g^2 n^2 x^2 \text{sgn}(F) \\ & + 12 \pi a a b^2 d^2 f^2 g^2 n^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 6 I \pi^2 a a b^2 d^2 f^2 g^2 n^2 x^2 - 12 \pi a a b^2 d^2 f^2 g^2 n^2 x^2 \log(\text{abs}(F)) + 12 I a a \\ & b^2 d^2 f^2 g^2 n^2 x^2 \log(\text{abs}(F))^2 + 12 I \pi^2 a a b^2 c d f^2 g^2 n^2 x \text{sgn}(F) + 24 \pi a a b^2 c d f^2 g^2 n^2 x \log(\text{abs}(F)) \text{sgn}(F) - 12 I \pi^2 a a b^2 \\ & c d f^2 g^2 n^2 x - 24 \pi a a b^2 c d f^2 g^2 n^2 x \log(\text{abs}(F)) + 24 I a a b^2 c d f^2 g^2 n^2 x \log(\text{abs}(F))^2 + 6 I \pi^2 a a b^2 c^2 f^2 g^2 n^2 \text{sgn}(F) + \\ & 12 \pi a a b^2 c^2 f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - 6 I \pi^2 a a b^2 c^2 f^2 g^2 n^2 - 12 \pi a a b^2 c^2 f^2 g^2 n^2 \log(\text{abs}(F)) + 12 I a a b^2 c^2 f^2 g^2 n^2 \\ & \log(\text{abs}(F))^2 - 6 \pi a a b^2 d^2 f g n x \text{sgn}(F) + 6 \pi a a b^2 d^2 f g n x - 12 I a a b^2 d^2 f g n x \log(\text{abs}(F)) - 6 \pi a a b^2 c d f g n \text{sgn}(F) + 6 \pi a a b^2 \\ & c d f g n - 12 I a a b^2 c d f g n \log(\text{abs}(F)) + 6 I a a b^2 d^2 e^{(-I \pi f g n x \text{sgn}(F) + I \pi f g n x - I \pi g n e \text{sgn}(F) + I \pi g n e) / (4 I \pi^3 f^3 \\ & g^3 n^3 \text{sgn}(F) + 12 \pi^2 f^3 g^3 n^3 \log(\text{abs}(F)) \text{sgn}(F) - 12 I \pi^2 f^3 g^3 n^3 \log(\text{abs}(F))^2 \text{sgn}(F) - 4 I \pi^3 f^3 g^3 n^3 - 12 \pi^2 f^3 g^3 n^3 \log(\text{abs}(F)) \\ & + 12 I \pi^2 f^3 g^3 n^3 \log(\text{abs}(F))^2 + 8 f^3 g^3 n^3 \log(\text{abs}(F))^3)} e^{(2f g n x \log(\text{abs}(F)) + 2g n e \log(\text{abs}(F)))} - 3 * ((2 * (\pi a^2 b d^2 f^2 g^2 n^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi a^2 b d^2 f^2 g^2 n^2 x^2 \log(\text{abs}(F)) + \\ & 2 \pi a^2 b c d f^2 g^2 n^2 x \log(\text{abs}(F)) \text{sgn}(F) - 2 \pi a^2 b c d f^2 g^2 n^2 x \log(\text{abs}(F)) + \pi a^2 b c^2 f^2 g^2 n^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi a^2 b c^2 \\ & ) \end{aligned}$$

$$\begin{aligned}
& 2f^2g^2n^2\log(\text{abs}(F)) - \pi a^2bd^2f^2g^2n^2\text{sgn}(F) + \pi a^2bd^2f^2g^2n^2x \\
& - \pi a^2b^2cd^2f^2g^2n^2\text{sgn}(F) + \pi a^2b^2cd^2f^2g^2n^2x(\pi^3f^3g^3n^3\text{sgn}(F) - 3\pi^3f^3g^3n^3\log(\text{abs}(F)) \\
& - 3\pi^3f^3g^3n^3\log(\text{abs}(F))^2\text{sgn}(F) - \pi^3f^3g^3n^3 + 3\pi^3f^3g^3n^3\log(\text{abs}(F))^2) / ((\pi^3f^3g^3n^3\text{sgn}(F) - 3\pi^3f^3g^3n^3\log(\text{abs}(F)) \\
& )^2\text{sgn}(F) - \pi^3f^3g^3n^3 + 3\pi^3f^3g^3n^3\log(\text{abs}(F))^2)^2 + (3\pi^2f^3g^3n^3\log(\text{abs}(F))\text{sgn}(F) - 3\pi^2f^3g^3n^3\log(\text{abs}(F)) + 2f^3g^3n^3\log(\text{abs}(F))^3)^2 \\
& - (\pi^2a^2bd^2f^2g^2n^2x^2\text{sgn}(F) - \pi^2a^2bd^2f^2g^2n^2x^2 + 2a^2bd^2f^2g^2n^2x^2\log(\text{abs}(F))^2 + 2\pi^2a^2b^2cd^2f^2g^2n^2x\text{sgn}(F) - 2\pi^2a^2b^2cd^2f^2g^2n^2x \\
& + 4a^2b^2cd^2f^2g^2n^2x\log(\text{abs}(F))^2 + \pi^2a^2b^2c^2f^2g^2n^2\text{sgn}(F) - \pi^2a^2b^2c^2f^2g^2n^2 + 2a^2b^2c^2f^2g^2n^2\log(\text{abs}(F))^2 - 4a^2bd^2f^2g^2n^2x\log(\text{abs}(F)) \\
& - 4a^2b^2cd^2f^2g^2n^2\log(\text{abs}(F)) + 4a^2bd^2)(3\pi^2f^3g^3n^3\log(\text{abs}(F))\text{sgn}(F) - 3\pi^2f^3g^3n^3\log(\text{abs}(F)) + 2f^3g^3n^3\log(\text{abs}(F))^3) / ((\pi^3f^3g^3n^3\text{sgn}(F) - 3\pi^3f^3g^3n^3\log(\text{abs}(F)) \\
& )^2\text{sgn}(F) - \pi^3f^3g^3n^3 + 3\pi^3f^3g^3n^3\log(\text{abs}(F))^2)^2 + (3\pi^2f^3g^3n^3\log(\text{abs}(F))\text{sgn}(F) - 3\pi^2f^3g^3n^3\log(\text{abs}(F)) + 2f^3g^3n^3\log(\text{abs}(F))^3)^2) \\
& )\cos(-1/2\pi f^2g^2n^2\text{sgn}(F) + 1/2\pi f^2g^2n^2x - 1/2\pi f^2g^2n^2x\text{sgn}(F) + 1/2\pi f^2g^2n^2x\log(\text{abs}(F))) - ((\pi^2a^2bd^2f^2g^2n^2x^2\text{sgn}(F) - \pi^2a^2bd^2f^2g^2n^2x^2 \\
& + 2a^2bd^2f^2g^2n^2x^2\log(\text{abs}(F))^2 + 2\pi^2a^2b^2cd^2f^2g^2n^2x\text{sgn}(F) - 2\pi^2a^2b^2cd^2f^2g^2n^2x + 4a^2b^2cd^2f^2g^2n^2x\log(\text{abs}(F))^2 + \pi^2a^2b^2c^2f^2g^2n^2\text{sgn}(F) \\
& - \pi^2a^2b^2c^2f^2g^2n^2 + 2a^2b^2c^2f^2g^2n^2\log(\text{abs}(F))^2 - 4a^2bd^2f^2g^2n^2x\log(\text{abs}(F)) - 4a^2b^2cd^2f^2g^2n^2\log(\text{abs}(F)) + 4a^2bd^2)(\pi^3f^3g^3n^3\text{sgn}(F) - 3\pi^3f^3g^3n^3\log(\text{abs}(F)) \\
& )^2\text{sgn}(F) - \pi^3f^3g^3n^3 + 3\pi^3f^3g^3n^3\log(\text{abs}(F))^2) / ((\pi^3f^3g^3n^3\text{sgn}(F) - 3\pi^3f^3g^3n^3\log(\text{abs}(F)) \\
& )^2\text{sgn}(F) - \pi^3f^3g^3n^3 + 3\pi^3f^3g^3n^3\log(\text{abs}(F))^2)^2 + (3\pi^2f^3g^3n^3\log(\text{abs}(F))\text{sgn}(F) - 3\pi^2f^3g^3n^3\log(\text{abs}(F)) + 2f^3g^3n^3\log(\text{abs}(F))^3)^2) + 2(\pi a^2bd^2f^2g^2n^2x^2\log(\text{abs}(F))\text{sgn}(F) - \pi a^2bd^2f^2g^2n^2x^2\log(\text{abs}(F)) + 2\pi a^2b^2cd^2f^2g^2n^2x\log(\text{abs}(F))\text{sgn}(F) - 2\pi a^2b^2cd^2f^2g^2n^2x\log(\text{abs}(F)) + \pi a^2b^2c^2f^2g^2n^2\log(\text{abs}(F))\text{sgn}(F) - \pi a^2b^2c^2f^2g^2n^2\log(\text{abs}(F)) - \pi a^2bd^2f^2g^2n^2\text{sgn}(F) + \pi a^2bd^2f^2g^2n^2x - \pi a^2b^2cd^2f^2g^2n^2\text{sgn}(F) + \pi a^2b^2cd^2f^2g^2n^2x)(3\pi^2f^3g^3n^3\log(\text{abs}(F))\text{sgn}(F) - 3\pi^2f^3g^3n^3\log(\text{abs}(F)) + 2f^3g^3n^3\log(\text{abs}(F))^3) / ((\pi^3f^3g^3n^3\text{sgn}(F) - 3\pi^3f^3g^3n^3\log(\text{abs}(F)) \\
& )^2\text{sgn}(F) - \pi^3f^3g^3n^3 + 3\pi^3f^3g^3n^3\log(\text{abs}(F))^2)^2 + (3\pi^2f^3g^3n^3\log(\text{abs}(F))\text{sgn}(F) - 3\pi^2f^3g^3n^3\log(\text{abs}(F)) + 2f^3g^3n^3\log(\text{abs}(F))^3)^2) \\
& )\sin(-1/2\pi f^2g^2n^2\text{sgn}(F) + 1/2\pi f^2g^2n^2x - 1/2\pi f^2g^2n^2x\text{sgn}(F) + 1/2\pi f^2g^2n^2x\log(\text{abs}(F)))e^{(f^2g^2n^2\log(\text{abs}(F)) + g^2n^2\log(\text{abs}(F)))} + 1/2I((1 \\
& 2I\pi^2a^2bd^2f^2g^2n^2x^2\text{sgn}(F) - 24\pi^2a^2bd^2f^2g^2n^2x^2\log(\text{abs}(F))\text{sgn}(F) - 12I\pi^2a^2bd^2f^2g^2n^2x^2 + 24\pi^2a^2bd^2f^2g^2n^2x^2\log(\text{abs}(F))^2 \\
& + 24I\pi^2a^2b^2cd^2f^2g^2n^2x\text{sgn}(F) - 48\pi^2a^2b^2cd^2f^2g^2n^2x\log(\text{abs}(F))\text{sgn}(F) - 24I\pi^2a^2b^2cd^2f^2g^2n^2x + 48\pi^2a^2b^2cd^2f^2g^2n^2x\log(\text{abs}(F)) + 48I\pi^2a^2b^2cd^2f^2g^2n^2x\log(\text{abs}(F))^2 + 12
\end{aligned}$$

$$\begin{aligned}
& I\pi^2 a^2 b^2 c^2 f^2 g^2 n^2 \operatorname{sgn}(F) - 24\pi a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 12I\pi^2 a^2 b^2 c^2 f^2 g^2 n^2 + 24\pi a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \\
& + 24I a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F))^2 + 24\pi a^2 b^2 d^2 f^2 g^2 n^2 \operatorname{sgn}(F) - 24\pi a^2 b^2 d^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \\
& + 24\pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 \operatorname{sgn}(F) - 24\pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 - 48I a^2 b^2 d^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \\
& + 24\pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 \operatorname{sgn}(F) - 24\pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 - 48I a^2 b^2 c^2 d^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \\
& + 48I a^2 b^2 d^2) e^{(1/2 I \pi f^2 g^2 n^2 \operatorname{sgn}(F) - 1/2 I \pi f^2 g^2 n^2)} \\
& + 1/2 I \pi f^2 g^2 n^2 e \operatorname{sgn}(F) - 1/2 I \pi f^2 g^2 n^2 e) / (-4 I \pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) + 12 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) \\
& + 12 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) + 4 I \pi^3 f^3 g^3 n^3 - 12 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) - 12 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \\
& + 8 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3) - (12 I \pi^2 a^2 b^2 d^2 f^2 g^2 n^2 x^2 \operatorname{sgn}(F) + 24 \pi a^2 b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) \\
& \operatorname{sgn}(F) - 12 I \pi^2 a^2 b^2 d^2 f^2 g^2 n^2 x^2 - 24 \pi a^2 b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + 24 I a^2 b^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F))^2 \\
& + 24 I \pi^2 a^2 b^2 c^2 d^2 f^2 g^2 n^2 x^2 \operatorname{sgn}(F) + 48 \pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 24 I \pi^2 a^2 b^2 c^2 d^2 f^2 g^2 n^2 x^2 \\
& - 48 \pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F)) + 48 I a^2 b^2 c^2 d^2 f^2 g^2 n^2 x^2 \log(\operatorname{abs}(F))^2 + 12 I \pi^2 a^2 b^2 c^2 f^2 g^2 n^2 \operatorname{sgn}(F) \\
& + 24 \pi a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 12 I \pi^2 a^2 b^2 c^2 f^2 g^2 n^2 - 24 \pi a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \\
& + 24 I a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F))^2 - 24 \pi a^2 b^2 d^2 f^2 g^2 n^2 \operatorname{sgn}(F) + 24 \pi a^2 b^2 d^2 f^2 g^2 n^2 \\
& - 48 I a^2 b^2 d^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) - 24 \pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 \operatorname{sgn}(F) + 24 \pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 \\
& - 48 I a^2 b^2 c^2 d^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) + 48 I a^2 b^2 c^2 d^2 f^2 g^2 n^2 \log(\operatorname{abs}(F))^2 + 12 I \pi^2 a^2 b^2 c^2 f^2 g^2 n^2 \operatorname{sgn}(F) \\
& + 24 \pi a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 12 I \pi^2 a^2 b^2 c^2 f^2 g^2 n^2 - 24 \pi a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) \\
& + 24 I a^2 b^2 c^2 f^2 g^2 n^2 \log(\operatorname{abs}(F))^2 - 24 \pi a^2 b^2 d^2 f^2 g^2 n^2 \operatorname{sgn}(F) + 24 \pi a^2 b^2 d^2 f^2 g^2 n^2 \\
& - 48 I a^2 b^2 d^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) - 24 \pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 \operatorname{sgn}(F) + 24 \pi a^2 b^2 c^2 d^2 f^2 g^2 n^2 \\
& - 48 I a^2 b^2 c^2 d^2 f^2 g^2 n^2 \log(\operatorname{abs}(F)) + 48 I a^2 b^2 d^2) e^{(-1/2 I \pi f^2 g^2 n^2 \operatorname{sgn}(F) + 1/2 I \pi f^2 g^2 n^2)} \\
& + 1/2 I \pi f^2 g^2 n^2 e \operatorname{sgn}(F) + 1/2 I \pi f^2 g^2 n^2 e) / (4 I \pi^3 f^3 g^3 n^3 \operatorname{sgn}(F) + 12 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) \\
& - 12 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - 4 I \pi^3 f^3 g^3 n^3 - 12 \pi^2 f^3 g^3 n^3 \log(\operatorname{abs}(F)) + 12 I \pi f^3 g^3 n^3 \log(\operatorname{abs}(F))^2 \\
& + 8 f^3 g^3 n^3 \log(\operatorname{abs}(F))^3) e^{(f^2 g^2 n^2 \log(\operatorname{abs}(F)) + g^2 n^2 \log(\operatorname{abs}(F)))}
\end{aligned}$$

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(fx+e)g} \right)^n + a \right)^3 (dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^3\*(d\*x+c)^2,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^3\*(d\*x+c)^2,x)

**maxima [A]** time = 1.81, size = 558, normalized size = 1.52

$$\frac{1}{3} a^3 d^2 x^3 + a^3 c d x^2 + a^3 c^2 x + \frac{3 (F^{fgx+eg})^n a^2 b c^2}{f g n \log(F)} + \frac{3 (F^{fgx+eg})^{2n} a b^2 c^2}{2 f g n \log(F)} + \frac{(F^{fgx+eg})^{3n} b^3 c^2}{3 f g n \log(F)} + \frac{6 ((F^{eg})^n f g n x \log(F) - (F^{eg})^n f^2 g^2 n^2 \log(F))}{f^2 g^2 n^2 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}a^3d^2x^3 + a^3cdx^2 + a^3c^2x + 3(F^{fgx+eg})^{3n}a^2b^2c^2 / (fgn \log(F)) + 3/2(F^{fgx+eg})^{2n}a^2b^2c^2 / (fgn \log(F)) + 1/3(F^{fgx+eg})^{3n}b^3c^2 / (fgn \log(F)) + 6((F^{eg})^n fgn x \log(F) - (F^{eg})^n) * (F^{fgx})^{2n} a^2 b^2 c^2 d / (f^2 g^2 n^2 \log(F)^2) + 3/2(2(F^{eg})^{2n} fgn x \log(F) - (F^{eg})^{2n}) * (F^{fgx})^{2n} a^2 b^2 c^2 d / (f^2 g^2 n^2 \log(F)^2) + 2/9(3(F^{eg})^{3n} fgn x \log(F) - (F^{eg})^{3n}) * (F^{fgx})^{3n} b^3 c^2 d / (f^2 g^2 n^2 \log(F)^2) + 3((F^{eg})^n f^2 g^2 n^2 x^2 \log(F)^2 - 2(F^{eg})^n fgn x \log(F) + 2(F^{eg})^n) * (F^{fgx})^{2n} a^2 b^2 d^2 / (f^3 g^3 n^3 \log(F)^3) + 3/4(2(F^{eg})^{2n} f^2 g^2 n^2 x^2 \log(F)^2 - 2(F^{eg})^{2n} fgn x \log(F) + (F^{eg})^{2n}) * (F^{fgx})^{2n} a^2 b^2 d^2 / (f^3 g^3 n^3 \log(F)^3) + 1/27(9(F^{eg})^{3n} f^2 g^2 n^2 x^2 \log(F)^2 - 6(F^{eg})^{3n} fgn x \log(F) + 2(F^{eg})^{3n}) * (F^{fgx})^{3n} b^3 d^2 / (f^3 g^3 n^3 \log(F)^3)$

mupad [B] time = 3.99, size = 399, normalized size = 1.09

$$(F^{fgx} F^{eg})^{3n} \left( \frac{b^3 (9c^2 f^2 g^2 n^2 \ln(F)^2 - 6cdfgn \ln(F) + 2d^2)}{27f^3 g^3 n^3 \ln(F)^3} + \frac{b^3 d^2 x^2}{3fgn \ln(F)} - \frac{2b^3 dx (d - 3cdfgn \ln(F))}{9f^2 g^2 n^2 \ln(F)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)^2,x)

[Out]  $(F^{fgx} F^{eg})^{3n} * ((b^3(2d^2 + 9c^2 f^2 g^2 n^2 \log(F)^2 - 6c^2 d fgn \log(F)) / (27f^3 g^3 n^3 \log(F)^3) + (b^3 d^2 x^2) / (3fgn \log(F)) - (2b^3 d x (d - 3cdfgn \log(F))) / (9f^2 g^2 n^2 \log(F)^2)) + (F^{fgx} F^{eg})^n * ((3a^2 b^2 (2d^2 + c^2 f^2 g^2 n^2 \log(F)^2 - 2c^2 d fgn \log(F))) / (f^3 g^3 n^3 \log(F)^3) + (3a^2 b^2 d^2 x^2) / (fgn \log(F)) - (6a^2 b^2 d x (d - c fgn \log(F))) / (f^2 g^2 n^2 \log(F)^2)) + (F^{fgx} F^{eg})^{2n} * ((3a^2 b^2 (d^2 + 2c^2 f^2 g^2 n^2 \log(F)^2 - 2c^2 d fgn \log(F))) / (4f^3 g^3 n^3 \log(F)^3) + (3a^2 b^2 d^2 x^2) / (2fgn \log(F)) - (3a^2 b^2 d x (d - 2cdfgn \log(F))) / (2f^2 g^2 n^2 \log(F)^2)) + a^3 c^2 x + (a^3 d^2 x^3) / 3 + a^3 c d x^2$

sympy [A] time = 0.48, size = 653, normalized size = 1.78

$$a^3 c^2 x + a^3 c d x^2 + \frac{a^3 d^2 x^3}{3} + \left\{ \frac{(36b^3 c^2 f^8 g^8 n^8 \log(F)^8 + 72b^3 c d f^8 g^8 n^8 x \log(F)^8 - 24b^3 c d f^7 g^7 n^7 \log(F)^7 + 36b^3 d^2 f^8 g^8 n^8 x^2 \log(F)^8 - 24b^3 d^2 f^7 g^7 n^7 x \log(F)^7 + 36b^3 d^2 f^6 g^6 n^6 \log(F)^6 - 24b^3 d^2 f^5 g^5 n^5 x \log(F)^5 + 36b^3 d^2 f^4 g^4 n^4 \log(F)^4 - 24b^3 d^2 f^3 g^3 n^3 x \log(F)^3 + 36b^3 d^2 f^2 g^2 n^2 \log(F)^2 - 24b^3 d^2 f g n \log(F) + 36b^3 d^2 \log(F))}{x^3 \left( a^2 b d^2 + a b^2 d^2 + \frac{b^3 d^2}{3} \right) + x^2 (3a^2 b c d + 3a b^2 c d + b^3 c d) + x (3a^2 b c^2 + 3a b^2 c^2 + b^3 c^2)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3\*(d\*x+c)\*\*2,x)

[Out]  $a^{**3}c^{**2}x + a^{**3}c*d*x^{**2} + a^{**3}d^{**2}*x^{**3}/3 + \text{Piecewise}(\left(\left(\left(36*b^{**3}c^{**2}f^{**8}g^{**8}n^{**8}\log(F)^{**8} + 72*b^{**3}c*d*f^{**8}g^{**8}n^{**8}x*\log(F)^{**8} - 24*b^{**3}c*d*f^{**7}g^{**7}n^{**7}\log(F)^{**7} + 36*b^{**3}d^{**2}f^{**8}g^{**8}n^{**8}x^{**2}\log(F)^{**8} - 24*b^{**3}d^{**2}f^{**7}g^{**7}n^{**7}x*\log(F)^{**7} + 8*b^{**3}d^{**2}f^{**6}g^{**6}n^{**6}\log(F)^{**6}\right)*(F^{**}(g*(e + f*x)))^{**}(3*n) + \left(162*a*b^{**2}c^{**2}f^{**8}g^{**8}n^{**8}\log(F)^{**8} + 324*a*b^{**2}c*d*f^{**8}g^{**8}n^{**8}x*\log(F)^{**8} - 162*a*b^{**2}c*d*f^{**7}g^{**7}n^{**7}\log(F)^{**7} + 162*a*b^{**2}d^{**2}f^{**8}g^{**8}n^{**8}x^{**2}\log(F)^{**8} - 162*a*b^{**2}d^{**2}f^{**7}g^{**7}n^{**7}x*\log(F)^{**7} + 81*a*b^{**2}d^{**2}f^{**6}g^{**6}n^{**6}\log(F)^{**6}\right)*(F^{**}(g*(e + f*x)))^{**}(2*n) + \left(324*a^{**2}b*c^{**2}f^{**8}g^{**8}n^{**8}\log(F)^{**8} + 648*a^{**2}b*c*d*f^{**8}g^{**8}n^{**8}x*\log(F)^{**8} - 648*a^{**2}b*c*d*f^{**7}g^{**7}n^{**7}\log(F)^{**7} + 324*a^{**2}b*d^{**2}f^{**8}g^{**8}n^{**8}x^{**2}\log(F)^{**8} - 648*a^{**2}b*d^{**2}f^{**7}g^{**7}n^{**7}x*\log(F)^{**7} + 648*a^{**2}b*d^{**2}f^{**6}g^{**6}n^{**6}\log(F)^{**6}\right)*(F^{**}(g*(e + f*x)))^{**}n\right)/(108*f^{**9}g^{**9}n^{**9}\log(F)^{**9}), \text{Ne}(108*f^{**9}g^{**9}n^{**9}\log(F)^{**9}, 0), (x^{**3}(a^{**2}b*d^{**2} + a*b^{**2}d^{**2} + b^{**3}d^{**2}/3) + x^{**2}(3*a^{**2}b*c*d + 3*a*b^{**2}c*d + b^{**3}c*d) + x(3*a^{**2}b*c^{**2} + 3*a*b^{**2}c^{**2} + b^{**3}c^{**2}), \text{True}))$

$$3.41 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx) dx$$

**Optimal.** Leaf size=236

$$\frac{a^3(c+dx)^2}{2d} + \frac{3a^2b(c+dx)(F^{eg+fgx})^n}{fgn \log(F)} - \frac{3a^2bd(F^{eg+fgx})^n}{f^2g^2n^2 \log^2(F)} + \frac{3ab^2(c+dx)(F^{eg+fgx})^{2n}}{2fgn \log(F)} - \frac{3ab^2d(F^{eg+fgx})^{2n}}{4f^2g^2n^2 \log^2(F)} + \frac{b^3(c+dx)}{3fgn}$$

[Out]  $\frac{1}{2}a^3(d*x+c)^2/d - 3a^2b*d*(F^(f*g*x+e*g))^n/f^2/g^2/n^2/\ln(F)^2 - 3/4*a*b^2*d*(F^(f*g*x+e*g))^(2*n)/f^2/g^2/n^2/\ln(F)^2 - 1/9*b^3*d*(F^(f*g*x+e*g))^(3*n)/f^2/g^2/n^2/\ln(F)^2 + 3*a^2*b*(F^(f*g*x+e*g))^n*(d*x+c)/f/g/n/\ln(F) + 3/2*a*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f/g/n/\ln(F) + 1/3*b^3*(F^(f*g*x+e*g))^(3*n)*(d*x+c)/f/g/n/\ln(F)$

**Rubi [A]** time = 0.24, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {2183, 2176, 2194}

$$\frac{3a^2b(c+dx)(F^{eg+fgx})^n}{fgn \log(F)} - \frac{3a^2bd(F^{eg+fgx})^n}{f^2g^2n^2 \log^2(F)} + \frac{a^3(c+dx)^2}{2d} + \frac{3ab^2(c+dx)(F^{eg+fgx})^{2n}}{2fgn \log(F)} - \frac{3ab^2d(F^{eg+fgx})^{2n}}{4f^2g^2n^2 \log^2(F)} + \frac{b^3(c+dx)}{3fgn}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]^3\*(c + d\*x), x]

[Out]  $(a^3*(c + d*x)^2)/(2*d) - (3*a^2*b*d*(F^(e*g + f*g*x))^n)/(f^2*g^2*n^2*\text{Log}[F]^2) - (3*a*b^2*d*(F^(e*g + f*g*x))^(2*n))/(4*f^2*g^2*n^2*\text{Log}[F]^2) - (b^3*d*(F^(e*g + f*g*x))^(3*n))/(9*f^2*g^2*n^2*\text{Log}[F]^2) + (3*a^2*b*(F^(e*g + f*g*x))^n*(c + d*x))/(f*g*n*\text{Log}[F]) + (3*a*b^2*(F^(e*g + f*g*x))^(2*n)*(c + d*x))/(2*f*g*n*\text{Log}[F]) + (b^3*(F^(e*g + f*g*x))^(3*n)*(c + d*x))/(3*f*g*n*\text{Log}[F])$

**Rule 2176**

Int[((b\_)\*(F\_)^((g\_)\*((e\_)+(f\_)\*(x\_))))^(n\_)\*((c\_)+(d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

**Rule 2183**

Int[((a\_)+(b\_)\*((F\_)^((g\_)\*((e\_)+(f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_)+(d\_)\*(x\_))^(m\_), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p], x]



$(g*(e + f*x))^n)^p, x], x] /; \text{FreeQ}[\{F, a, b, c, d, e, f, g, m, n\}, x] \ \&\& \text{IGtQ}[p, 0]$

### Rule 2194

$\text{Int}[(F^((c_.)*(a_.) + (b_.)*(x_.)))^(n_.), x\_Symbol] := \text{Simp}[(F^(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

### Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx) dx &= \int \left( a^3(c + dx) + 3a^2b \left( F^{eg+fgx} \right)^n (c + dx) + 3ab^2 \left( F^{eg+fgx} \right)^{2n} (c + dx) + b^3 \left( F^{eg+fgx} \right)^{3n} (c + dx) \right) dx \\ &= \frac{a^3(c + dx)^2}{2d} + (3a^2b) \int \left( F^{eg+fgx} \right)^n (c + dx) dx + (3ab^2) \int \left( F^{eg+fgx} \right)^{2n} (c + dx) dx + b^3 \int \left( F^{eg+fgx} \right)^{3n} (c + dx) dx \\ &= \frac{a^3(c + dx)^2}{2d} + \frac{3a^2b \left( F^{eg+fgx} \right)^n (c + dx)}{fgn \log(F)} + \frac{3ab^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)}{2fgn \log(F)} + \frac{b^3 \left( F^{eg+fgx} \right)^{3n} (c + dx)}{3fgn \log(F)} \\ &= \frac{a^3(c + dx)^2}{2d} - \frac{3a^2bd \left( F^{eg+fgx} \right)^n}{f^2g^2n^2 \log^2(F)} - \frac{3ab^2d \left( F^{eg+fgx} \right)^{2n}}{4f^2g^2n^2 \log^2(F)} - \frac{b^3d \left( F^{eg+fgx} \right)^{3n}}{9f^2g^2n^2 \log^2(F)} + \dots \end{aligned}$$

**Mathematica [A]** time = 0.38, size = 161, normalized size = 0.68

$$\frac{18a^3f^2g^2n^2x \log^2(F)(2c + dx) + 6bfgn \log(F)(c + dx) \left( F^{g(e+fx)} \right)^n \left( 18a^2 + 9ab \left( F^{g(e+fx)} \right)^n + 2b^2 \left( F^{g(e+fx)} \right)^{2n} \right) - b^3 \left( F^{g(e+fx)} \right)^{3n} (c + dx)}{36f^2g^2n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^3\*(c + d\*x), x]

[Out]  $(- (b*d*(F^(g*(e + f*x)))^n*(108*a^2 + 27*a*b*(F^(g*(e + f*x)))^n + 4*b^2*(F^(g*(e + f*x)))^(2*n))) + 6*b*f*(F^(g*(e + f*x)))^n*(18*a^2 + 9*a*b*(F^(g*(e + f*x)))^n + 2*b^2*(F^(g*(e + f*x)))^(2*n))*g*n*(c + d*x)*\text{Log}[F] + 18*a^3*f^2*g^2*n^2*x*(2*c + d*x)*\text{Log}[F]^2)/(36*f^2*g^2*n^2*\text{Log}[F]^2)$

**fricas [A]** time = 0.43, size = 192, normalized size = 0.81

$$\frac{18 \left( a^3df^2g^2n^2x^2 + 2a^3cf^2g^2n^2x \right) \log(F)^2 - 4 \left( b^3d - 3 \left( b^3dfgnx + b^3cfdgn \right) \log(F) \right) F^3fgnx+3egn - 27 \left( ab^2d - 2 \left( b^2cfdgn + b^2cfdgn \right) \log(F) \right) F^2fgnx+2egn}{36f^2g^2n^2 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c),x, algorithm="fricas")

[Out]  $\frac{1}{36} \cdot (18 \cdot (a^3 \cdot d \cdot f^2 \cdot g^2 \cdot n^2 \cdot x^2 + 2 \cdot a^3 \cdot c \cdot f^2 \cdot g^2 \cdot n^2 \cdot x) \cdot \log(F)^2 - 4 \cdot (b^3 \cdot d - 3 \cdot (b^3 \cdot d \cdot f \cdot g \cdot n \cdot x + b^3 \cdot c \cdot f \cdot g \cdot n) \cdot \log(F)) \cdot F^{(3 \cdot f \cdot g \cdot n \cdot x + 3 \cdot e \cdot g \cdot n)} - 27 \cdot (a \cdot b^2 \cdot d - 2 \cdot (a \cdot b^2 \cdot d \cdot f \cdot g \cdot n \cdot x + a \cdot b^2 \cdot c \cdot f \cdot g \cdot n) \cdot \log(F)) \cdot F^{(2 \cdot f \cdot g \cdot n \cdot x + 2 \cdot e \cdot g \cdot n)} - 108 \cdot (a^2 \cdot b \cdot d - (a^2 \cdot b \cdot d \cdot f \cdot g \cdot n \cdot x + a^2 \cdot b \cdot c \cdot f \cdot g \cdot n) \cdot \log(F)) \cdot F^{(f \cdot g \cdot n \cdot x + e \cdot g \cdot n)}) / (f^2 \cdot g^2 \cdot n^2 \cdot \log(F)^2)$

**giac** [C] time = 1.33, size = 3572, normalized size = 15.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c),x, algorithm="giac")

[Out]  $\frac{1}{2} \cdot a^3 \cdot d \cdot x^2 + a^3 \cdot c \cdot x + \frac{1}{9} \cdot (2 \cdot ((3 \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x \cdot \log(\text{abs}(F)) + 3 \cdot b^3 \cdot c \cdot f \cdot g \cdot n \cdot \log(\text{abs}(F)) - b^3 \cdot d) \cdot (\pi^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 + 2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2) / ((\pi^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 + 2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2)^2 + 4 \cdot (\pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)))^2) + 3 \cdot (\pi \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) - \pi \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x + \pi \cdot b^3 \cdot c \cdot f \cdot g \cdot n \cdot \text{sgn}(F) - \pi \cdot b^3 \cdot c \cdot f \cdot g \cdot n) \cdot (\pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)))) / ((\pi^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 + 2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2)^2 + 4 \cdot (\pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)))^2) \cdot \cos(-3/2 \cdot \pi \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) + 3/2 \cdot \pi \cdot f \cdot g \cdot n \cdot x - 3/2 \cdot \pi \cdot g \cdot n \cdot e \cdot \text{sgn}(F) + 3/2 \cdot \pi \cdot g \cdot n \cdot e) + (3 \cdot (\pi \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) - \pi \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x + \pi \cdot b^3 \cdot c \cdot f \cdot g \cdot n \cdot \text{sgn}(F) - \pi \cdot b^3 \cdot c \cdot f \cdot g \cdot n) \cdot (\pi^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 + 2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2) / ((\pi^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 + 2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2)^2 + 4 \cdot (\pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)))^2) - 4 \cdot (3 \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x \cdot \log(\text{abs}(F)) + 3 \cdot b^3 \cdot c \cdot f \cdot g \cdot n \cdot \log(\text{abs}(F)) - b^3 \cdot d) \cdot (\pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)))) / ((\pi^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 + 2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2)^2 + 4 \cdot (\pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)))^2) \cdot \sin(-3/2 \cdot \pi \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) + 3/2 \cdot \pi \cdot f \cdot g \cdot n \cdot x - 3/2 \cdot \pi \cdot g \cdot n \cdot e \cdot \text{sgn}(F) + 3/2 \cdot \pi \cdot g \cdot n \cdot e)) \cdot e^{(3 \cdot f \cdot g \cdot n \cdot x \cdot \log(\text{abs}(F)) + 3 \cdot g \cdot n \cdot e \cdot \log(\text{abs}(F)))} - \frac{1}{2} \cdot I \cdot ((6 \cdot \pi \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) - 6 \cdot \pi \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x - 12 \cdot I \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x \cdot \log(\text{abs}(F)) + 6 \cdot \pi \cdot b^3 \cdot c \cdot f \cdot g \cdot n \cdot \text{sgn}(F) - 6 \cdot \pi \cdot b^3 \cdot c \cdot f \cdot g \cdot n - 12 \cdot I \cdot b^3 \cdot c \cdot f \cdot g \cdot n \cdot \log(\text{abs}(F)) + 4 \cdot I \cdot b^3 \cdot d) \cdot e^{(3/2 \cdot I \cdot \pi \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) - 3/2 \cdot I \cdot \pi \cdot f \cdot g \cdot n \cdot x + 3/2 \cdot I \cdot \pi \cdot g \cdot n \cdot e \cdot \text{sgn}(F) - 3/2 \cdot I \cdot \pi \cdot g \cdot n \cdot e)} / (18 \cdot \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) + 36 \cdot I \cdot \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - 18 \cdot \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 - 36 \cdot I \cdot \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) + 36 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))^2) + (6 \cdot \pi \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) - 6 \cdot \pi \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x + 12 \cdot I \cdot b^3 \cdot d \cdot f \cdot g \cdot n \cdot x \cdot \log(\text{abs}(F)) + 6 \cdot \pi \cdot b^3 \cdot c \cdot f \cdot g \cdot n \cdot \text{sgn}(F) - 6 \cdot \pi \cdot b^3 \cdot c \cdot f \cdot g \cdot n + 12 \cdot I \cdot b^3 \cdot c \cdot f \cdot g \cdot n \cdot \log(\text{abs}(F)) - 4 \cdot I \cdot b^3 \cdot d) \cdot e^{(-3/2 \cdot I \cdot \pi \cdot f \cdot g \cdot n \cdot x \cdot \text{sgn}(F) + 3/2 \cdot I \cdot \pi \cdot f \cdot g \cdot n \cdot x - 3/2 \cdot I \cdot \pi \cdot g \cdot n \cdot e \cdot \text{sgn}(F) + 3/2 \cdot I \cdot \pi \cdot g \cdot n \cdot e)} / (18 \cdot \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 \cdot \text{sgn}(F) - 36 \cdot I \cdot \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F)) \cdot \text{sgn}(F) - 18 \cdot \pi^2 \cdot f^2 \cdot g^2 \cdot n^2 + 36 \cdot I \cdot \pi \cdot f^2 \cdot g^2 \cdot n^2 \cdot \log(\text{abs}(F))$



$$\begin{aligned} & \log(\text{abs}(F))^2 - 4*(a^2*b*d*f*g*n*x*\log(\text{abs}(F)) + a^2*b*c*f*g*n*\log(\text{abs}(F)) \\ & - a^2*b*d)*(pi*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) - pi*f^2*g^2*n^2*\log(\text{abs}(F))) \\ & /((pi^2*f^2*g^2*n^2*\text{sgn}(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*\log(\text{abs}(F)))^2 \\ & + 4*(pi*f^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) - pi*f^2*g^2*n^2*\log(\text{abs}(F)))^2) \\ & * \sin(-1/2*pi*f*g*n*x*\text{sgn}(F) + 1/2*pi*f*g*n*x - 1/2*pi*g*n*e*\text{sgn}(F) + 1/2 \\ & *pi*g*n*e)) * e^{(f*g*n*x*\log(\text{abs}(F)) + g*n*e*\log(\text{abs}(F)))} - 1/2*I*((6*pi*a^2*b \\ & *d*f*g*n*x*\text{sgn}(F) - 6*pi*a^2*b*d*f*g*n*x - 12*I*a^2*b*d*f*g*n*x*\log(\text{abs}(F)) \\ & + 6*pi*a^2*b*c*f*g*n*\text{sgn}(F) - 6*pi*a^2*b*c*f*g*n - 12*I*a^2*b*c*f*g*n*\log \\ & (\text{abs}(F)) + 12*I*a^2*b*d)*e^{(1/2*I*pi*f*g*n*x*\text{sgn}(F) - 1/2*I*pi*f*g*n*x + 1/2 \\ & *I*pi*g*n*e*\text{sgn}(F) - 1/2*I*pi*g*n*e)/(2*pi^2*f^2*g^2*n^2*\text{sgn}(F) + 4*I*pi*f \\ & ^2*g^2*n^2*\log(\text{abs}(F))*\text{sgn}(F) - 2*pi^2*f^2*g^2*n^2 - 4*I*pi*f^2*g^2*n^2*\log \\ & (\text{abs}(F)) + 4*f^2*g^2*n^2*\log(\text{abs}(F))^2) + (6*pi*a^2*b*d*f*g*n*x*\text{sgn}(F) - 6 \\ & pi*a^2*b*d*f*g*n*x + 12*I*a^2*b*d*f*g*n*x*\log(\text{abs}(F)) + 6*pi*a^2*b*c*f*g*n* \\ & \text{sgn}(F) - 6*pi*a^2*b*c*f*g*n + 12*I*a^2*b*c*f*g*n*\log(\text{abs}(F)) - 12*I*a^2*b*d) \\ & ) * e^{(-1/2*I*pi*f*g*n*x*\text{sgn}(F) + 1/2*I*pi*f*g*n*x - 1/2*I*pi*g*n*e*\text{sgn}(F) + \\ & 1/2*I*pi*g*n*e)/(2*pi^2*f^2*g^2*n^2*\text{sgn}(F) - 4*I*pi*f^2*g^2*n^2*\log(\text{abs}(F)) \\ & *\text{sgn}(F) - 2*pi^2*f^2*g^2*n^2 + 4*I*pi*f^2*g^2*n^2*\log(\text{abs}(F)) + 4*f^2*g^2*n \\ & ^2*\log(\text{abs}(F))^2)} * e^{(f*g*n*x*\log(\text{abs}(F)) + g*n*e*\log(\text{abs}(F)))} \end{aligned}$$

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(f x + e) g} \right)^n + a \right)^3 (dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^3\*(d\*x+c),x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^3\*(d\*x+c),x)

**maxima [A]** time = 1.42, size = 284, normalized size = 1.20

$$\frac{1}{2} a^3 dx^2 + a^3 cx + \frac{3 (F^{fgx+eg})^n a^2 bc}{fgn \log(F)} + \frac{3 (F^{fgx+eg})^{2n} ab^2 c}{2 fgn \log(F)} + \frac{(F^{fgx+eg})^{3n} b^3 c}{3 fgn \log(F)} + \frac{3 ((F^{eg})^n fgnx \log(F) - (F^{eg})^n) (F^{fgx})^n a^2 bc}{f^2 g^2 n^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c),x, algorithm="maxima")

[Out]  $\frac{1}{2} a^3 d x^2 + a^3 c x + 3 (F^{(f g x + e g)})^n a^2 b c / (f g n \log(F)) + 3 / 2 (F^{(f g x + e g)})^{(2 n)} a b^2 c / (f g n \log(F)) + 1 / 3 (F^{(f g x + e g)})^{(3 n)} b^3 c / (f g n \log(F)) + 3 ((F^{(e g)})^n f g n x \log(F) - (F^{(e g)})^n) (F^{(f g x)})^n a^2 b d / (f^2 g^2 n^2 \log(F)^2) + 3 / 4 (2 (F^{(e g)})^{(2 n)} f g n x \log(F) - (F^{(e g)})^{(2 n)}) (F^{(f g x)})^{(2 n)} a b^2 d / (f^2 g^2 n^2 \log(F)^2) + 1 / 9 (3 (F^{(e g)})^{(3 n)} f g n x \log(F) - (F^{(e g)})^{(3 n)}) (F^{(f g x)})^{(3 n)} b^3 d / (f^2 g^2 n^2 \log(F)^2)$

**mupad** [B] time = 3.77, size = 219, normalized size = 0.93

$$\frac{a^3 dx^2}{2} - (F^{fgx} F^{eg})^{2n} \left( \frac{3ab^2(d - 2c f g n \ln(F))}{4f^2 g^2 n^2 \ln(F)^2} - \frac{3ab^2 dx}{2f g n \ln(F)} \right) - (F^{fgx} F^{eg})^{3n} \left( \frac{b^3(d - 3c f g n \ln(F))}{9f^2 g^2 n^2 \ln(F)^2} - \frac{3ab^2 dx}{2f g n \ln(F)} \right) - \frac{3ab^2 dx}{2f g n \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*(F^(g*(e + f*x))))^n)^3*(c + d*x), x`

[Out]  $(a^3 d x^2) / 2 - (F^{(f g x)} F^{(e g)})^{(2 n)} * ((3 a b^2 (d - 2 c f g n \log(F))) / (4 f^2 g^2 n^2 \log(F)^2) - (3 a b^2 d x) / (2 f g n \log(F))) - (F^{(f g x)} F^{(e g)})^{(3 n)} * ((b^3 (d - 3 c f g n \log(F))) / (9 f^2 g^2 n^2 \log(F)^2) - (b^3 d x) / (3 f g n \log(F))) - (F^{(f g x)} F^{(e g)})^n * ((3 a^2 b (d - c f g n \log(F))) / (f^2 g^2 n^2 \log(F)^2) - (3 a^2 b d x) / (f g n \log(F))) + a^3 c x$

**sympy** [A] time = 0.34, size = 350, normalized size = 1.48

$$a^3 c x + \frac{a^3 dx^2}{2} + \left\{ \frac{(12b^3 c f^5 g^5 n^5 \log(F)^5 + 12b^3 d f^5 g^5 n^5 x \log(F)^5 - 4b^3 d f^4 g^4 n^4 \log(F)^4) (F^{g(e+fx)})^{3n} + (54ab^2 c f^5 g^5 n^5 \log(F)^5 + 54ab^2 d f^5 g^5 n^5 x \log(F)^5 - 27a^2 b^2 c f^4 g^4 n^4 \log(F)^4) (F^{g(e+fx)})^{2n} + (108a^2 b^2 c f^5 g^5 n^5 \log(F)^5 + 108a^2 b^2 d f^5 g^5 n^5 x \log(F)^5 - 108a^2 b^2 d f^4 g^4 n^4 \log(F)^4) (F^{g(e+fx)})^n}{36f^6 g^6 n^6 \log(F)^6}, \text{Ne}(36f^6 g^6 n^6 \log(F)^6, 0), (x^2(3a^2 b d / 2 + 3a b^2 d / 2 + b^3 d / 2) + x(3a^2 b c + 3a b^2 c + b^3 c), \text{True}) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F**(g*(f*x+e))))**n)**3*(d*x+c), x`

[Out]  $a^{**3} c x + a^{**3} d x^{**2} / 2 + \text{Piecewise}((((12 * b^{**3} * c * f^{**5} * g^{**5} * n^{**5} * \log(F)^{**5} + 12 * b^{**3} * d * f^{**5} * g^{**5} * n^{**5} * x * \log(F)^{**5} - 4 * b^{**3} * d * f^{**4} * g^{**4} * n^{**4} * \log(F)^{**4}) * (F^{**}(g * (e + f * x)))^{**}(3 * n) + (54 * a * b^{**2} * c * f^{**5} * g^{**5} * n^{**5} * \log(F)^{**5} + 54 * a * b^{**2} * d * f^{**5} * g^{**5} * n^{**5} * x * \log(F)^{**5} - 27 * a * b^{**2} * d * f^{**4} * g^{**4} * n^{**4} * \log(F)^{**4}) * (F^{**}(g * (e + f * x)))^{**}(2 * n) + (108 * a^{**2} * b * c * f^{**5} * g^{**5} * n^{**5} * \log(F)^{**5} + 108 * a^{**2} * b * d * f^{**5} * g^{**5} * n^{**5} * x * \log(F)^{**5} - 108 * a^{**2} * b * d * f^{**4} * g^{**4} * n^{**4} * \log(F)^{**4}) * (F^{**}(g * (e + f * x)))^{**}n) / (36 * f^{**6} * g^{**6} * n^{**6} * \log(F)^{**6}), \text{Ne}(36 * f^{**6} * g^{**6} * n^{**6} * \log(F)^{**6}, 0)), (x^{**2} * (3 * a^{**2} * b * d / 2 + 3 * a * b^{**2} * d / 2 + b^{**3} * d / 2) + x * (3 * a^{**2} * b * c + 3 * a * b^{**2} * c + b^{**3} * c), \text{True}))$

$$3.42 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 dx$$

**Optimal.** Leaf size=103

$$a^3x + \frac{3a^2b \left( F^{g(e+fx)} \right)^n}{fgn \log(F)} + \frac{3ab^2 \left( F^{g(e+fx)} \right)^{2n}}{2fgn \log(F)} + \frac{b^3 \left( F^{g(e+fx)} \right)^{3n}}{3fgn \log(F)}$$

[Out]  $a^3x + 3a^2b(F^{g(f*x+e)})^n/f/g/n/\ln(F) + 3/2*a*b^2*(F^{g(f*x+e)})^{(2*n)}/f/g/n/\ln(F) + 1/3*b^3*(F^{g(f*x+e)})^{(3*n)}/f/g/n/\ln(F)$

**Rubi [A]** time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2282, 266, 43}

$$\frac{3a^2b \left( F^{g(e+fx)} \right)^n}{fgn \log(F)} + a^3x + \frac{3ab^2 \left( F^{g(e+fx)} \right)^{2n}}{2fgn \log(F)} + \frac{b^3 \left( F^{g(e+fx)} \right)^{3n}}{3fgn \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^3, x]

[Out]  $a^3x + (3a^2b*(F^{g*(e + f*x)})^n)/(f*g*n*\text{Log}[F]) + (3a*b^2*(F^{g*(e + f*x)})^{(2*n)})/(2*f*g*n*\text{Log}[F]) + (b^3*(F^{g*(e + f*x)})^{(3*n)})/(3*f*g*n*\text{Log}[F])$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 dx &= \frac{\text{Subst} \left( \int \frac{(a+bx^n)^3}{x} dx, x, F^{g(e+fx)} \right)}{fg \log(F)} \\ &= \frac{\text{Subst} \left( \int \frac{(a+bx)^3}{x} dx, x, \left( F^{g(e+fx)} \right)^n \right)}{fgn \log(F)} \\ &= \frac{\text{Subst} \left( \int \left( 3a^2b + \frac{a^3}{x} + 3ab^2x + b^3x^2 \right) dx, x, \left( F^{g(e+fx)} \right)^n \right)}{fgn \log(F)} \\ &= a^3x + \frac{3a^2b \left( F^{g(e+fx)} \right)^n}{fgn \log(F)} + \frac{3ab^2 \left( F^{g(e+fx)} \right)^{2n}}{2fgn \log(F)} + \frac{b^3 \left( F^{g(e+fx)} \right)^{3n}}{3fgn \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 74, normalized size = 0.72

$$a^3x + \frac{b \left( F^{g(e+fx)} \right)^n \left( 18a^2 + 9ab \left( F^{g(e+fx)} \right)^n + 2b^2 \left( F^{g(e+fx)} \right)^{2n} \right)}{6fgn \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^3, x]

[Out] a^3\*x + (b\*(F^(g\*(e + f\*x)))^n\*(18\*a^2 + 9\*a\*b\*(F^(g\*(e + f\*x)))^n + 2\*b^2\*(F^(g\*(e + f\*x)))^(2\*n)))/(6\*f\*g\*n\*Log[F])

**fricas [A]** time = 0.43, size = 84, normalized size = 0.82

$$\frac{6a^3fgnx \log(F) + 18Ffgnx+egn a^2b + 9F^2fgnx+2egn ab^2 + 2F^3fgnx+3egn b^3}{6fgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)^3, x, algorithm="fricas")

[Out] 1/6\*(6\*a^3\*f\*g\*n\*x\*log(F) + 18\*F^(f\*g\*n\*x + e\*g\*n)\*a^2\*b + 9\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n)\*a\*b^2 + 2\*F^(3\*f\*g\*n\*x + 3\*e\*g\*n)\*b^3)/(f\*g\*n\*log(F))

**giac** [A] time = 0.44, size = 102, normalized size = 0.99

$$\frac{18 F^{fgnx} F^{gne} a^2 b + 9 F^2 f^{gnx} F^{2gne} a b^2 + 2 F^3 f^{gnx} F^{3gne} b^3 + 6 a^3 \log(|F|^{fgnx} |F|^{gne})}{6 f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="giac")

[Out]  $\frac{1}{6} * (18 * F^{(f * g * n * x)} * F^{(g * n * e)} * a^2 * b + 9 * F^{(2 * f * g * n * x)} * F^{(2 * g * n * e)} * a * b^2 + 2 * F^{(3 * f * g * n * x)} * F^{(3 * g * n * e)} * b^3 + 6 * a^3 * \log(\text{abs}(F)^{(f * g * n * x)} * \text{abs}(F)^{(g * n * e)})) / (f * g * n * \log(F))$

**maple** [A] time = 0.01, size = 124, normalized size = 1.20

$$\frac{a^3 \ln\left(\left(F^{(f x + e) g}\right)^n\right)}{f g n \ln(F)} + \frac{3 a^2 b \left(F^{(f x + e) g}\right)^n}{f g n \ln(F)} + \frac{3 a b^2 \left(F^{(f x + e) g}\right)^{2 n}}{2 f g n \ln(F)} + \frac{b^3 \left(F^{(f x + e) g}\right)^{3 n}}{3 f g n \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g)))^n+a)^3,x)

[Out]  $\frac{1}{3} / g / f / \ln(F) / n * b^3 * (F^{((f * x + e) * g)})^n)^{3 + 3/2} / g / f / \ln(F) / n * a * b^2 * (F^{((f * x + e) * g)})^n)^{2 + 3} * a^2 * b * (F^{((f * x + e) * g)})^n / f / g / n / \ln(F) + 1 / g / f / \ln(F) / n * a^3 * \ln((F^{((f * x + e) * g)})^n)$

**maxima** [A] time = 1.56, size = 115, normalized size = 1.12

$$a^3 x + \frac{3 (F^{fgx})^n (F^{eg})^n a^2 b}{f g n \log(F)} + \frac{3 (F^{fgx})^{2n} (F^{eg})^{2n} a b^2}{2 f g n \log(F)} + \frac{(F^{fgx})^{3n} (F^{eg})^{3n} b^3}{3 f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="maxima")

[Out]  $a^3 * x + 3 * (F^{(f * g * x)})^n * (F^{(e * g)})^n * a^2 * b / (f * g * n * \log(F)) + 3/2 * (F^{(f * g * x)})^{(2 * n)} * (F^{(e * g)})^{(2 * n)} * a * b^2 / (f * g * n * \log(F)) + 1/3 * (F^{(f * g * x)})^{(3 * n)} * (F^{(e * g)})^{(3 * n)} * b^3 / (f * g * n * \log(F))$

**mupad** [B] time = 3.78, size = 124, normalized size = 1.20

$$\frac{a^3 \ln(F^{fgx})}{f g \ln(F)} + \frac{b^3 (F^{fgx} F^{eg})^{3n}}{3 f g n \ln(F)} + \frac{3 a^2 b (F^{fgx} F^{eg})^n}{f g n \ln(F)} + \frac{3 a b^2 (F^{fgx} F^{eg})^{2n}}{2 f g n \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((a + b*(F^(g*(e + f*x)))^n)^3,x)`

[Out]  $(a^3 \log(F^{f*g*x})) / (f*g*\log(F)) + (b^3 * (F^{f*g*x} * F^{e*g})^{3*n}) / (3*f*g*n*\log(F)) + (3*a^2*b*(F^{f*g*x} * F^{e*g})^n) / (f*g*n*\log(F)) + (3*a*b^2*(F^{f*g*x} * F^{e*g})^{2*n}) / (2*f*g*n*\log(F))$

sympy [A] time = 0.21, size = 153, normalized size = 1.49

$$a^3 x + \begin{cases} \frac{18a^2 b f^2 g^2 n^2 \left(F^{g(e+fx)}\right)^n \log(F)^2 + 9ab^2 f^2 g^2 n^2 \left(F^{g(e+fx)}\right)^{2n} \log(F)^2 + 2b^3 f^2 g^2 n^2 \left(F^{g(e+fx)}\right)^{3n} \log(F)^2}{6f^3 g^3 n^3 \log(F)^3} & \text{for } 6f^3 g^3 n^3 \log(F)^3 \neq 0 \\ x(3a^2 b + 3ab^2 + b^3) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F**(g*(f*x+e))))**n)**3,x)`

[Out]  $a**3*x + \text{Piecewise}(((18*a**2*b*f**2*g**2*n**2*(F**(g*(e + f*x))))**n*\log(F)**2 + 9*a*b**2*f**2*g**2*n**2*(F**(g*(e + f*x))))**(2*n)*\log(F)**2 + 2*b**3*f**2*g**2*n**2*(F**(g*(e + f*x))))**(3*n)*\log(F)**2)/(6*f**3*g**3*n**3*\log(F)**3), \text{Ne}(6*f**3*g**3*n**3*\log(F)**3, 0)), (x*(3*a**2*b + 3*a*b**2 + b**3), \text{True}))$

$$3.43 \quad \int \frac{\left(a + b(F^g(e+fx))^n\right)^3}{c+dx} dx$$

Optimal. Leaf size=200

$$\frac{a^3 \log(c+dx)}{d} + \frac{3a^2 b (F^{eg+fgx})^n F^{gn\left(\frac{e-cf}{d}\right) - gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{3ab^2 (F^{eg+fgx})^{2n} F^{2gn\left(\frac{e-cf}{d}\right) - 2gn(e+fx)} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$

[Out]  $3*a^2*b*F^{((e-c*f/d)*g*n-g*n*(f*x+e))*(F^{(f*g*x+e*g)})^n*Ei(f*g*n*(d*x+c)*\ln(F)/d)/d+3*a*b^2*F^{(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^{(f*g*x+e*g)})^{(2*n)*Ei(2*f*g*n*(d*x+c)*\ln(F)/d)/d+b^3*F^{(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(F^{(f*g*x+e*g)})^{(3*n)*Ei(3*f*g*n*(d*x+c)*\ln(F)/d)/d+a^3*\ln(d*x+c)/d}$

Rubi [A] time = 0.35, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2183, 2182, 2178}

$$\frac{3a^2 b (F^{eg+fgx})^n F^{gn\left(\frac{e-cf}{d}\right) - gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{a^3 \log(c+dx)}{d} + \frac{3ab^2 (F^{eg+fgx})^{2n} F^{2gn\left(\frac{e-cf}{d}\right) - 2gn(e+fx)} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^3/(c + d\*x), x]

[Out]  $(3*a^2*b*F^{((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^{(e*g + f*g*x)})^n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d])/d + (3*a*b^2*F^{(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^{(e*g + f*g*x)})^{(2*n)*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d])/d + (b^3*F^{(3*(e - (c*f)/d)*g*n - 3*g*n*(e + f*x))*(F^{(e*g + f*g*x)})^{(3*n)*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d])/d + (a^3*Log[c + d*x])/d$

Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2182

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^n)/((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Dist[(b\*F^(g\*(e + f\*x)))^n/F^(g\*n\*(e + f\*x)), Int[(c + d\*x)^m\*F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

Rule 2183

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + b(F^{g(e+fx)})^n)^3}{c + dx} dx &= \int \left( \frac{a^3}{c + dx} + \frac{3a^2b(F^{eg+fgx})^n}{c + dx} + \frac{3ab^2(F^{eg+fgx})^{2n}}{c + dx} + \frac{b^3(F^{eg+fgx})^{3n}}{c + dx} \right) dx \\ &= \frac{a^3 \log(c + dx)}{d} + (3a^2b) \int \frac{(F^{eg+fgx})^n}{c + dx} dx + (3ab^2) \int \frac{(F^{eg+fgx})^{2n}}{c + dx} dx + b^3 \int \frac{(F^{eg+fgx})^{3n}}{c + dx} dx \\ &= \frac{a^3 \log(c + dx)}{d} + \left( 3a^2bF^{-n(eg+fgx)} (F^{eg+fgx})^n \right) \int \frac{F^{n(eg+fgx)}}{c + dx} dx + \left( 3ab^2F^{-2n(eg+fgx)} (F^{eg+fgx})^{2n} \right) \int \frac{F^{2n(eg+fgx)}}{c + dx} dx \\ &= \frac{3a^2bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)} (F^{eg+fgx})^n \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + 3ab^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)} (F^{eg+fgx})^{2n} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.50, size = 160, normalized size = 0.80

$$\frac{a^3 \log(c + dx) + 3a^2b (F^{g(e+fx)})^n F^{-\frac{fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + 3ab^2 (F^{g(e+fx)})^{2n} F^{-\frac{2fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n]^3/(c + d\*x), x]

[Out] ((3\*a^2\*b\*(F^(g\*(e + f\*x))))^n\*ExpIntegralEi[(f\*g\*n\*(c + d\*x)\*Log[F])/d])/F^((f\*g\*n\*(c + d\*x))/d) + (3\*a\*b^2\*(F^(g\*(e + f\*x))))^(2\*n)\*ExpIntegralEi[(2\*f\*g\*n\*(c + d\*x)\*Log[F])/d])/F^((2\*f\*g\*n\*(c + d\*x))/d) + (b^3\*(F^(g\*(e + f\*x))))^(3\*n)\*ExpIntegralEi[(3\*f\*g\*n\*(c + d\*x)\*Log[F])/d])/F^((3\*f\*g\*n\*(c + d\*x))/d) + a^3\*Log[c + d\*x])/d

**fricas [A]** time = 0.44, size = 140, normalized size = 0.70

$$\frac{F^{\frac{3(de-cf)gn}{d}} b^3 \operatorname{Ei}\left(\frac{3(dfgnx+cfgn)\log(F)}{d}\right) + 3F^{\frac{2(de-cf)gn}{d}} ab^2 \operatorname{Ei}\left(\frac{2(dfgnx+cfgn)\log(F)}{d}\right) + 3F^{\frac{(de-cf)gn}{d}} a^2 b \operatorname{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c),x, algorithm="fricas")

[Out] (F^(3\*(d\*e - c\*f)\*g\*n/d)\*b^3\*Ei(3\*(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + 3\*F^(2\*(d\*e - c\*f)\*g\*n/d)\*a\*b^2\*Ei(2\*(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + 3\*F^((d\*e - c\*f)\*g\*n/d)\*a^2\*b\*Ei((d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + a^3\*log(d\*x + c))/d

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left( \left( F^{(f^{x+e})g} \right)^n b + a \right)^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c),x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^3/(d\*x + c), x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left( b \left( F^{(f^{x+e})g} \right)^n + a \right)^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^3/(d\*x+c),x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^3/(d\*x+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^{3n} b^3 \int \frac{(F^{fgx})^{3n}}{dx + c} dx + 3(F^{eg})^{2n} ab^2 \int \frac{(F^{fgx})^{2n}}{dx + c} dx + 3(F^{eg})^n a^2 b \int \frac{(F^{fgx})^n}{dx + c} dx + \frac{a^3 \log(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c),x, algorithm="maxima")

[Out] (F^(e\*g))^(3\*n)\*b^3\*integrate((F^(f\*g\*x))^(3\*n)/(d\*x + c), x) + 3\*(F^(e\*g))^(2\*n)\*a\*b^2\*integrate((F^(f\*g\*x))^(2\*n)/(d\*x + c), x) + 3\*(F^(e\*g))^n\*a^2\*b\*integrate((F^(f\*g\*x))^n/(d\*x + c), x) + a^3\*log(d\*x + c)/d

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x)))^n)^3/(c + d\*x), x)

[Out] int((a + b\*(F^(g\*(e + f\*x)))^n)^3/(c + d\*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \left(F^{egfgx}\right)^n\right)^3}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3/(d\*x+c), x)

[Out] Integral((a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))))\*\*n)\*\*3/(c + d\*x), x)

$$3.44 \quad \int \frac{\left(a + b(F^{g(e+fx)})^n\right)^3}{(c+dx)^2} dx$$

**Optimal.** Leaf size=305

$$-\frac{a^3}{d(c+dx)} + \frac{3a^2bfgn \log(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{3a^2b (F^{eg+fgx})^n}{d(c+dx)} + \frac{6ab^2fgn \log(F) (F^{eg+fgx})^n}{d(c+dx)}$$

[Out]  $-a^3/d/(d*x+c) - 3*a^2*b*(F^(f*g*x+e*g))^n/d/(d*x+c) - 3*a*b^2*(F^(f*g*x+e*g))^(2*n)/d/(d*x+c) - b^3*(F^(f*g*x+e*g))^(3*n)/d/(d*x+c) + 3*a^2*b*f*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g*n*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2 + 6*a*b^2*f*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g*n*Ei(2*f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2 + 3*b^3*f*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(F^(f*g*x+e*g))^(3*n)*g*n*Ei(3*f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2$

**Rubi [A]** time = 0.49, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {2183, 2177, 2182, 2178}

$$\frac{3a^2bfgn \log(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{3a^2b (F^{eg+fgx})^n}{d(c+dx)} - \frac{a^3}{d(c+dx)} + \frac{6ab^2fgn \log(F) (F^{eg+fgx})^n}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^3/(c + d\*x)^2, x]

[Out]  $-(a^3/(d*(c + d*x))) - (3*a^2*b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)) - (3*a*b^2*(F^(e*g + f*g*x))^(2*n))/(d*(c + d*x)) - (b^3*(F^(e*g + f*g*x))^(3*n))/(d*(c + d*x)) + (3*a^2*b*f*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*g*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2 + (6*a*b^2*f*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*g*n*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2 + (3*b^3*f*F^(3*(e - (c*f)/d)*g*n - 3*g*n*(e + f*x))*(F^(e*g + f*g*x))^(3*n)*g*n*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2$

**Rule 2177**

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n)/(d\*(m + 1)), x] - Dist[(f\*g\*n\*Log[F])/d\*(m + 1), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !\$UseGamma == True

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2182

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] := Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2183

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^((p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3}{(c + dx)^2} dx &= \int \left( \frac{a^3}{(c + dx)^2} + \frac{3a^2b \left(F^{g+fgx}\right)^n}{(c + dx)^2} + \frac{3ab^2 \left(F^{g+fgx}\right)^{2n}}{(c + dx)^2} + \frac{b^3 \left(F^{g+fgx}\right)^{3n}}{(c + dx)^2} \right) dx \\ &= -\frac{a^3}{d(c + dx)} + (3a^2b) \int \frac{\left(F^{g+fgx}\right)^n}{(c + dx)^2} dx + (3ab^2) \int \frac{\left(F^{g+fgx}\right)^{2n}}{(c + dx)^2} dx + b^3 \int \frac{\left(F^{g+fgx}\right)^{3n}}{(c + dx)^2} dx \\ &= -\frac{a^3}{d(c + dx)} - \frac{3a^2b \left(F^{g+fgx}\right)^n}{d(c + dx)} - \frac{3ab^2 \left(F^{g+fgx}\right)^{2n}}{d(c + dx)} - \frac{b^3 \left(F^{g+fgx}\right)^{3n}}{d(c + dx)} + \frac{(3a^2bfgn \log(F)) \left(F^{g+fgx}\right)^n}{d(c + dx)} \\ &= -\frac{a^3}{d(c + dx)} - \frac{3a^2b \left(F^{g+fgx}\right)^n}{d(c + dx)} - \frac{3ab^2 \left(F^{g+fgx}\right)^{2n}}{d(c + dx)} - \frac{b^3 \left(F^{g+fgx}\right)^{3n}}{d(c + dx)} + \frac{(3a^2bfF^{-n(eg+f)} \log(F)) \left(F^{g+fgx}\right)^n}{d(c + dx)} \\ &= -\frac{a^3}{d(c + dx)} - \frac{3a^2b \left(F^{g+fgx}\right)^n}{d(c + dx)} - \frac{3ab^2 \left(F^{g+fgx}\right)^{2n}}{d(c + dx)} - \frac{b^3 \left(F^{g+fgx}\right)^{3n}}{d(c + dx)} + \frac{3a^2bfF^{\left(\frac{e-cf}{d}\right)g} \log(F) \left(F^{g+fgx}\right)^n}{d(c + dx)} \end{aligned}$$

**Mathematica [A]** time = 1.32, size = 250, normalized size = 0.82

$$\frac{a^3 d - 3a^2 b f g n \log(F) (c + dx) \left(F^{g(e+fx)}\right)^n F^{-\frac{fgn(c+dx)}{d}} \operatorname{Ei}\left(\frac{fgn(c+dx) \log(F)}{d}\right) + 3a^2 b d \left(F^{g(e+fx)}\right)^n - 6ab^2 f g n \log(F) (c + dx) \left(F^{g(e+fx)}\right)^{2n} + b^3 \left(F^{g(e+fx)}\right)^{3n}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n]^3/(c + d\*x)^2,x]

[Out] -((a^3\*d + 3\*a^2\*b\*d\*(F^(g\*(e + f\*x))))^n + 3\*a\*b^2\*d\*(F^(g\*(e + f\*x)))^(2\*n) + b^3\*d\*(F^(g\*(e + f\*x)))^(3\*n) - (3\*a^2\*b\*f\*(F^(g\*(e + f\*x))))^n\*g\*n\*(c + d\*x)\*ExpIntegralEi[(f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F])/F^((f\*g\*n\*(c + d\*x))/d) - (6\*a\*b^2\*f\*(F^(g\*(e + f\*x)))^(2\*n)\*g\*n\*(c + d\*x)\*ExpIntegralEi[(2\*f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F])/F^((2\*f\*g\*n\*(c + d\*x))/d) - (3\*b^3\*f\*(F^(g\*(e + f\*x)))^(3\*n)\*g\*n\*(c + d\*x)\*ExpIntegralEi[(3\*f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F])/F^((3\*f\*g\*n\*(c + d\*x))/d))/(d^2\*(c + d\*x))

**fricas** [A] time = 0.44, size = 259, normalized size = 0.85

$$3 F^{fgnx+egn} a^2 b d + 3 F^{2fgnx+2egn} a b^2 d + F^{3fgnx+3egn} b^3 d + a^3 d - 3 (b^3 d f g n x + b^3 c f g n) F^{\frac{3(de-cf)gn}{d}} \text{Ei}\left(\frac{3(df g n x + c f g n)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c)^2,x, algorithm="fricas")

[Out] -(3\*F^(f\*g\*n\*x + e\*g\*n)\*a^2\*b\*d + 3\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n)\*a\*b^2\*d + F^(3\*f\*g\*n\*x + 3\*e\*g\*n)\*b^3\*d + a^3\*d - 3\*(b^3\*d\*f\*g\*n\*x + b^3\*c\*f\*g\*n)\*F^(3\*(d\*e - c\*f)\*g\*n/d)\*Ei(3\*(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d)\*log(F) - 6\*(a\*b^2\*d\*f\*g\*n\*x + a\*b^2\*c\*f\*g\*n)\*F^(2\*(d\*e - c\*f)\*g\*n/d)\*Ei(2\*(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d)\*log(F) - 3\*(a^2\*b\*d\*f\*g\*n\*x + a^2\*b\*c\*f\*g\*n)\*F^((d\*e - c\*f)\*g\*n/d)\*Ei((d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d)\*log(F))/(d^3\*x + c\*d^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left( \left( F^{(f x + e) g} \right)^n b + a \right)^3}{(d x + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^3/(d\*x + c)^2, x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left( b \left( F^{(f x + e) g} \right)^n + a \right)^3}{(d x + c)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*(F^((f*x+e)*g))^n+a)^3/(d*x+c)^2,x)`

[Out] `int((b*(F^((f*x+e)*g))^n+a)^3/(d*x+c)^2,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^{3n} b^3 \int \frac{(Ffgx)^{3n}}{d^2x^2 + 2cdx + c^2} dx + 3(F^{eg})^{2n} ab^2 \int \frac{(Ffgx)^{2n}}{d^2x^2 + 2cdx + c^2} dx + 3(F^{eg})^n a^2 b \int \frac{(Ffgx)^n}{d^2x^2 + 2cdx + c^2} dx - \frac{a^3}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F^(g*(f*x+e))))^n)^3/(d*x+c)^2,x, algorithm="maxima")`

[Out] `(F^(e*g))^(3*n)*b^3*integrate((F^(f*g*x))^(3*n)/(d^2*x^2 + 2*c*d*x + c^2), x) + 3*(F^(e*g))^(2*n)*a*b^2*integrate((F^(f*g*x))^(2*n)/(d^2*x^2 + 2*c*d*x + c^2), x) + 3*(F^(e*g))^n*a^2*b*integrate((F^(f*g*x))^n/(d^2*x^2 + 2*c*d*x + c^2), x) - a^3/(d^2*x + c*d)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*(F^(g*(e + f*x))))^n)^3/(c + d*x)^2,x)`

[Out] `int((a + b*(F^(g*(e + f*x))))^n)^3/(c + d*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \left(F^{eg} Ffgx\right)^n\right)^3}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(F**(g*(f*x+e))))**n)**3/(d*x+c)**2,x)`

[Out] `Integral((a + b*(F**(e*g)*F**(f*g*x))))**n)**3/(c + d*x)**2, x)`

$$3.45 \quad \int \frac{\left(a + b(F^{g(e+fx)})^n\right)^3}{(c+dx)^3} dx$$

**Optimal.** Leaf size=447

$$\frac{a^3}{2d(c+dx)^2} + \frac{3a^2bf^2g^2n^2 \log^2(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{2d^3} - \frac{3a^2bfgn \log(F) (F^{eg+fgx})^n}{2d^2(c+dx)}$$

[Out]  $-1/2*a^3/d/(d*x+c)^2-3/2*a^2*b*(F^(f*g*x+e*g))^n/d/(d*x+c)^2-3/2*a*b^2*(F^(f*g*x+e*g))^(2*n)/d/(d*x+c)^2-1/2*b^3*(F^(f*g*x+e*g))^(3*n)/d/(d*x+c)^2-3/2*a^2*b*f*(F^(f*g*x+e*g))^n*g*n*\ln(F)/d^2/(d*x+c)-3*a*b^2*f*(F^(f*g*x+e*g))^(2*n)*g*n*\ln(F)/d^2/(d*x+c)-3/2*b^3*f*(F^(f*g*x+e*g))^(3*n)*g*n*\ln(F)/d^2/(d*x+c)+3/2*a^2*b*f^2*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g^2*n^2*Ei(f*g*n*(d*x+c)*\ln(F)/d)*\ln(F)^2/d^3+6*a*b^2*f^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g^2*n^2*Ei(2*f*g*n*(d*x+c)*\ln(F)/d)*\ln(F)^2/d^3+9/2*b^3*f^2*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(F^(f*g*x+e*g))^(3*n)*g^2*n^2*Ei(3*f*g*n*(d*x+c)*\ln(F)/d)*\ln(F)^2/d^3$

**Rubi [A]** time = 0.69, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {2183, 2177, 2182, 2178}

$$\frac{3a^2bf^2g^2n^2 \log^2(F) (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{2d^3} - \frac{3a^2bfgn \log(F) (F^{eg+fgx})^n}{2d^2(c+dx)} - \frac{3a^2b (F^{eg+fgx})^n}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^3/(c + d\*x)^3, x]

[Out]  $-a^3/(2*d*(c+d*x)^2) - (3*a^2*b*(F^(e*g+f*g*x))^n)/(2*d*(c+d*x)^2) - (3*a*b^2*(F^(e*g+f*g*x))^(2*n))/(2*d*(c+d*x)^2) - (b^3*(F^(e*g+f*g*x))^(3*n))/(2*d*(c+d*x)^2) - (3*a^2*b*f*(F^(e*g+f*g*x))^n*g*n*\operatorname{Log}[F])/(2*d^2*(c+d*x)) - (3*a*b^2*f*(F^(e*g+f*g*x))^(2*n)*g*n*\operatorname{Log}[F])/(d^2*(c+d*x)) - (3*b^3*f*(F^(e*g+f*g*x))^(3*n)*g*n*\operatorname{Log}[F])/(2*d^2*(c+d*x)) + (3*a^2*b*f^2*F^((e-(c*f)/d)*g*n-g*n*(e+f*x))*(F^(e*g+f*g*x))^n*g^2*n^2*\operatorname{ExpIntegralEi}[(f*g*n*(c+d*x)*\operatorname{Log}[F])/d]*\operatorname{Log}[F]^2)/(2*d^3) + (6*a*b^2*f^2*F^(2*(e-(c*f)/d)*g*n-2*g*n*(e+f*x))*(F^(e*g+f*g*x))^(2*n)*g^2*n^2*\operatorname{ExpIntegralEi}[(2*f*g*n*(c+d*x)*\operatorname{Log}[F])/d]*\operatorname{Log}[F]^2)/d^3 + (9*b^3*f^2*F^(3*(e-(c*f)/d)*g*n-3*g*n*(e+f*x))*(F^(e*g+f*g*x))^(3*n)*g^2*n^2*\operatorname{ExpIntegralEi}[(3*f*g*n*(c+d*x)*\operatorname{Log}[F])/d]*\operatorname{Log}[F]^2)/(2*d^3)$

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1)), x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !$UseGamma === True
```

### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2182

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

### Rule 2183

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b(Fg^{e+fx}))^n)^3}{(c + dx)^3} dx &= \int \left( \frac{a^3}{(c + dx)^3} + \frac{3a^2b(Fg^{e+fx})^n}{(c + dx)^3} + \frac{3ab^2(Fg^{e+fx})^{2n}}{(c + dx)^3} + \frac{b^3(Fg^{e+fx})^{3n}}{(c + dx)^3} \right) dx \\
&= -\frac{a^3}{2d(c + dx)^2} + (3a^2b) \int \frac{(Fg^{e+fx})^n}{(c + dx)^3} dx + (3ab^2) \int \frac{(Fg^{e+fx})^{2n}}{(c + dx)^3} dx + b^3 \int \frac{(Fg^{e+fx})^{3n}}{(c + dx)^3} dx \\
&= -\frac{a^3}{2d(c + dx)^2} - \frac{3a^2b(Fg^{e+fx})^n}{2d(c + dx)^2} - \frac{3ab^2(Fg^{e+fx})^{2n}}{2d(c + dx)^2} - \frac{b^3(Fg^{e+fx})^{3n}}{2d(c + dx)^2} + \frac{(3a^2bfgn \log(F)(c + dx))}{2d^2(c + dx)^2} \\
&= -\frac{a^3}{2d(c + dx)^2} - \frac{3a^2b(Fg^{e+fx})^n}{2d(c + dx)^2} - \frac{3ab^2(Fg^{e+fx})^{2n}}{2d(c + dx)^2} - \frac{b^3(Fg^{e+fx})^{3n}}{2d(c + dx)^2} - \frac{3a^2bf(Fg^{e+fx})^n \log(F)(c + dx)}{2d^2(c + dx)^2} \\
&= -\frac{a^3}{2d(c + dx)^2} - \frac{3a^2b(Fg^{e+fx})^n}{2d(c + dx)^2} - \frac{3ab^2(Fg^{e+fx})^{2n}}{2d(c + dx)^2} - \frac{b^3(Fg^{e+fx})^{3n}}{2d(c + dx)^2} - \frac{3a^2bf(Fg^{e+fx})^n \log(F)(c + dx)}{2d^2(c + dx)^2} \\
&= -\frac{a^3}{2d(c + dx)^2} - \frac{3a^2b(Fg^{e+fx})^n}{2d(c + dx)^2} - \frac{3ab^2(Fg^{e+fx})^{2n}}{2d(c + dx)^2} - \frac{b^3(Fg^{e+fx})^{3n}}{2d(c + dx)^2} - \frac{3a^2bf(Fg^{e+fx})^n \log(F)(c + dx)}{2d^2(c + dx)^2}
\end{aligned}$$

**Mathematica** [A] time = 1.06, size = 325, normalized size = 0.73

$$\frac{a^3 d^2 - 3a^2 b f^2 g^2 n^2 \log^2(F)(c + dx)^2 (Fg^{e+fx})^n F^{-\frac{fgn(c+dx)}{d}} \text{Ei}\left(\frac{fgn(c+dx) \log(F)}{d}\right) + 3a^2 b d (Fg^{e+fx})^n (fgn \log(F)(c + dx))}{d^3 (c + dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n]^3/(c + d\*x)^3,x]

[Out] -1/2\*(a^3\*d^2 - (3\*a^2\*b\*f^2\*(F^(g\*(e + f\*x))))^n\*g^2\*n^2\*(c + d\*x)^2\*ExpIntegralEi[(f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F]^2)/F^((f\*g\*n\*(c + d\*x))/d) - (12\*a\*b^2\*f^2\*(F^(g\*(e + f\*x))))^(2\*n)\*g^2\*n^2\*(c + d\*x)^2\*ExpIntegralEi[(2\*f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F]^2)/F^((2\*f\*g\*n\*(c + d\*x))/d) - (9\*b^3\*f^2\*(F^(g\*(e + f\*x))))^(3\*n)\*g^2\*n^2\*(c + d\*x)^2\*ExpIntegralEi[(3\*f\*g\*n\*(c + d\*x)\*Log[F])/d]\*Log[F]^2)/F^((3\*f\*g\*n\*(c + d\*x))/d) + 3\*a^2\*b\*d\*(F^(g\*(e + f\*x)))^n\*(d + f\*g\*n\*(c + d\*x)\*Log[F]) + 3\*a\*b^2\*d\*(F^(g\*(e + f\*x)))^(2\*n)\*(d + 2\*f\*g\*n\*(c + d\*x)\*Log[F]) + b^3\*d\*(F^(g\*(e + f\*x)))^(3\*n)\*(d + 3\*f\*g\*n\*(c + d\*x)\*Log[F])/(d^3\*(c + d\*x)^2)

**fricas** [A] time = 0.43, size = 475, normalized size = 1.06

$$a^3 d^2 - 9 \left( b^3 d^2 f^2 g^2 n^2 x^2 + 2 b^3 c d f^2 g^2 n^2 x + b^3 c^2 f^2 g^2 n^2 \right) F^{\frac{3(de-cf)gn}{d}} \operatorname{Ei} \left( \frac{3(dfgnx+cfgn)\log(F)}{d} \right) \log(F)^2 - 12 \left( ab^2 d^2 f \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c)^3,x, algorithm="fricas")

[Out] 
$$-1/2*(a^3*d^2 - 9*(b^3*d^2*f^2*g^2*n^2*x^2 + 2*b^3*c*d*f^2*g^2*n^2*x + b^3*c^2*f^2*g^2*n^2))*F^{3*(d*e - c*f)*g*n/d}*Ei(3*(d*f*g*n*x + c*f*g*n)*\log(F)/d)*\log(F)^2 - 12*(a*b^2*d^2*f^2*g^2*n^2*x^2 + 2*a*b^2*c*d*f^2*g^2*n^2*x + a*b^2*c^2*f^2*g^2*n^2)*F^{2*(d*e - c*f)*g*n/d}*Ei(2*(d*f*g*n*x + c*f*g*n)*\log(F)/d)*\log(F)^2 - 3*(a^2*b*d^2*f^2*g^2*n^2*x^2 + 2*a^2*b*c*d*f^2*g^2*n^2*x + a^2*b*c^2*f^2*g^2*n^2)*F^{(d*e - c*f)*g*n/d}*Ei((d*f*g*n*x + c*f*g*n)*\log(F)/d)*\log(F)^2 + (b^3*d^2 + 3*(b^3*d^2*f*g*n*x + b^3*c*d*f*g*n)*\log(F))*F^{3*f*g*n*x + 3*e*g*n} + 3*(a*b^2*d^2 + 2*(a*b^2*d^2*f*g*n*x + a*b^2*c*d*f*g*n)*\log(F))*F^{2*f*g*n*x + 2*e*g*n} + 3*(a^2*b*d^2 + (a^2*b*d^2*f*g*n*x + a^2*b*c*d*f*g*n)*\log(F))*F^{f*g*n*x + e*g*n})/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left( \left( F^{(fx+e)g} \right)^n b + a \right)^3}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c)^3,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^3/(d\*x + c)^3, x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left( b \left( F^{(fx+e)g} \right)^n + a \right)^3}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^3/(d\*x+c)^3,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^3/(d\*x+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^{3n} b^3 \int \frac{(Ffgx)^{3n}}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3} dx + 3(F^{eg})^{2n} ab^2 \int \frac{(Ffgx)^{2n}}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3} dx + 3(F^{eg})^n a^2 b \int \frac{1}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c)^3,x, algorithm="maxima")

[Out] (F^(e\*g))^(3\*n)\*b^3\*integrate((F^(f\*g\*x))^(3\*n)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x) + 3\*(F^(e\*g))^(2\*n)\*a\*b^2\*integrate((F^(f\*g\*x))^(2\*n)/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x) + 3\*(F^(e\*g))^n\*a^2\*b\*integrate((F^(f\*g\*x))^n/(d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x + c^3), x) - 1/2\*a^3/(d^3\*x^2 + 2\*c\*d^2\*x + c^2\*d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \left(F^g(e+fx)\right)^n\right)^3}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^3/(c + d\*x)^3,x)

[Out] int((a + b\*(F^(g\*(e + f\*x))))^n)^3/(c + d\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \left(F^{eg}Ffgx\right)^n\right)^3}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3/(d\*x+c)\*\*3,x)

[Out] Integral((a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))))\*\*n)\*\*3/(c + d\*x)\*\*3, x)

$$3.46 \quad \int \frac{(c+dx)^3}{a+b(Fg^{(e+fx)})^n} dx$$

**Optimal.** Leaf size=192

$$\frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(\frac{b(Fg^{(e+fx)})^n}{a}+1\right)}{afgn\log(F)} - \frac{6d^3\text{Li}_4\left(-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^4g^4n^4\log^4(F)}$$

[Out]  $1/4*(d*x+c)^4/a/d-(d*x+c)^3*\ln(1+b*(F^(g*(f*x+e)))^n/a)/a/f/g/n/\ln(F)-3*d*(d*x+c)^2*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a/f^2/g^2/n^2/\ln(F)^2+6*d^2*(d*x+c)*polylog(3,-b*(F^(g*(f*x+e)))^n/a)/a/f^3/g^3/n^3/\ln(F)^3-6*d^3*polylog(4,-b*(F^(g*(f*x+e)))^n/a)/a/f^4/g^4/n^4/\ln(F)^4$

**Rubi [A]** time = 0.33, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\text{PolyLog}\left(2,-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^2g^2n^2\log^2(F)} - \frac{6d^3\text{PolyLog}\left(4,-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^4g^4n^4\log^4(F)} - \frac{(c+dx)^3\log\left(\frac{b(Fg^{(e+fx)})^n}{a}+1\right)}{afgn\log(F)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x)))^n), x]

[Out]  $(c+d*x)^4/(4*a*d) - ((c+d*x)^3*\text{Log}[1+(b*(F^(g*(e+f*x)))^n)/a])/(a*f*g*n*\text{Log}[F]) - (3*d*(c+d*x)^2*\text{PolyLog}[2,-((b*(F^(g*(e+f*x)))^n)/a)])/(a*f^2*g^2*n^2*\text{Log}[F]^2) + (6*d^2*(c+d*x)*\text{PolyLog}[3,-((b*(F^(g*(e+f*x)))^n)/a)])/(a*f^3*g^3*n^3*\text{Log}[F]^3) - (6*d^3*\text{PolyLog}[4,-((b*(F^(g*(e+f*x)))^n)/a)])/(a*f^4*g^4*n^4*\text{Log}[F]^4)$

**Rule 2184**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2190**

Int[((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*\text{Log}[1+(b\*(F^(g\*(e+f\*x)))^n)/a])/(b\*f\*g\*n\*\text{Log}[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*\text{Log}[F]), Int[(c + d\*x)^(m - 1)\*\text{Log}[1+(b\*(F^(g\*(e+f\*x)))^n)/a], x], x]

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :=> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :=> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(p\_)], x\_Symbol] :=> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{(c+dx)^3}{a+b(F^{g(e+fx)})^n} dx &= \frac{(c+dx)^4}{4ad} - \frac{b \int \frac{(F^{g(e+fx)})^n (c+dx)^3}{a+b(F^{g(e+fx)})^n} dx}{a} \\
&= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} + \frac{(3d) \int (c+dx)^2 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right) dx}{afgn \log(F)} \\
&= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} + \frac{(6d^2) \int (c+dx) \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right) dx}{afgn \log(F)} \\
&= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} + \frac{6d^2(c+dx) \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} \\
&= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} + \frac{6d^2(c+dx) \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} \\
&= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} + \frac{6d^2(c+dx) \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 166, normalized size = 0.86

$$\frac{3d \left( f^2g^2n^2 \log^2(F)(c+dx)^2 \text{Li}_2\left(-\frac{a(F^{g(e+fx)})^{-n}}{b}\right) + 2d \left( fgn \log(F)(c+dx) \text{Li}_3\left(-\frac{a(F^{g(e+fx)})^{-n}}{b}\right) + d \text{Li}_4\left(-\frac{a(F^{g(e+fx)})^{-n}}{b}\right) \right) \right)}{f^3g^3n^3 \log^3(F)} - (c+dx)^3 \log\left(\frac{a(F^{g(e+fx)})}{b}\right)$$


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$$afgn \log(F)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x)))^n), x]

[Out] (-((c + d\*x)^3\*Log[1 + a/(b\*(F^(g\*(e + f\*x)))^n])) + (3\*d\*(f^2\*g^2\*n^2\*(c + d\*x)^2\*Log[F]^2\*PolyLog[2, -a/(b\*(F^(g\*(e + f\*x)))^n])) + 2\*d\*(f\*g\*n\*(c + d\*x)\*Log[F]\*PolyLog[3, -a/(b\*(F^(g\*(e + f\*x)))^n])) + d\*PolyLog[4, -a/(b\*(F^(g\*(e + f\*x)))^n)])))/(f^3\*g^3\*n^3\*Log[F]^3)/(a\*f\*g\*n\*Log[F])

**fricas** [C] time = 0.44, size = 413, normalized size = 2.15

$$4(d^3e^3 - 3cd^2e^2f + 3c^2def^2 - c^3f^3)g^3n^3 \log(Ffg^{nx+egn}b + a) \log(F)^3 + (d^3f^4g^4n^4x^4 + 4cd^2f^4g^4n^4x^3 + 6c^2df^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e))))^n),x, algorithm="fricas")

[Out] 1/4\*(4\*(d^3\*e^3 - 3\*c\*d^2\*e^2\*f + 3\*c^2\*d\*e\*f^2 - c^3\*f^3)\*g^3\*n^3\*log(F^(f\*g\*n\*x + e\*g\*n)\*b + a)\*log(F)^3 + (d^3\*f^4\*g^4\*n^4\*x^4 + 4\*c\*d^2\*f^4\*g^4\*n^4\*x^3 + 6\*c^2\*d\*f^4\*g^4\*n^4\*x^2 + 4\*c^3\*f^4\*g^4\*n^4\*x)\*log(F)^4 - 4\*(d^3\*f^3\*g^3\*n^3\*x^3 + 3\*c\*d^2\*f^3\*g^3\*n^3\*x^2 + 3\*c^2\*d\*f^3\*g^3\*n^3\*x + (d^3\*e^3 - 3\*c\*d^2\*e^2\*f + 3\*c^2\*d\*e\*f^2)\*g^3\*n^3)\*log(F)^3\*log((F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a) - 12\*(d^3\*f^2\*g^2\*n^2\*x^2 + 2\*c\*d^2\*f^2\*g^2\*n^2\*x + c^2\*d\*f^2\*g^2\*n^2)\*dilog(-(F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a + 1)\*log(F)^2 - 24\*d^3\*polylog(4, -F^(f\*g\*n\*x + e\*g\*n)\*b/a) + 24\*(d^3\*f\*g\*n\*x + c\*d^2\*f\*g\*n)\*log(F)\*polylog(3, -F^(f\*g\*n\*x + e\*g\*n)\*b/a))/(a\*f^4\*g^4\*n^4\*log(F)^4)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{\left(F^{(f*x+e)g}\right)^n b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e))))^n),x, algorithm="giac")

[Out] integrate((d\*x + c)^3/((F^((f\*x + e)\*g))^n\*b + a), x)

**maple** [B] time = 0.09, size = 3075, normalized size = 16.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3/(b\*(F^((f\*x+e)\*g)))^n+a),x)

[Out] 3\*c\*d^2/a\*x^3+3\*c^2\*d/a\*x^2-3/f/g/ln(F)\*d^3/a\*ln(F^((f\*x+e)\*g))\*x^3+9/2/f^2/g^2/ln(F)^2\*d^3/a\*ln(F^((f\*x+e)\*g))^2\*x^2-2/f^3/g^3/ln(F)^3\*c\*d^2/a\*ln(F^((f\*x+e)\*g))^3+3/2/f^2/g^2/ln(F)^2\*c^2\*d/a\*ln(F^((f\*x+e)\*g))^2-6/n^4/f^4/g^4/ln(F)^4\*d^3/a\*polylog(4,-b\*F^(n\*f\*g\*x)\*F^(-n\*f\*g\*x)\*(F^((f\*x+e)\*g))^n/a)+1/f^3/g^3/ln(F)^3\*d^3/a\*x\*(ln(F^((f\*x+e)\*g))-(f\*x+e)\*g\*ln(F))^3-1/n/f/g/ln(F)\*c^3/a\*ln(a+b\*F^(n\*f\*g\*x)\*F^(-n\*f\*g\*x)\*(F^((f\*x+e)\*g))^n)+1/n/f/g/ln(F)\*c^3/a\*ln(F^(n\*f\*g\*x)\*F^(-n\*f\*g\*x)\*(F^((f\*x+e)\*g))^n)+d^3/a\*x^4+3/f\*c^2\*d/a\*x\*

$$\begin{aligned}
& e^{-3/f^2} c^d a^x e^{2+3/4/f^4/g^4/\ln(F)^4} d^3/a \ln(F^{((f*x+e)*g)})^{4+1/f^3} d^3/a^x e^{-6/n^2/f^2/g^2/\ln(F)^2} c^d a^x \text{polylog}(2, -bF^{(nf*gx)} F^{-(nf*gx)}) \\
& * (F^{((f*x+e)*g)})^n/a * x + 3/n/f^3/g/\ln(F) * c^d a^x e^{2/a} \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) \\
& * (F^{((f*x+e)*g)})^n/a - 3/n/f^4/g^2/\ln(F)^2 d^3 e^{2*} (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F)) \\
& /a \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a - 3/n/f^4/g^3/\ln(F)^3 d^3 e^* (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F))^2/a \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a - 6/f^2/g/\ln(F) * c^d a^x e^* (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F)) + 3/n/f^3/g^3/\ln(F)^3 c^d a^x e^{2*} (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F))^2/a \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a - 3/n/f/g/\ln(F) * c^2 d/a \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a * x + 3/n/f^4/g^3/\ln(F)^3 d^3 e^* (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F))^2/a \ln(a+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n - 3/n/f^4/g^2/\ln(F)^2 d^3 e^{2*} (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F)) /a \ln(a+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n - 3/n/f^2/g/\ln(F) * c^2 d e/a \ln(F^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n + 3/n/f^2/g/\ln(F) * c^2 d e/a \ln(a+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n - 3/n/f^3/g/\ln(F) * c^d a^x e^{2/a} \ln(a+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n + 3/n/f^2/g^2/\ln(F)^2 c^2 d^* (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F)) /a \ln(a+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n - 3/n/f/g/\ln(F) * c^d a^x \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a * x^2 - 3/n/f^2/g^2/\ln(F)^2 c^2 d^* (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F)) /a \ln(F^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n - 3/n/f^4/g^3/\ln(F)^3 d^3 e^* (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F))^2/a \ln(F^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n + 3/n/f^3/g^3/\ln(F)^3 c^d a^x e^{2*} (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F))^2/a \ln(F^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n - 3/n/f^3/g^3/\ln(F)^3 c^d a^x e^{2/a} \ln(F^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n + 3/n/f^3/g^3/\ln(F)^3 d^3/a \ln(F^{((f*x+e)*g)})^3 * x - 3/n/f^2/g/\ln(F) * c^2 d/a \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a * e^{-3/n/f^2/g^2/\ln(F)^2} c^2 d/a \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F)) + 3/f^3/g/\ln(F) * d^3/a^x e^{2*} (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F)) + 3/f^3/g^2/\ln(F)^2 d^3/a^x e^* (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F))^2 - 1/n/f^4/g^4/\ln(F)^4 d^3 * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F))^3/a \ln(F^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n + 1/n/f^4/g/\ln(F) * d^3 e^3/a \ln(a+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n - 1/n/f^4/g/\ln(F) * d^3 e^3/a \ln(F^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n - 3/n^2/f^2/g^2/\ln(F)^2 c^2 d/a \text{polylog}(2, -bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a + 6/n^3/f^3/g^3/\ln(F)^3 c^d a^x \text{polylog}(3, -bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a + 3/f/g/\ln(F) * c^2 d/a^x * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F)) - 3/f^2/g^2/\ln(F)^2 c^d a^x * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g \ln(F))^2 - 6/f/g/\ln(F) * c^d a^x \ln(F^{((f*x+e)*g)}) * x^2 + 6/f^2/g^2/\ln(F)^2 c^d a^x \ln(F^{((f*x+e)*g)})^2 * x - 3/f/g/\ln(F) * c^2 d/a \ln(F^{((f*x+e)*g)}) * x + 6/n^3/f^3/g^3/\ln(F)^3 d^3/a \text{polylog}(3, -bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a * x - 1/n/f/g/\ln(F) * d^3/a \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a * x^3 - 1/n/f^4/g/\ln(F) * d^3/a \ln(1+bF^{(nf*gx)} F^{-(nf*gx)}) * (F^{((f*x+e)*g)})^n/a * e^{-3-1/n/f^4/g^4/\ln}
\end{aligned}$$

$(F)^4 d^3/a \ln(1+bF^{(nf*gx)} * F^{(-nf*gx)} * (F^{((f*x+e)*g)})^n/a) * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g*\ln(F))^{-3} - 3/n^2/f^2/g^2/\ln(F)^2 d^3/a \text{polylog}(2, -bF^{(nf*gx)} * F^{(-nf*gx)} * (F^{((f*x+e)*g)})^n/a) * x^2 + 1/n/f^4/g^4/\ln(F)^4 d^3 * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g*\ln(F))^{-3} / a \ln(a+bF^{(nf*gx)} * F^{(-nf*gx)} * (F^{((f*x+e)*g)})^n) - 6/n/f^3/g^2/\ln(F)^2 * c*d^2 * e * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g*\ln(F)) / a \ln(a+bF^{(nf*gx)} * F^{(-nf*gx)} * (F^{((f*x+e)*g)})^n) + 6/n/f^3/g^2/\ln(F)^2 * c*d^2 * e / a \ln(1+bF^{(nf*gx)} * F^{(-nf*gx)} * (F^{((f*x+e)*g)})^n/a) * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g*\ln(F)) + 6/n/f^3/g^2/\ln(F)^2 * c*d^2 * e * (\ln(F^{((f*x+e)*g)}) - (f*x+e)*g*\ln(F)) / a \ln(F^{(nf*gx)} * F^{(-nf*gx)} * (F^{((f*x+e)*g)})^n)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \left( \frac{\log\left(\left(F^{fgx+eg}\right)^n b + a\right)}{afgn \log(F)} - \frac{\log\left(\left(F^{fgx+eg}\right)^n\right)}{afgn \log(F)} \right) + \int \frac{d^3 x^3 + 3cd^2 x^2 + 3c^2 dx}{\left(F^{fgx}\right)^n \left(F^{eg}\right)^n b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e))))^n, x, algorithm="maxima")

[Out] -c^3\*(log((F^(f\*gx + e\*g))^n\*b + a)/(a\*f\*g\*n\*log(F)) - log((F^(f\*gx + e\*g))^n)/(a\*f\*g\*n\*log(F))) + integrate((d^3\*x^3 + 3\*c\*d^2\*x^2 + 3\*c^2\*d\*x)/((F^(f\*gx))^n\*(F^(e\*g))^n\*b + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx)^3}{a + b \left(F^{g(e+fx)}\right)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x))))^n, x)

[Out] int((c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x))))^n, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^3}{a + b \left(F^{eg} F^{fgx}\right)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n, x)

[Out] Integral((c + d\*x)\*\*3/(a + b\*(F\*\*(e\*g)\*F\*\*(f\*gx))))\*\*n, x)

$$3.47 \quad \int \frac{(c+dx)^2}{a+b(Fg^{(e+fx)})^n} dx$$

**Optimal.** Leaf size=145

$$\frac{2d(c+dx)\text{Li}_2\left(-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(\frac{b(Fg^{(e+fx)})^n}{a}+1\right)}{afgn\log(F)} + \frac{2d^2\text{Li}_3\left(-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^3g^3n^3\log^3(F)} + \frac{(c+dx)^3}{3ad}$$

[Out]  $1/3*(d*x+c)^3/a/d-(d*x+c)^2*\ln(1+b*(F^(g*(f*x+e)))^n/a)/a/f/g/n/\ln(F)-2*d*(d*x+c)*\text{polylog}(2,-b*(F^(g*(f*x+e)))^n/a)/a/f^2/g^2/n^2/\ln(F)^2+2*d^2*\text{polylog}(3,-b*(F^(g*(f*x+e)))^n/a)/a/f^3/g^3/n^3/\ln(F)^3$

**Rubi [A]** time = 0.28, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2184, 2190, 2531, 2282, 6589}

$$\frac{2d(c+dx)\text{PolyLog}\left(2,-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{2d^2\text{PolyLog}\left(3,-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{(c+dx)^2\log\left(\frac{b(Fg^{(e+fx)})^n}{a}+1\right)}{afgn\log(F)} + \frac{(c+dx)^3}{3ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^2/(a + b*(F^(g*(e + f*x))))^n, x]$

[Out]  $(c + d*x)^3/(3*a*d) - ((c + d*x)^2*\text{Log}[1 + (b*(F^(g*(e + f*x))))^n/a])/(a*f*g^n*\text{Log}[F]) - (2*d*(c + d*x)*\text{PolyLog}[2, -((b*(F^(g*(e + f*x))))^n/a])/(a*f^2*g^2*n^2*\text{Log}[F]^2) + (2*d^2*\text{PolyLog}[3, -((b*(F^(g*(e + f*x))))^n/a])/(a*f^3*g^3*n^3*\text{Log}[F]^3)$

**Rule 2184**

$\text{Int}[(c + d*x)^m/(a + b*(F^(g*(e + f*x))))^n, x] := \text{Simp}[(c + d*x)^{m+1}/(a*d*(m+1)), x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m*(F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x))))^n, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \ \text{IGtQ}[m, 0]$

**Rule 2190**

$\text{Int}[(c + d*x)^m*\text{Log}[1 + (b*(F^(g*(e + f*x))))^n/a], x] := \text{Simp}[(c + d*x)^{m+1}/(a*d*(m+1)), x] - \text{Dist}[(d*m)/(b*f*g^n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^(g*(e + f*x))))^n/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{(c+dx)^2}{a+b(F^{g(e+fx)})^n} dx &= \frac{(c+dx)^3}{3ad} - \frac{b \int \frac{(F^{g(e+fx)})^n (c+dx)^2}{a+b(F^{g(e+fx)})^n} dx}{a} \\ &= \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} + \frac{(2d) \int (c+dx) \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right) dx}{afgn \log(F)} \\ &= \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} - \frac{2d(c+dx) \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} + \frac{(2d^2) \int \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right) dx}{af^2g^2} \\ &= \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} - \frac{2d(c+dx) \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} + \frac{(2d^2) \text{Subst}\left[\text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right), \frac{b(F^{g(e+fx)})^n}{a}\right]}{af^2g^2} \\ &= \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn \log(F)} - \frac{2d(c+dx) \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} + \frac{2d^2 \text{Li}_3\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^3g^3n^3 \log^3(F)} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 121, normalized size = 0.83

$$\frac{-f^2 g^2 n^2 \log^2(F)(c + dx)^2 \log\left(\frac{a(Fg^{e+fx})^{-n}}{b} + 1\right) + 2dfgn \log(F)(c + dx) \operatorname{Li}_2\left(-\frac{a(Fg^{e+fx})^{-n}}{b}\right) + 2d^2 \operatorname{Li}_3\left(-\frac{a(Fg^{e+fx})^{-n}}{b}\right)}{af^3 g^3 n^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x)))^n), x]

[Out]  $(-(f^2 g^2 n^2 (c + d x)^2 \operatorname{Log}[F]^2 \operatorname{Log}[1 + a/(b(F^{g(e + f x)})^n])]) + 2 d f g n (c + d x) \operatorname{Log}[F] \operatorname{PolyLog}[2, -(a/(b(F^{g(e + f x)})^n)])] + 2 d^2 \operatorname{PolyLog}[3, -(a/(b(F^{g(e + f x)})^n)])]) / (a f^3 g^3 n^3 \operatorname{Log}[F]^3)$

**fricas [C]** time = 0.41, size = 271, normalized size = 1.87

$$\frac{3(d^2 e^2 - 2 c d e f + c^2 f^2) g^2 n^2 \log(F f g^{n x + e g^n} b + a) \log(F)^2 - (d^2 f^3 g^3 n^3 x^3 + 3 c d f^3 g^3 n^3 x^2 + 3 c^2 f^3 g^3 n^3 x) \log(F)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e)))^n), x, algorithm="fricas")

[Out]  $-1/3*(3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*g^2*n^2*\log(F^{(f*g*n*x + e*g*n)*b + a})*\log(F)^2 - (d^2*f^3*g^3*n^3*x^3 + 3*c*d*f^3*g^3*n^3*x^2 + 3*c^2*f^3*g^3*n^3*x)*\log(F)^3 + 3*(d^2*f^2*g^2*n^2*x^2 + 2*c*d*f^2*g^2*n^2*x - (d^2*e^2 - 2*c*d*e*f)*g^2*n^2)*\log(F)^2*\log((F^{(f*g*n*x + e*g*n)*b + a})/a) + 6*(d^2*f*g*n*x + c*d*f*g*n)*\operatorname{dilog}(-(F^{(f*g*n*x + e*g*n)*b + a})/a + 1)*\log(F) - 6*d^2*\operatorname{polylog}(3, -F^{(f*g*n*x + e*g*n)*b/a})/(a*f^3*g^3*n^3*\log(F)^3)$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{(F^{(fx+e)g})^n b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e)))^n), x, algorithm="giac")

[Out] integrate((d\*x + c)^2/((F^((f\*x + e)\*g))^n\*b + a), x)

**maple [B]** time = 0.11, size = 1677, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*x+c)^2/(b*(F((f*x+e)*g))^n+a),x)$

[Out] 
$$-1/f^2*d^2/a*x*e^{-2-2/3/\ln(F)^3/f^3/g^3*d^2/a*\ln(F((f*x+e)*g))^3-1/\ln(F)^2/f^2/g^2*d^2/a*x*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))^2+1/\ln(F)/f/g/n*c^2/a*\ln(F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})-1/\ln(F)/f/g/n*c^2/a*\ln(a+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})+2/\ln(F)^3/f^3/g^3/n^3*d^2/a*\text{polylog}(3,-b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a)-2/\ln(F)^2/f^2/g^2/n*c*d/a*\ln(1+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a)*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))+2/\ln(F)^2/f^3/g^2/n*d^2*e/a*\ln(1+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a)*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))-2/\ln(F)/f/g/n*c*d/a*\ln(1+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a)*x-2/\ln(F)^2/f^3/g^2/n*d^2*e*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))/a*\ln(a+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})+2/\ln(F)^2/f^2/g^2/n*c*d*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))/a*\ln(a+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})+2/\ln(F)^2/f^3/g^2/n*d^2*e*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))/a*\ln(F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})+2/\ln(F)/f^2/g/n*c*d*e/a*\ln(F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})+2/\ln(F)/f^2/g/n*c*d*e/a*\ln(a+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})+2/\ln(F)/f^2/g/n*c*d/a*\ln(1+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a)*e+d^2/a*x^3-2/\ln(F)^2/f^2/g^2/n*c*d*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))/a*\ln(F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})+2*c*d/a*x^2+2/f*c*d/a*x*e+1/\ln(F)^2/f^2/g^2*c*d/a*\ln(F((f*x+e)*g))^2-2/\ln(F)/f/g*d^2/a*\ln(F((f*x+e)*g))*x^2+2/\ln(F)^2/f^2/g^2*d^2/a*\ln(F((f*x+e)*g))^2*x-2/\ln(F)/f/g*c*d/a*\ln(F((f*x+e)*g))*x-1/\ln(F)/f/g/n*d^2/a*\ln(1+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a))*x^2-2/\ln(F)^2/f^2/g^2/n^2*d^2/a*\text{polylog}(2,-b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a))*x-1/\ln(F)/f^3/g/n*d^2*e^2/a*\ln(a+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})+1/\ln(F)/f^3/g/n*d^2*e^2/a*\ln(F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})-2/\ln(F)/f^2/g*d^2/a*x*e*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))+2/\ln(F)/f/g*c*d/a*x*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))-1/\ln(F)^3/f^3/g^3/n*d^2*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))^2/a*\ln(a+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})-2/\ln(F)^2/f^2/g^2/n^2*c*d/a*\text{polylog}(2,-b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a))+1/\ln(F)/f^3/g/n*d^2*e^2/a*\ln(1+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a))+1/\ln(F)^3/f^3/g^3/n*d^2*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))^2/a*\ln(1+b*F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}}/a))+1/\ln(F)^3/f^3/g^3/n*d^2*(\ln(F((f*x+e)*g))-(f*x+e)*g*\ln(F))^2/a*\ln(F^{(n*f*g*x)*F^{(-n*f*g*x)*(F((f*x+e)*g))^n}})$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^2 \left( \frac{\log\left(\left(F^{fgx+eg}\right)^n b + a\right)}{afgn \log(F)} - \frac{\log\left(\left(F^{fgx+eg}\right)^n\right)}{afgn \log(F)} \right) + \int \frac{d^2x^2 + 2cdx}{\left(F^{fgx}\right)^n \left(F^{eg}\right)^n b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e)))^n),x, algorithm="maxima")

[Out]  $-c^2 \cdot (\log((F^{f \cdot g \cdot x + e \cdot g})^n \cdot b + a) / (a \cdot f \cdot g \cdot n \cdot \log(F)) - \log((F^{f \cdot g \cdot x + e \cdot g})^n) / (a \cdot f \cdot g \cdot n \cdot \log(F))) + \int (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x) / ((F^{f \cdot g \cdot x})^n \cdot (F^{e \cdot g})^n \cdot b + a) dx$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx)^2}{a + b \left( F^{g(e+fx)} \right)^n dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x)))^n),x)

[Out] int((c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x)))^n), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^2}{a + b \left( F^{eg} F^{fgx} \right)^n dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n),x)

[Out] Integral((c + d\*x)\*\*2/(a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))))\*\*n, x)

$$3.48 \quad \int \frac{c+dx}{a+b(Fg^{(e+fx)})^n} dx$$

**Optimal.** Leaf size=98

$$-\frac{(c+dx) \log\left(\frac{b(Fg^{(e+fx)})^n}{a} + 1\right)}{afgn \log(F)} - \frac{d \operatorname{Li}_2\left(-\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} + \frac{(c+dx)^2}{2ad}$$

[Out] 1/2\*(d\*x+c)^2/a/d-(d\*x+c)\*ln(1+b\*(F^(g\*(f\*x+e)))^n/a)/a/f/g/n/ln(F)-d\*polylog(2,-b\*(F^(g\*(f\*x+e)))^n/a)/a/f^2/g^2/n^2/ln(F)^2

**Rubi [A]** time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {2184, 2190, 2279, 2391}

$$-\frac{d \operatorname{PolyLog}\left(2, -\frac{b(Fg^{(e+fx)})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} - \frac{(c+dx) \log\left(\frac{b(Fg^{(e+fx)})^n}{a} + 1\right)}{afgn \log(F)} + \frac{(c+dx)^2}{2ad}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n], x]

[Out] (c + d\*x)^2/(2\*a\*d) - ((c + d\*x)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n/a])/(a\*f\*g\*n\*Log[F]) - (d\*PolyLog[2, -(b\*(F^(g\*(e + f\*x))))^n/a])/(a\*f^2\*g^2\*n^2\*Log[F]^2)

**Rule 2184**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n/(a + b\*(F^(g\*(e + f\*x))))^n], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2190**

Int[((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2279**

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))
]^(n)], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{c + dx}{a + b(Fg^{e+fx})^n} dx &= \frac{(c + dx)^2}{2ad} - \frac{b \int \frac{(Fg^{e+fx})^{n(c+dx)}}{a + b(Fg^{e+fx})^n} dx}{a} \\ &= \frac{(c + dx)^2}{2ad} - \frac{(c + dx) \log\left(1 + \frac{b(Fg^{e+fx})^n}{a}\right)}{afgn \log(F)} + \frac{d \int \log\left(1 + \frac{b(Fg^{e+fx})^n}{a}\right) dx}{afgn \log(F)} \\ &= \frac{(c + dx)^2}{2ad} - \frac{(c + dx) \log\left(1 + \frac{b(Fg^{e+fx})^n}{a}\right)}{afgn \log(F)} + \frac{d \operatorname{Subst}\left(\int \frac{\log\left(1 + \frac{bx}{a}\right)}{x} dx, x, (Fg^{e+fx})^n\right)}{af^2g^2n^2 \log^2(F)} \\ &= \frac{(c + dx)^2}{2ad} - \frac{(c + dx) \log\left(1 + \frac{b(Fg^{e+fx})^n}{a}\right)}{afgn \log(F)} - \frac{d \operatorname{Li}_2\left(-\frac{b(Fg^{e+fx})^n}{a}\right)}{af^2g^2n^2 \log^2(F)} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 74, normalized size = 0.76

$$\frac{d \operatorname{Li}_2\left(-\frac{a(Fg^{e+fx})^{-n}}{b}\right) - fgn \log(F)(c + dx) \log\left(\frac{a(Fg^{e+fx})^{-n}}{b} + 1\right)}{af^2g^2n^2 \log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x))))^n, x]
```

```
[Out] (-(f*g*n*(c + d*x)*Log[F]*Log[1 + a/(b*(F^(g*(e + f*x))))^n])) + d*PolyLog[2
, -(a/(b*(F^(g*(e + f*x))))^n)]/(a*f^2*g^2*n^2*Log[F]^2)
```

**fricas** [A] time = 0.42, size = 147, normalized size = 1.50

$$\frac{2(de - cf)gn \log(F^{fgnx+egn}b + a) \log(F) + (df^2g^2n^2x^2 + 2cf^2g^2n^2x) \log(F)^2 - 2(dfgnx + degn) \log(F) \log(F^{fgnx+egn}b + a)}{2af^2g^2n^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n),x, algorithm="fricas")

[Out] 1/2\*(2\*(d\*e - c\*f)\*g\*n\*log(F^(f\*g\*n\*x + e\*g\*n)\*b + a)\*log(F) + (d\*f^2\*g^2\*n^2\*x^2 + 2\*c\*f^2\*g^2\*n^2\*x)\*log(F)^2 - 2\*(d\*f\*g\*n\*x + d\*e\*g\*n)\*log(F)\*log((F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a) - 2\*d\*dilog(-(F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a + 1))/(a\*f^2\*g^2\*n^2\*log(F)^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left(F^{(f*x+e)g}\right)^n b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n),x, algorithm="giac")

[Out] integrate((d\*x + c)/((F^((f\*x + e)\*g))^n\*b + a), x)

**maple** [B] time = 0.09, size = 679, normalized size = 6.93

$$\frac{\frac{dx^2}{a} + \frac{dex}{af} - \frac{dx \ln(F^{(f*x+e)g})}{afg \ln(F)} - \frac{dx \ln\left(\frac{b F^{fgnx} F^{-fgnx} (F^{(f*x+e)g})^n}{a} + 1\right)}{afgn \ln(F)} + \frac{\left(- (f*x + e)g \ln(F) + \ln(F^{(f*x+e)g})\right) dx}{afg \ln(F)} + \frac{c \ln(F^{fgnx})}{a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(b\*(F^((f\*x+e)\*g))^(n+a)),x)

[Out] 1/ln(F)/f/g/n\*c/a\*ln(F^(f\*g\*n\*x)\*F^(-f\*g\*n\*x)\*(F^((f\*x+e)\*g))^n)-1/ln(F)/f/g/n\*c/a\*ln(b\*F^(f\*g\*n\*x)\*F^(-f\*g\*n\*x)\*(F^((f\*x+e)\*g))^(n+a)+d/a\*x^2+1/f\*d/a\*x\*e+1/ln(F)/f/g\*d/a\*x\*(-(f\*x+e)\*g\*ln(F)+ln(F^((f\*x+e)\*g)))-1/ln(F)/f/g\*d/a\*ln(F^((f\*x+e)\*g))\*x+1/2/ln(F)^2/f^2/g^2\*d/a\*ln(F^((f\*x+e)\*g))^2-1/ln(F)/f/g/n\*d/a\*ln(1/a\*b\*F^(f\*g\*n\*x)\*F^(-f\*g\*n\*x)\*(F^((f\*x+e)\*g))^(n+1))\*x-1/ln(F)/f^2/g/n\*d/a\*ln(1/a\*b\*F^(f\*g\*n\*x)\*F^(-f\*g\*n\*x)\*(F^((f\*x+e)\*g))^(n+1))\*e-1/ln(F)^2/f^2/g^2/n\*d/a\*ln(1/a\*b\*F^(f\*g\*n\*x)\*F^(-f\*g\*n\*x)\*(F^((f\*x+e)\*g))^(n+1))\*(-(f\*x+e)\*g\*ln(F)+ln(F^((f\*x+e)\*g)))-1/ln(F)^2/f^2/g^2/n^2\*d/a\*polylog(2,-1/a\*b\*F^(f\*g\*n\*x)\*F^(-f\*g\*n\*x)\*(F^((f\*x+e)\*g))^n)-1/ln(F)/f^2/g/n\*d\*e/a\*ln(F^(f\*g

$*n*x)*F^{(-f*g*n*x)}*(F^{((f*x+e)*g)})^n+1/\ln(F)/f^2/g/n*d*e/a*\ln(b*F^{(f*g*n*x)})*F^{(-f*g*n*x)}*(F^{((f*x+e)*g)})^{n+a}-1/\ln(F)^2/f^2/g^2/n*d*(-(f*x+e)*g*\ln(F)+\ln(F^{((f*x+e)*g)}))/a*\ln(F^{(f*g*n*x)})*F^{(-f*g*n*x)}*(F^{((f*x+e)*g)})^n+1/\ln(F)^2/f^2/g^2/n*d*(-(f*x+e)*g*\ln(F)+\ln(F^{((f*x+e)*g)}))/a*\ln(b*F^{(f*g*n*x)})*F^{(-f*g*n*x)}*(F^{((f*x+e)*g)})^{n+a}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \left( \frac{\log\left(\left(F^{fgx+eg}\right)^n b + a\right)}{afgn \log(F)} - \frac{\log\left(\left(F^{fgx+eg}\right)^n\right)}{afgn \log(F)} \right) + d \int \frac{x}{\left(F^{fgx}\right)^n \left(F^{eg}\right)^n b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n),x, algorithm="maxima")

[Out] -c\*(log((F^(f\*g\*x + e\*g))^n\*b + a)/(a\*f\*g\*n\*log(F)) - log((F^(f\*g\*x + e\*g))^n)/(a\*f\*g\*n\*log(F))) + d\*integrate(x/((F^(f\*g\*x))^n\*(F^(e\*g))^n\*b + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + dx}{a + b \left( F^{g(e+fx)} \right)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n),x)

[Out] int((c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{a + b \left( F^{eg} F^{fgx} \right)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n),x)

[Out] Integral((c + d\*x)/(a + b\*(F\*\*(e\*g)\*F\*\*(f\*g\*x))))\*\*n, x)

$$3.49 \quad \int \frac{1}{a+b(F^{g(e+fx)})^n} dx$$

Optimal. Leaf size=40

$$\frac{x}{a} - \frac{\log\left(a + b\left(F^{g(e+fx)}\right)^n\right)}{afgn \log(F)}$$

[Out] x/a-ln(a+b\*(F^(g\*(f\*x+e)))^n)/a/f/g/n/ln(F)

**Rubi [A]** time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {2282, 266, 36, 29, 31}

$$\frac{x}{a} - \frac{\log\left(a + b\left(F^{g(e+fx)}\right)^n\right)}{afgn \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^(-1), x]

[Out] x/a - Log[a + b\*(F^(g\*(e + f\*x)))^n]/(a\*f\*g\*n\*Log[F])

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_)\*(x\_))(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{a + b (F^{g(e+fx)})^n} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^n)} dx, x, F^{g(e+fx)}\right)}{fg \log(F)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, (F^{g(e+fx)})^n\right)}{fgn \log(F)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, (F^{g(e+fx)})^n\right)}{afgn \log(F)} - \frac{b \text{Subst}\left(\int \frac{1}{a+bx} dx, x, (F^{g(e+fx)})^n\right)}{afgn \log(F)} \\ &= \frac{x}{a} - \frac{\log\left(a + b (F^{g(e+fx)})^n\right)}{afgn \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 1.00

$$\frac{x}{a} - \frac{\log\left(a + b (F^{g(e+fx)})^n\right)}{afgn \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^(-1), x]

[Out] x/a - Log[a + b\*(F^(g\*(e + f\*x)))^n]/(a\*f\*g\*n\*Log[F])

**fricas [A]** time = 0.43, size = 44, normalized size = 1.10

$$\frac{fgnx \log(F) - \log\left(F^{fgnx+egn} b + a\right)}{afgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e)))^n), x, algorithm="fricas")

[Out]  $(f \cdot g \cdot n \cdot x \cdot \log(F) - \log(F^{(f \cdot g \cdot n \cdot x + e \cdot g \cdot n) \cdot b + a})) / (a \cdot f \cdot g \cdot n \cdot \log(F))$

**giac** [A] time = 0.35, size = 74, normalized size = 1.85

$$\frac{\log(|F|^{f g n x} |F|^{g n e})}{a f g n \log(F)} - \frac{\log(|F|^{f g n x} F^{g n e} b + a)}{a f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")`

[Out]  $\log(\text{abs}(F)^{(f \cdot g \cdot n \cdot x)} \cdot \text{abs}(F)^{(g \cdot n \cdot e)}) / (a \cdot f \cdot g \cdot n \cdot \log(F)) - \log(\text{abs}(F^{(f \cdot g \cdot n \cdot x)} \cdot F^{(g \cdot n \cdot e)} \cdot b + a)) / (a \cdot f \cdot g \cdot n \cdot \log(F))$

**maple** [A] time = 0.02, size = 65, normalized size = 1.62

$$\frac{\ln\left(\left(F^{(f x + e) g}\right)^n\right)}{a f g n \ln(F)} - \frac{\ln\left(b \left(F^{(f x + e) g}\right)^n + a\right)}{a f g n \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*(F^((f*x+e)*g))^n+a),x)`

[Out]  $1/g/f/\ln(F)/n/a \cdot \ln((F^{((f \cdot x + e) \cdot g)})^n) - \ln(b \cdot (F^{((f \cdot x + e) \cdot g)})^n + a) / a / f / g / n / \ln(F)$

**maxima** [A] time = 1.14, size = 66, normalized size = 1.65

$$-\frac{\log\left(\left(F^{f g x + e g}\right)^n b + a\right)}{a f g n \log(F)} + \frac{\log\left(\left(F^{f g x + e g}\right)^n\right)}{a f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")`

[Out]  $-\log((F^{(f \cdot g \cdot x + e \cdot g)})^n \cdot b + a) / (a \cdot f \cdot g \cdot n \cdot \log(F)) + \log((F^{(f \cdot g \cdot x + e \cdot g)})^n) / (a \cdot f \cdot g \cdot n \cdot \log(F))$

**mupad** [B] time = 3.45, size = 44, normalized size = 1.10

$$-\frac{\ln\left(a + b \left(F^{e g + f g x}\right)^n\right) - f g n x \ln(F)}{a f g n \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/(a + b*(F^(g*(e + f*x)))^n),x)`

[Out] `-(log(a + b*(F^(e*g + f*g*x))^n) - f*g*n*x*log(F))/(a*f*g*n*log(F))`

**sympy [A]** time = 0.16, size = 27, normalized size = 0.68

$$\frac{x}{a} - \frac{\log\left(\frac{a}{b} + \left(F^{g(e+fx)}\right)^n\right)}{afgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*(F**(g*(f*x+e)))**n),x)`

[Out] `x/a - log(a/b + (F**(g*(e + f*x)))**n)/(a*f*g*n*log(F))`

$$3.50 \quad \int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)(c+dx)} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{1}{(c+dx) \left(a+b(F^{eg+fgx})^n\right)}, x \right)$$

[Out] Unintegrable(1/(a+b\*(F^(f\*g\*x+e\*g))^n)/(d\*x+c), x)

**Rubi [A]** time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*(F^(g\*(e + f\*x))))^n)\*(c + d\*x)], x]

[Out] Defer[Int][1/((a + b\*(F^(e\*g + f\*g\*x)))^n)\*(c + d\*x)], x]

Rubi steps

$$\int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)(c+dx)} dx = \int \frac{1}{\left(a+b(F^{eg+fgx})^n\right)(c+dx)} dx$$

**Mathematica [A]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)\*(c + d\*x)], x]

[Out] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)\*(c + d\*x)], x]

**fricas [A]** time = 0.41, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{adx + (bdx + bc)(F^{fgx+eg})^n + ac}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c),x, algorithm="fricas")

[Out] integral(1/(a\*d\*x + (b\*d\*x + b\*c)\*(F^(f\*g\*x + e\*g))^n + a\*c), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(F^{(f x+e) g}\right)^n b+a\right)(d x+c)} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c),x, algorithm="giac")

[Out] integrate(1/(((F^((f\*x + e)\*g))^n\*b + a)\*(d\*x + c)), x)

**maple** [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(F^{(f x+e) g}\right)^n+a\right)(d x+c)} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*(F^((f\*x+e)\*g))^n+a)/(d\*x+c),x)

[Out] int(1/(b\*(F^((f\*x+e)\*g))^n+a)/(d\*x+c),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(F^{(f x+e) g}\right)^n b+a\right)(d x+c)} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)/(d\*x+c),x, algorithm="maxima")

[Out] integrate(1/(((F^((f\*x + e)\*g))^n\*b + a)\*(d\*x + c)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a+b\left(F^{g(e+f x)}\right)^n\right)(c+d x)} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)), x)`

[Out] `int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \left(F^{egFfgx}\right)^n\right) (c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*(F**(g*(f*x+e)))**n)/(d*x+c), x)`

[Out] `Integral(1/((a + b*(F**(e*g)*F**(f*g*x))**n)*(c + d*x)), x)`

$$3.51 \quad \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2} dx$$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{1}{(c+dx)^2\left(a+b\left(F^{eg+fgx}\right)^n\right)}, x\right)$$

[Out] Unintegrable(1/(a+b\*(F^(f\*g\*x+e\*g))^n)/(d\*x+c)^2,x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*(F^(g\*(e + f\*x))))^n)\*(c + d\*x)^2], x]

[Out] Defer[Int][1/((a + b\*(F^(e\*g + f\*g\*x))^n)\*(c + d\*x)^2), x]

Rubi steps

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2} dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)^2} dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)\*(c + d\*x)^2], x]

[Out] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)\*(c + d\*x)^2], x]

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ad^2x^2 + 2acdx + ac^2 + (bd^2x^2 + 2bcdx + bc^2)\left(F^{fgx+eg}\right)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e)))^n)/(d\*x+c)^2,x, algorithm="fricas")

[Out] integral(1/(a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + (b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*(F^(f\*g\*x + e\*g))^n), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( \left( F^{(f^{x+e})g} \right)^n b + a \right) (dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e)))^n)/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(1/(((F^((f\*x + e)\*g))^n\*b + a)\*(d\*x + c)^2), x)

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( b \left( F^{(f^{x+e})g} \right)^n + a \right) (dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*(F^((f\*x+e)\*g))^n+a)/(d\*x+c)^2,x)

[Out] int(1/(b\*(F^((f\*x+e)\*g))^n+a)/(d\*x+c)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( \left( F^{(f^{x+e})g} \right)^n b + a \right) (dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e)))^n)/(d\*x+c)^2,x, algorithm="maxima")

[Out] integrate(1/(((F^((f\*x + e)\*g))^n\*b + a)\*(d\*x + c)^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(F^(g*(e + f*x))))^n)*(c + d*x)^2), x)
```

```
[Out] int(1/((a + b*(F^(g*(e + f*x))))^n)*(c + d*x)^2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(F**(g*(f*x+e))))**n)/(d*x+c)**2, x)
```

```
[Out] Timed out
```

$$3.52 \quad \int \frac{(c+dx)^3}{\left(a+b(F^g(e+fx))^n\right)^2} dx$$

**Optimal.** Leaf size=388

$$\frac{6d^2(c+dx)\text{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^3 g^3 n^3 \log^3(F)} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^3 g^3 n^3 \log^3(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)} + \frac{3d(c+dx)^2 \log\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)}$$

[Out]  $\frac{1}{4}*(d*x+c)^4/a^2/d-(d*x+c)^3/a^2/f/g/n/\ln(F)+(d*x+c)^3/a/f/(a+b*(F^(g*(f*x+e)))^n)/g/n/\ln(F)+3*d*(d*x+c)^2*\ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^2/\ln(F)^2-(d*x+c)^3*\ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f/g/n/\ln(F)+6*d^2*(d*x+c)*\text{polylog}(2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/\ln(F)^3-3*d*(d*x+c)^2*\text{polylog}(2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^2/\ln(F)^2-6*d^3*\text{polylog}(3,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^4/g^4/n^4/\ln(F)^4+6*d^2*(d*x+c)*\text{polylog}(3,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/\ln(F)^3-6*d^3*\text{polylog}(4,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^4/g^4/n^4/\ln(F)^4$

**Rubi [A]** time = 0.87, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$ , Rules used = {2185, 2184, 2190, 2531, 6609, 2282, 6589, 2191}

$$\frac{6d^2(c+dx)\text{PolyLog}\left(2,-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^3 g^3 n^3 \log^3(F)} + \frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^3 g^3 n^3 \log^3(F)} - \frac{3d(c+dx)^2 \text{PolyLog}\left(2,-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x)))^n)^2,x]

[Out]  $(c+d*x)^4/(4*a^2*d) - (c+d*x)^3/(a^2*f*g*n*\text{Log}[F]) + (c+d*x)^3/(a*f*(a+b*(F^(g*(e+f*x)))^n)*g*n*\text{Log}[F]) + (3*d*(c+d*x)^2*\text{Log}[1+(b*(F^(g*(e+f*x)))^n)/a])/(a^2*f^2*g^2*n^2*\text{Log}[F]^2) - ((c+d*x)^3*\text{Log}[1+(b*(F^(g*(e+f*x)))^n)/a])/(a^2*f*g*n*\text{Log}[F]) + (6*d^2*(c+d*x)*\text{PolyLog}[2,-((b*(F^(g*(e+f*x)))^n)/a)])/(a^2*f^3*g^3*n^3*\text{Log}[F]^3) - (3*d*(c+d*x)^2*\text{PolyLog}[2,-((b*(F^(g*(e+f*x)))^n)/a)])/(a^2*f^2*g^2*n^2*\text{Log}[F]^2) - (6*d^3*\text{PolyLog}[3,-((b*(F^(g*(e+f*x)))^n)/a)])/(a^2*f^4*g^4*n^4*\text{Log}[F]^4) + (6*d^2*(c+d*x)*\text{PolyLog}[3,-((b*(F^(g*(e+f*x)))^n)/a)])/(a^2*f^3*g^3*n^3*\text{Log}[F]^3) - (6*d^3*\text{PolyLog}[4,-((b*(F^(g*(e+f*x)))^n)/a)])/(a^2*f^4*g^4*n^4*\text{Log}[F]^4)$

Rule 2184



```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2185

```
Int[((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int((((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2191

```
Int(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Log[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a + b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*(f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f
```

, g, n}, x] && GtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{\left(a+b(F^{g(e+fx)})^n\right)^2} dx &= \frac{\int \frac{(c+dx)^3}{a+b(F^{g(e+fx)})^n} dx}{a} - \frac{b \int \frac{(F^{g(e+fx)})^n (c+dx)^3}{\left(a+b(F^{g(e+fx)})^n\right)^2} dx}{a} \\
&= \frac{(c+dx)^4}{4a^2d} + \frac{(c+dx)^3}{af\left(a+b(F^{g(e+fx)})^n\right)gn\log(F)} - \frac{b \int \frac{(F^{g(e+fx)})^n (c+dx)^3}{a+b(F^{g(e+fx)})^n} dx}{a^2} - \frac{(3d) \int \frac{(c+dx)^3}{a+b(F^{g(e+fx)})^n} dx}{afgn\log(F)} \\
&= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b(F^{g(e+fx)})^n\right)gn\log(F)} - \frac{(c+dx)^3 \log\left(1+\frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2fgn\log(F)} \\
&= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b(F^{g(e+fx)})^n\right)gn\log(F)} + \frac{3d(c+dx)^2 \log\left(1+\frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2f^2g^2n^2\log(F)} \\
&= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b(F^{g(e+fx)})^n\right)gn\log(F)} + \frac{3d(c+dx)^2 \log\left(1+\frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2f^2g^2n^2\log(F)} \\
&= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b(F^{g(e+fx)})^n\right)gn\log(F)} + \frac{3d(c+dx)^2 \log\left(1+\frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2f^2g^2n^2\log(F)} \\
&= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b(F^{g(e+fx)})^n\right)gn\log(F)} + \frac{3d(c+dx)^2 \log\left(1+\frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2f^2g^2n^2\log(F)}
\end{aligned}$$

**Mathematica [F]** time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{\left(a+b(F^{g(e+fx)})^n\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x)))^n)^2,x]

[Out] Integrate[(c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x)))^n)^2, x]

**fricas** [C] time = 0.46, size = 1390, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e)))^n)^2,x, algorithm="fricas")

[Out] 
$$-1/4*(4*(a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*g^3*n^3 * \log(F)^3 - (a*d^3*f^4*g^4*n^4*x^4 + 4*a*c*d^2*f^4*g^4*n^4*x^3 + 6*a*c^2*d*f^4*g^4*n^4*x^2 + 4*a*c^3*f^4*g^4*n^4*x - (a*d^3*e^4 - 4*a*c*d^2*e^3*f + 6*a*c^2*d*e^2*f^2 - 4*a*c^3*e*f^3)*g^4*n^4)*\log(F)^4 - ((b*d^3*f^4*g^4*n^4*x^4 + 4*b*c*d^2*f^4*g^4*n^4*x^3 + 6*b*c^2*d*f^4*g^4*n^4*x^2 + 4*b*c^3*f^4*g^4*n^4*x - (b*d^3*e^4 - 4*b*c*d^2*e^3*f + 6*b*c^2*d*e^2*f^2 - 4*b*c^3*e*f^3)*g^4*n^4)*\log(F)^4 - 4*(b*d^3*f^3*g^3*n^3*x^3 + 3*b*c*d^2*f^3*g^3*n^3*x^2 + 3*b*c^2*d*f^3*g^3*n^3*x + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*g^3*n^3)*\log(F)^3)*F^(f*g*n*x + e*g*n) + 12*((a*d^3*f^2*g^2*n^2*x^2 + 2*a*c*d^2*f^2*g^2*n^2*x + a*c^2*d*f^2*g^2*n^2)*\log(F)^2 + ((b*d^3*f^2*g^2*n^2*x^2 + 2*b*c*d^2*f^2*g^2*n^2*x + b*c^2*d*f^2*g^2*n^2)*\log(F)^2 - 2*(b*d^3*f*g*n*x + b*c*d^2*f*g*n)*\log(F))*F^(f*g*n*x + e*g*n) - 2*(a*d^3*f*g*n*x + a*c*d^2*f*g*n)*\log(F))*\operatorname{dilog}(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1) - 4*((a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*g^3*n^3*\log(F)^3 + 3*(a*d^3*e^2 - 2*a*c*d^2*e*f + a*c^2*d*f^2)*g^2*n^2*\log(F)^2 + ((b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*g^3*n^3*\log(F)^3 + 3*(b*d^3*e^2 - 2*b*c*d^2*e*f + b*c^2*d*f^2)*g^2*n^2*\log(F)^2)*F^(f*g*n*x + e*g*n))*\log(F^(f*g*n*x + e*g*n)*b + a) + 4*((a*d^3*f^3*g^3*n^3*x^3 + 3*a*c*d^2*f^3*g^3*n^3*x^2 + 3*a*c^2*d*f^3*g^3*n^3*x + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2)*g^3*n^3)*\log(F)^3 - 3*(a*d^3*f^2*g^2*n^2*x^2 + 2*a*c*d^2*f^2*g^2*n^2*x - (a*d^3*e^2 - 2*a*c*d^2*e*f)*g^2*n^2)*\log(F)^2 + ((b*d^3*f^3*g^3*n^3*x^3 + 3*b*c*d^2*f^3*g^3*n^3*x^2 + 3*b*c^2*d*f^3*g^3*n^3*x + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*g^3*n^3)*\log(F)^3 - 3*(b*d^3*f^2*g^2*n^2*x^2 + 2*b*c*d^2*f^2*g^2*n^2*x - (b*d^3*e^2 - 2*b*c*d^2*e*f)*g^2*n^2)*\log(F)^2)*F^(f*g*n*x + e*g*n))*\log((F^(f*g*n*x + e*g*n)*b + a)/a) + 24*(F^(f*g*n*x + e*g*n)*b*d^3 + a*d^3)*\operatorname{polylog}(4, -F^(f*g*n*x + e*g*n)*b/a) + 24*(a*d^3 + (b*d^3 - (b*d^3*f*g*n*x + b*c*d^2*f*g*n)*\log(F))*F^(f*g*n*x + e*g*n) - (a*d^3*f*g*n*x + a*c*d^2*f*g*n)*\log(F))*\operatorname{polylog}(3, -F^(f*g*n*x + e*g*n)*b/a))/(F^(f*g*n*x + e*g*n)*a^2*b*f^4*g^4*n^4*\log(F)^4 + a^3*f^4*g^4*n^4*\log(F)^4)$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{\left(\left(F^{(f*x+e)g}\right)^n b + a\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^3/((F^((f\*x + e)\*g))^n\*b + a)^2, x)

**maple [B]** time = 0.08, size = 3319, normalized size = 8.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3/(b\*(F^((f\*x+e)\*g))^n+a)^2,x)

[Out] 
$$\frac{1}{n} \frac{f}{g} \frac{\ln(F)}{a} \frac{(d^3 x^3 + 3 c d^2 x^2 + 3 c^2 d x + c^3)}{(b (F^((f x + e) g))^n + a)} + \frac{3}{4} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^2 d^3} \frac{\ln(F^((f x + e) g))^{4-6/n}}{f^2/g^2} \frac{\ln(F)^{2/a}}{a^2 c} d^2 \ln(F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) \ln(F^((f x + e) g))^{x+6/n} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 c} d^2 \ln(b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + a) \ln(F^((f x + e) g))^{x-6/n} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 c} d^2 \ln(1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + 1) \ln(F^((f x + e) g))^{x+3/n} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 d^3} \ln(b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + a) \frac{x^2+3}{n^2} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^2 d^3} \ln(b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + a) \ln(F^((f x + e) g))^{2+6/n} \frac{f^3}{g^3} \frac{\ln(F)^3}{a^2 d^3} \text{polylog}(2, -1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) \frac{x-3}{n^2} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 c} d^2 \ln(F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) + \frac{6}{n^3} \frac{f^3}{g^3} \frac{\ln(F)^3}{a^2 c} d^2 \text{polylog}(3, -1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) + \frac{6}{n^3} \frac{f^3}{g^3} \frac{\ln(F)^3}{a^2 c} d^2 \text{polylog}(2, -1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) - \frac{3}{n^2} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 c} d^2 \text{polylog}(2, -1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) - \frac{3}{n^2} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 d^3} \ln(F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) \frac{x^2-3}{n^2} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^2 d^3} \ln(F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) \ln(F^((f x + e) g))^{2-3/n} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^2 d^3} \ln(1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + 1) \ln(F^((f x + e) g))^{2+1/n} \frac{f}{g} \frac{\ln(F)}{a^2 d^3} \ln(F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) \frac{x^3-1}{n} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^2 d^3} \ln(F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) \ln(F^((f x + e) g))^{3+6/n} \frac{f^3}{g^3} \frac{\ln(F)^3}{a^2 d^3} \text{polylog}(3, -1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) \frac{x+3}{n^2} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 c} d^2 \ln(b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + a) - \frac{3}{n^2} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 d^3} \text{polylog}(2, -1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n) \frac{x^2-3}{n} \frac{f^3}{g^3} \frac{\ln(F)^3}{a^2 c} d^2 \ln(F^((f x + e) g))^{2-3/n} \frac{f^3}{g^3} \frac{\ln(F)^3}{a^2 d^3} \ln(F^((f x + e) g))^{2x-1} \frac{f}{g} \frac{\ln(F)}{a^2 d^3} \ln(b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + a) \frac{x^3+1}{n} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^2 d^3} \ln(b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + a) \ln(F^((f x + e) g))^{3-1/n} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^2 d^3} \ln(1/a b F^{(f g n x)} F^{(-f g n x)} (F^((f x + e) g))^n + 1) \ln(F^((f x + e) g))^{3+3/n} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 c} d^2 \ln(F^((f x + e) g))^{2x+3/2} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 c} d^2 \ln(F^((f x + e) g))^{2+3/2} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2 d^3} \ln(F^((f x + e) g))^{2x^2-2} \frac{f^3}{g^3} \frac{\ln(F)^3}{a^2 d^3} \ln(F^((f x + e) g))^{3x-2} \frac{f^3}{g^3} \frac{\ln(F)^3}{a^2 c} d^2 \ln(F^(($$

$$\begin{aligned}
& f*x+e)*g))^3+2/n/f^4/g^4/\ln(F)^4/a^2*d^3*\ln(F^((f*x+e)*g))^3-1/n/f/g/\ln(F)/ \\
& a^2*c^3*\ln(b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+a)+1/n/f/g/\ln(F)/a^ \\
& 2*c^3*\ln(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)-6/n^4/f^4/g^4/\ln(F)^4/ \\
& a^2*d^3*polylog(4,-1/a*b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)-6/n^4/ \\
& f^4/g^4/\ln(F)^4/a^2*d^3*polylog(3,-1/a*b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+ \\
& e)*g))^n)+3/n/f^3/g^3/\ln(F)^3/a^2*d^3*\ln(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+ \\
& e)*g))^n)*\ln(F^((f*x+e)*g))^2*x+6/n^2/f^2/g^2/\ln(F)^2/a^2*c*d^2*\ln(b*F^(f*g \\
& *n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+a)*x-6/n^2/f^3/g^3/\ln(F)^3/a^2*c*d^2*1 \\
& n(b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+a)*\ln(F^((f*x+e)*g))-6/n^2/f \\
& ^2/g^2/\ln(F)^2/a^2*c*d^2*\ln(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)*x+6 \\
& /n^2/f^3/g^3/\ln(F)^3/a^2*c*d^2*\ln(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^ \\
& n)*\ln(F^((f*x+e)*g))+6/n^2/f^3/g^3/\ln(F)^3/a^2*d^3*\ln(1/a*b*F^(f*g*n*x)*F^ \\
& (-f*g*n*x)*(F^((f*x+e)*g))^n+1)*\ln(F^((f*x+e)*g))*x-6/n^2/f^3/g^3/\ln(F)^3/a^ \\
& 2*d^3*\ln(b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+a)*\ln(F^((f*x+e)*g))* \\
& x+6/n^2/f^3/g^3/\ln(F)^3/a^2*d^3*\ln(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g)) \\
& ^n)*\ln(F^((f*x+e)*g))*x-6/n^2/f^2/g^2/\ln(F)^2/a^2*c*d^2*polylog(2,-1/a*b*F^ \\
& (f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)*x+3/n/f/g/\ln(F)/a^2*c*d^2*\ln(F^(f \\
& *g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)*x^2+3/n/f^3/g^3/\ln(F)^3/a^2*c*d^2*1 \\
& n(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)*\ln(F^((f*x+e)*g))^2-3/n/f/g/l \\
& n(F)/a^2*c*d^2*\ln(b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+a)*x^2-3/n/f \\
& ^3/g^3/\ln(F)^3/a^2*c*d^2*\ln(b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+a) \\
& *\ln(F^((f*x+e)*g))^2+3/n/f^3/g^3/\ln(F)^3/a^2*c*d^2*\ln(1/a*b*F^(f*g*n*x)*F^ \\
& (-f*g*n*x)*(F^((f*x+e)*g))^n+1)*\ln(F^((f*x+e)*g))^2-3/n/f/g/\ln(F)/a^2*c^2*d* \\
& \ln(b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+a)*x+6/n^2/f^3/g^3/\ln(F)^3/ \\
& a^2*c*d^2*\ln(1/a*b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+1)*\ln(F^((f*x \\
& +e)*g))+3/n/f^2/g^2/\ln(F)^2/a^2*c^2*d*\ln(b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f* \\
& x+e)*g))^n+a)*\ln(F^((f*x+e)*g))-3/n/f^2/g^2/\ln(F)^2/a^2*c^2*d*\ln(1/a*b*F^(f \\
& *g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+1)*\ln(F^((f*x+e)*g))+3/n/f/g/\ln(F)/a \\
& ^2*c^2*d*\ln(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)*x-3/n/f^2/g^2/\ln(F) \\
& ^2/a^2*c^2*d*\ln(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)*\ln(F^((f*x+e)*g \\
& ))+3/n/f^2/g^2/\ln(F)^2/a^2*d^3*\ln(b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g) \\
& ))^n+a)*\ln(F^((f*x+e)*g))*x^2-3/n/f^3/g^3/\ln(F)^3/a^2*d^3*\ln(b*F^(f*g*n*x)*F \\
& ^(-f*g*n*x)*(F^((f*x+e)*g))^n+a)*\ln(F^((f*x+e)*g))^2*x-3/n/f^2/g^2/\ln(F)^2/ \\
& a^2*d^3*\ln(1/a*b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n+1)*\ln(F^((f*x+e \\
& )*g))*x^2+3/n/f^3/g^3/\ln(F)^3/a^2*d^3*\ln(1/a*b*F^(f*g*n*x)*F^(-f*g*n*x)*(F^ \\
& ((f*x+e)*g))^n+1)*\ln(F^((f*x+e)*g))^2*x-3/n/f^2/g^2/\ln(F)^2/a^2*d^3*\ln(F^(f \\
& *g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)*\ln(F^((f*x+e)*g))*x^2
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$c^3 \left( \frac{1}{\left( (F^{fgx+eg})^n abn + a^2n \right) fg \log(F)} + \frac{\log(F^{fgx+eg})}{a^2 fg \log(F)} - \frac{\log\left(\frac{(F^{fgx+eg})^n b+a}{b}\right)}{a^2 fgn \log(F)} \right) + \frac{d^3 x^3 + 3 cd^2 x^2 + 3 c^2 dx}{(F^{fgx})^n (F^{eg})^n abfgn \log(F) + a^2 fgn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e)))^n)^2,x, algorithm="maxima")

[Out]  $c^3*(1/(((F^(f*g*x + e*g))^n*a*b*n + a^2*n)*f*g*\log(F)) + \log(F^(f*g*x + e*g)))/(a^2*f*g*\log(F)) - \log(((F^(f*g*x + e*g))^n*b + a)/b)/(a^2*f*g*n*\log(F)) + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)/((F^(f*g*x))^n*(F^(e*g))^n*a*b*f*g*n*\log(F) + a^2*f*g*n*\log(F)) + \text{integrate}((d^3*f*g*n*x^3*\log(F) - 3*c^2*d + 3*(c*d^2*f*g*n*\log(F) - d^3)*x^2 + 3*(c^2*d*f*g*n*\log(F) - 2*c*d^2)*x)/((F^(f*g*x))^n*(F^(e*g))^n*a*b*f*g*n*\log(F) + a^2*f*g*n*\log(F)), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^3}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x)))^n)^2,x)

[Out] int((c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x)))^n)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}{a^2fgn \log(F) + abfgn \left(F^{g(e+fx)}\right)^n \log(F)} + \frac{\int \left(-\frac{3c^2d}{a+be^{egn \log(F)}e^{fgnx \log(F)}}\right) dx + \int \left(-\frac{3d^3x^2}{a+be^{egn \log(F)}e^{fgnx \log(F)}}\right) dx + \int \left(-\frac{3d^2x}{a+be^{egn \log(F)}e^{fgnx \log(F)}}\right) dx}{a^2fgn \log(F) + abfgn \left(F^{g(e+fx)}\right)^n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3/(a+b\*(F\*\*(g\*(f\*x+e)))\*\*n)\*\*2,x)

[Out]  $(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)/(a**2*f*g*n*\log(F) + a*b*f*g*n*(F**(g*(e + f*x)))**n*\log(F)) + (\text{Integral}(-3*c**2*d/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) + \text{Integral}(-3*d**3*x**2/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) + \text{Integral}(-6*c*d**2*x/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) + \text{Integral}(c**3*f*g*n*\log(F)/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) + \text{Integral}(d**3*f*g*n*x**3*\log(F)/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) + \text{Integral}(3*c*d**2*f*g*n*x**2*\log(F)/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) + \text{Integral}(3*c**2*d*f*g*n*x*\log(F)/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x))/(a*f*g*n*\log(F))$

$$3.53 \quad \int \frac{(c+dx)^2}{\left(a+b(F^g(e+fx))^n\right)^2} dx$$

**Optimal.** Leaf size=294

$$\frac{2d(c+dx)\text{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)} + \frac{2d(c+dx) \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^2 f^2 g^2 n^2 \log^2(F)} - \frac{(c+dx)^2 \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^2 f g n \log(F)} + \frac{2d^2 \text{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^3 g^3 n^3 \log^3(F)}$$

[Out]  $\frac{1}{3} \frac{(d*x+c)^3}{a^2 d} - \frac{(d*x+c)^2}{a^2 f/g/n/\ln(F)} + \frac{(d*x+c)^2}{a f} \frac{1}{(a+b*(F^{g*(f*x+e)})^n)/g/n/\ln(F)} + 2*d*(d*x+c)*\ln(1+b*(F^{g*(f*x+e)})^n/a)/a^2 f^2/g^2/n^2/\ln(F)^2 - \frac{(d*x+c)^2*\ln(1+b*(F^{g*(f*x+e)})^n/a)}{a^2 f/g/n/\ln(F)} + 2*d^2*\text{polylog}(2, -b*(F^{g*(f*x+e)})^n/a)/a^2 f^3/g^3/n^3/\ln(F)^3 - 2*d*(d*x+c)*\text{polylog}(2, -b*(F^{g*(f*x+e)})^n/a)/a^2 f^2/g^2/n^2/\ln(F)^2 + 2*d^2*\text{polylog}(3, -b*(F^{g*(f*x+e)})^n/a)/a^2 f^3/g^3/n^3/\ln(F)^3$

**Rubi [A]** time = 0.67, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$ , Rules used = {2185, 2184, 2190, 2531, 2282, 6589, 2191, 2279, 2391}

$$\frac{2d(c+dx)\text{PolyLog}\left(2, -\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)} + \frac{2d^2 \text{PolyLog}\left(2, -\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^3 g^3 n^3 \log^3(F)} + \frac{2d^2 \text{PolyLog}\left(3, -\frac{b(F^g(e+fx))^n}{a}\right)}{a^2 f^3 g^3 n^3 \log^3(F)} + \frac{2d(c+dx) \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^2 f^2 g^2 n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] `Int[(c + d*x)^2/(a + b*(F^(g*(e + f*x))))^2, x]`

[Out]  $(c + d*x)^3/(3*a^2*d) - (c + d*x)^2/(a^2*f*g*n*\text{Log}[F]) + (c + d*x)^2/(a*f*(a + b*(F^{g*(e + f*x)})^n)*g*n*\text{Log}[F]) + (2*d*(c + d*x)*\text{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a]]/(a^2*f^2*g^2*n^2*\text{Log}[F]^2) - ((c + d*x)^2*\text{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a]]/(a^2*f*g*n*\text{Log}[F]) + (2*d^2*\text{PolyLog}[2, -((b*(F^{g*(e + f*x)})^n)/a)]]/(a^2*f^3*g^3*n^3*\text{Log}[F]^3) - (2*d*(c + d*x)*\text{PolyLog}[2, -((b*(F^{g*(e + f*x)})^n)/a)]]/(a^2*f^2*g^2*n^2*\text{Log}[F]^2) + (2*d^2*\text{PolyLog}[3, -((b*(F^{g*(e + f*x)})^n)/a)]]/(a^2*f^3*g^3*n^3*\text{Log}[F]^3)$

**Rule 2184**

`Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x))))^n/(a + b*(F^(g*(e + f*x))))^n], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`



Rule 2185

```
Int[((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))^(p_)*((c_) +
(d_)*(x_))^(m_), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
f*x))))^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x))))^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((a_) + (b_)*((F_)^((g_)*
(e_) + (f_)*(x_))))^(p_)*((c_) + (d_)*(x_))^(m_), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x))))^n)^(p + 1)]/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x))))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\int \frac{(c + dx)^2}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2} dx = \frac{\int \frac{(c+dx)^2}{a+b(F^{g(e+fx)})^n} dx}{a} - \frac{b \int \frac{(F^{g(e+fx)})^n (c+dx)^2}{(a+b(F^{g(e+fx)})^n)^2} dx}{a}$$

$$= \frac{(c + dx)^3}{3a^2d} + \frac{(c + dx)^2}{af \left(a + b \left(F^{g(e+fx)}\right)^n\right) gn \log(F)} - \frac{b \int \frac{(F^{g(e+fx)})^n (c+dx)^2}{a+b(F^{g(e+fx)})^n} dx}{a^2} - \frac{(2d) \int \frac{c+}{a+b(F^{g(e+fx)})^n} dx}{af gn \log(F)}$$

$$= \frac{(c + dx)^3}{3a^2d} - \frac{(c + dx)^2}{a^2 f gn \log(F)} + \frac{(c + dx)^2}{af \left(a + b \left(F^{g(e+fx)}\right)^n\right) gn \log(F)} - \frac{(c + dx)^2 \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2 f gn \log(F)}$$

$$= \frac{(c + dx)^3}{3a^2d} - \frac{(c + dx)^2}{a^2 f gn \log(F)} + \frac{(c + dx)^2}{af \left(a + b \left(F^{g(e+fx)}\right)^n\right) gn \log(F)} + \frac{2d(c + dx) \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)}$$

$$= \frac{(c + dx)^3}{3a^2d} - \frac{(c + dx)^2}{a^2 f gn \log(F)} + \frac{(c + dx)^2}{af \left(a + b \left(F^{g(e+fx)}\right)^n\right) gn \log(F)} + \frac{2d(c + dx) \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)}$$

$$= \frac{(c + dx)^3}{3a^2d} - \frac{(c + dx)^2}{a^2 f gn \log(F)} + \frac{(c + dx)^2}{af \left(a + b \left(F^{g(e+fx)}\right)^n\right) gn \log(F)} + \frac{2d(c + dx) \log\left(1 + \frac{b(F^{g(e+fx)})^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)}$$

**Mathematica** [F] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^2}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x)))^n)^2, x]

[Out] Integrate[(c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x)))^n)^2, x]

**fricas** [C] time = 0.44, size = 834, normalized size = 2.84

$$3(ad^2e^2 - 2acdef + ac^2f^2)g^2n^2 \log(F)^2 + (ad^2f^3g^3n^3x^3 + 3acdf^3g^3n^3x^2 + 3ac^2f^3g^3n^3x + (ad^2e^3 - 3acde^2f$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e)))^n)^2,x, algorithm="fricas")

[Out] 1/3\*(3\*(a\*d^2\*e^2 - 2\*a\*c\*d\*e\*f + a\*c^2\*f^2)\*g^2\*n^2\*log(F)^2 + (a\*d^2\*f^3\*g^3\*n^3\*x^3 + 3\*a\*c\*d\*f^3\*g^3\*n^3\*x^2 + 3\*a\*c^2\*f^3\*g^3\*n^3\*x + (a\*d^2\*e^3 - 3\*a\*c\*d\*e^2\*f + 3\*a\*c^2\*e\*f^2)\*g^3\*n^3)\*log(F)^3 + ((b\*d^2\*f^3\*g^3\*n^3\*x^3 + 3\*b\*c\*d\*f^3\*g^3\*n^3\*x^2 + 3\*b\*c^2\*f^3\*g^3\*n^3\*x + (b\*d^2\*e^3 - 3\*b\*c\*d\*e^2\*f + 3\*b\*c^2\*e\*f^2)\*g^3\*n^3)\*log(F)^3 - 3\*(b\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*b\*c\*d\*f^2\*g^2\*n^2\*x - (b\*d^2\*e^2 - 2\*b\*c\*d\*e\*f)\*g^2\*n^2)\*log(F)^2)\*F^(f\*g\*n\*x + e\*g\*n) + 6\*(a\*d^2 + (b\*d^2 - (b\*d^2\*f\*g\*n\*x + b\*c\*d\*f\*g\*n)\*log(F))\*F^(f\*g\*n\*x + e\*g\*n) - (a\*d^2\*f\*g\*n\*x + a\*c\*d\*f\*g\*n)\*log(F))\*dilog(-(F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a + 1) - 3\*((a\*d^2\*e^2 - 2\*a\*c\*d\*e\*f + a\*c^2\*f^2)\*g^2\*n^2\*log(F)^2 + 2\*(a\*d^2\*e - a\*c\*d\*f)\*g\*n\*log(F) + ((b\*d^2\*e^2 - 2\*b\*c\*d\*e\*f + b\*c^2\*f^2)\*g^2\*n^2\*log(F)^2 + 2\*(b\*d^2\*e - b\*c\*d\*f)\*g\*n\*log(F))\*F^(f\*g\*n\*x + e\*g\*n))\*log(F^(f\*g\*n\*x + e\*g\*n)\*b + a) - 3\*((a\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*a\*c\*d\*f^2\*g^2\*n^2\*x - (a\*d^2\*e^2 - 2\*a\*c\*d\*e\*f)\*g^2\*n^2)\*log(F)^2 + ((b\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*b\*c\*d\*f^2\*g^2\*n^2\*x - (b\*d^2\*e^2 - 2\*b\*c\*d\*e\*f)\*g^2\*n^2)\*log(F)^2 - 2\*(b\*d^2\*f\*g\*n\*x + b\*d^2\*e\*g\*n)\*log(F))\*F^(f\*g\*n\*x + e\*g\*n) - 2\*(a\*d^2\*f\*g\*n\*x + a\*d^2\*e\*g\*n)\*log(F))\*log((F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a) + 6\*(F^(f\*g\*n\*x + e\*g\*n)\*b\*d^2 + a\*d^2)\*polylog(3, -F^(f\*g\*n\*x + e\*g\*n)\*b/a)/(F^(f\*g\*n\*x + e\*g\*n)\*a^2\*b\*f^3\*g^3\*n^3\*log(F)^3 + a^3\*f^3\*g^3\*n^3\*log(F)^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{\left(\left(F^{(fx+e)g}\right)^n b + a\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^2/((F^((f\*x + e)\*g))^n\*b + a)^2, x)

**maple [B]** time = 0.08, size = 1650, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2/(b\*(F^((f\*x+e)\*g))^(n+a))^2,x)

[Out] 
$$\frac{1}{n} \frac{f}{g} \frac{\ln(F)}{a} \frac{(d^2 x^2 + 2 c d x + c^2)}{(b (F^{(f x + e) g})^{n+a})^{-2/3} f^{3/3} g^{3/1} n(F)^{3/a^2 d^2 \ln(F)^{3+2/n} f^{2/g^2} \ln(F)^2/a^2 c d \ln(b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+a} \ln(F^{(f x + e) g})^{-2/n} f^{2/g^2} \ln(F)^2/a^2 c d \ln(1/a b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+1} \ln(F^{(f x + e) g}) + 2/n} f/g \ln(F)/a^2 c d \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) x^{-2/n} f^{2/g^2} \ln(F)^2/a^2 c d \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) \ln(F^{(f x + e) g})^{-2/n} f^{2/g^2} \ln(F)^2/a^2 d^2 \ln(1/a b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+1} \ln(F^{(f x + e) g}) x^{-2/n} f^{2/g^2} \ln(F)^2/a^2 d^2 \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) \ln(F^{(f x + e) g}) x + 2/n} f^{2/g^2} \ln(F)^2/a^2 d^2 \ln(b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+a} \ln(F^{(f x + e) g}) x^{-2/n} f/g \ln(F)/a^2 c d \ln(b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+a} x^{-2/n^2} f^{2/g^2} \ln(F)^2/a^2 c d \operatorname{polylog}(2, -1/a b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+2/n^2} f^{2/g^2} \ln(F)^2/a^2 c d \ln(b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+a} - 2/n^2} f^{2/g^2} \ln(F)^2/a^2 d^2 \operatorname{polylog}(2, -1/a b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) x + 1/n} f^{3/g^3} \ln(F)^3/a^2 d^2 \ln(1/a b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+1} \ln(F^{(f x + e) g})^2 + 1/n} f/g \ln(F)/a^2 d^2 \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) x^2 + 1/n} f^{3/g^3} \ln(F)^3/a^2 d^2 \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) \ln(F^{(f x + e) g})^2 - 1/n} f/g \ln(F)/a^2 d^2 \ln(b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+a} x^2 - 1/n} f^{3/g^3} \ln(F)^3/a^2 d^2 \ln(b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+a} \ln(F^{(f x + e) g})^2 + 2/n^2} f^{3/g^3} \ln(F)^3/a^2 d^2 \ln(1/a b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+1} \ln(F^{(f x + e) g})^{-2/n^2} f^{2/g^2} \ln(F)^2/a^2 d^2 \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) x + 2/n^2} f^{3/g^3} \ln(F)^3/a^2 d^2 \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) \ln(F^{(f x + e) g})^{-2/n^2} f^{3/g^3} \ln(F)^3/a^2 d^2 \ln(b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+a} x^{-2/n^2} f^{2/g^2} \ln(F)^2/a^2 c d \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) - 1/n} f/g \ln(F)/a^2 c^2 \ln(b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^{n+a} + 1/n} f/g \ln(F)/a^2 c^2 \ln(F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) - 1/n} f^{3/g^3} \ln(F)^3/a^2 d^2 \ln(F^{(f x + e) g})^2 + 2/n^3} f^{3/g^3} \ln(F)^3/a^2 d^2 \operatorname{polylog}(3, -1/a b F^{(f g n x)} F^{-f g n x}) F^{(f x + e) g})^n) + 2/n^3} f^{3/g^3} \ln(F)^3/a^2 d^2$$

$\frac{c^2 \text{polylog}(2, -1/a*b*F^{(f*g*x)}*F^{-(f*g*x)}*(F^{((f*x+e)*g)})^n) + 1/f^2/g^2/\ln(F)^2/a^2*d^2*\ln(F^{((f*x+e)*g)})^2*x + 1/f^2/g^2/\ln(F)^2/a^2*c*d*\ln(F^{((f*x+e)*g)})^2}{1}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left( \frac{1}{\left( (F^{g*x+e*g})^n abn + a^2n \right) fg \log(F)} + \frac{\log(F^{fg*x+e*g})}{a^2 fg \log(F)} - \frac{\log\left(\frac{(F^{fg*x+e*g})^n b+a}{b}\right)}{a^2 fg n \log(F)} \right) + \frac{d^2 x^2 + 2 c d x}{(F^{g*x})^n (F^{e*g})^n ab fg n \log(F) + a^2 fg n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="maxima")

[Out]  $c^2*(1/(((F^{(f*g*x + e*g)})^n*a*b*n + a^2*n)*f*g*\log(F)) + \log(F^{(f*g*x + e*g)})/(a^2*f*g*\log(F)) - \log(((F^{(f*g*x + e*g)})^n*b + a)/b)/(a^2*f*g*n*\log(F))) + (d^2*x^2 + 2*c*d*x)/((F^{(f*g*x)})^n*(F^{(e*g)})^n*a*b*f*g*n*\log(F) + a^2*f*g*n*\log(F)) + \text{integrate}((d^2*f*g*n*x^2*\log(F) - 2*c*d + 2*(c*d*f*g*n*\log(F) - d^2)*x)/((F^{(f*g*x)})^n*(F^{(e*g)})^n*a*b*f*g*n*\log(F) + a^2*f*g*n*\log(F)), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d x)^2}{\left( a + b \left( F^{g(e+f x)} \right)^n \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x))))^n)^2,x

[Out] int((c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x))))^n)^2, x

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2 + 2cdx + d^2x^2}{a^2fgn \log(F) + abfgn \left( F^{g(e+f x)} \right)^n \log(F)} + \frac{\int \left( -\frac{2cd}{a+be^{egn \log(F)} e^{fgnx \log(F)}} \right) dx + \int \left( -\frac{2d^2x}{a+be^{egn \log(F)} e^{fgnx \log(F)}} \right) dx + \int \frac{1}{a+b \left( F^{g(e+f x)} \right)^n}}{afg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*2,x

[Out]  $(c**2 + 2*c*d*x + d**2*x**2)/(a**2*f*g*n*\log(F) + a*b*f*g*n*(F**(g*(e + f*x))))**n*\log(F) + (\text{Integral}(-2*c*d/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))$

$$\begin{aligned}
 & F))), x) + \text{Integral}(-2*d**2*x/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) \\
 & + \text{Integral}(c**2*f*g*n*\log(F)/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) \\
 & + \text{Integral}(d**2*f*g*n*x**2*\log(F)/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) \\
 & + \text{Integral}(2*c*d*f*g*n*x*\log(F)/(a + b*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) \\
 & )/(a*f*g*n*\log(F))
 \end{aligned}$$

$$3.54 \quad \int \frac{c+dx}{\left(a+b(Fg^{e+fx})^n\right)^2} dx$$

**Optimal.** Leaf size=191

$$\frac{(c+dx) \log\left(\frac{b(Fg^{e+fx})^n}{a} + 1\right)}{a^2 f g n \log(F)} - \frac{d \operatorname{Li}_2\left(-\frac{b(Fg^{e+fx})^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)} + \frac{d \log\left(a + b(Fg^{e+fx})^n\right)}{a^2 f^2 g^2 n^2 \log^2(F)} + \frac{(c+dx)^2}{2a^2 d} - \frac{dx}{a^2 f g n \log(F)} + \frac{dx}{a f g n \log(F)}$$

[Out]  $\frac{1}{2} * (d*x+c)^2 / a^2 / d - d*x / a^2 / f / g / n / \ln(F) + (d*x+c) / a / f / (a+b*(F^{(g*(f*x+e))))^n) / g / n / \ln(F) + d * \ln(a+b*(F^{(g*(f*x+e))))^n) / a^2 / f^2 / g^2 / n^2 / \ln(F)^2 - (d*x+c) * \ln(1+b*(F^{(g*(f*x+e))))^n / a) / a^2 / f / g / n / \ln(F) - d * \operatorname{polylog}(2, -b*(F^{(g*(f*x+e))))^n / a) / a^2 / f^2 / g^2 / n^2 / \ln(F)^2$

**Rubi [A]** time = 0.34, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {2185, 2184, 2190, 2279, 2391, 2191, 2282, 266, 36, 29, 31}

$$\frac{d \operatorname{PolyLog}\left(2, -\frac{b(Fg^{e+fx})^n}{a}\right)}{a^2 f^2 g^2 n^2 \log^2(F)} - \frac{(c+dx) \log\left(\frac{b(Fg^{e+fx})^n}{a} + 1\right)}{a^2 f g n \log(F)} + \frac{d \log\left(a + b(Fg^{e+fx})^n\right)}{a^2 f^2 g^2 n^2 \log^2(F)} + \frac{(c+dx)^2}{2a^2 d} - \frac{dx}{a^2 f g n \log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)/(a + b*(F^{(g*(e + f*x))))^n]^2, x]$

[Out]  $(c + d*x)^2 / (2*a^2*d) - (d*x) / (a^2*f*g*n*\operatorname{Log}[F]) + (c + d*x) / (a*f*(a + b*(F^{(g*(e + f*x))))^n)*g*n*\operatorname{Log}[F]) + (d*\operatorname{Log}[a + b*(F^{(g*(e + f*x))))^n]) / (a^2*f^2*g^2*n^2*\operatorname{Log}[F]^2) - ((c + d*x)*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x))))^n]/a]) / (a^2*f*g*n*\operatorname{Log}[F]) - (d*\operatorname{PolyLog}[2, -((b*(F^{(g*(e + f*x))))^n]/a)]) / (a^2*f^2*g^2*n^2*\operatorname{Log}[F]^2)$

**Rule 29**

$\operatorname{Int}[(x_)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[x], x]$

**Rule 31**

$\operatorname{Int}[(a_) + (b_)*(x_)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /;$   $\operatorname{FreeQ}\{a, b\}, x]$

**Rule 36**

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 2184

```
Int[(((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2185

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2191

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*
(e_.) + (f_.)*(x_))))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

### Rule 2279



```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx &= \frac{\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n} dx}{a} - \frac{b \int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx}{a} \\
&= \frac{(c+dx)^2}{2a^2d} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{b \int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)}{a+b\left(F^{g(e+fx)}\right)^n} dx}{a^2} - \frac{d \int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}}{afgn\log(F)} \\
&= \frac{(c+dx)^2}{2a^2d} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} - \frac{d \operatorname{Subst}\left(\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\right)}{afgn\log(F)} \\
&= \frac{(c+dx)^2}{2a^2d} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} + \frac{d \operatorname{Subst}\left(\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\right)}{afgn\log(F)} \\
&= \frac{(c+dx)^2}{2a^2d} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} - \frac{d \operatorname{Li}_2\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a+b\left(F^{g(e+fx)}\right)^n}\right)}{a^2f^2g^2n^2\log^2(F)} \\
&= \frac{(c+dx)^2}{2a^2d} - \frac{dx}{a^2fgn\log(F)} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{d \log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)}
\end{aligned}$$

**Mathematica [F]** time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n]^2, x]

[Out] Integrate[(c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n]^2, x]

**fricas** [B] time = 0.43, size = 400, normalized size = 2.09

$$2(adf^2g^2n^2x^2 + 2acdf^2g^2n^2x - (ade^2 - 2acef)g^2n^2)\log(F)^2 - ((bdf^2g^2n^2x^2 + 2bcf$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="fricas")

[Out]  $-1/2*(2*(a*d*e - a*c*f)*g*n*\log(F) - (a*d*f^2*g^2*n^2*x^2 + 2*a*c*f^2*g^2*n^2*x - (a*d*e^2 - 2*a*c*e*f)*g^2*n^2)*\log(F)^2 - ((b*d*f^2*g^2*n^2*x^2 + 2*b*c*f^2*g^2*n^2*x - (b*d*e^2 - 2*b*c*e*f)*g^2*n^2)*\log(F)^2 - 2*(b*d*f*g*n*x + b*d*e*g*n)*\log(F))*F^(f*g*n*x + e*g*n) + 2*(F^(f*g*n*x + e*g*n)*b*d + a*d)*\operatorname{dilog}(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1) - 2*((a*d*e - a*c*f)*g*n*\log(F) + ((b*d*e - b*c*f)*g*n*\log(F) + b*d)*F^(f*g*n*x + e*g*n) + a*d)*\log(F^(f*g*n*x + e*g*n)*b + a) + 2*((b*d*f*g*n*x + b*d*e*g*n)*F^(f*g*n*x + e*g*n)*\log(F) + (a*d*f*g*n*x + a*d*e*g*n)*\log(F))*\log((F^(f*g*n*x + e*g*n)*b + a)/a) / (F^(f*g*n*x + e*g*n)*a^2*b*f^2*g^2*n^2*\log(F)^2 + a^3*f^2*g^2*n^2*\log(F)^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left(\left(F^{(fx+e)g}\right)^n b + a\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)/((F^((f\*x + e)\*g))^n\*b + a)^2, x)

**maple** [B] time = 0.11, size = 591, normalized size = 3.09

$$\frac{dx \ln\left(F^{fgnx} F^{-fgnx} \left(F^{(fx+e)g}\right)^n\right)}{a^2 fgn \ln(F)} - \frac{dx \ln\left(b F^{fgnx} F^{-fgnx} \left(F^{(fx+e)g}\right)^n + a\right)}{a^2 fgn \ln(F)} + \frac{c \ln\left(F^{fgnx} F^{-fgnx} \left(F^{(fx+e)g}\right)^n\right)}{a^2 fgn \ln(F)} - \frac{c \ln\left(b F^{fgnx} F^{-fgnx} \left(F^{(fx+e)g}\right)^n + a\right)}{a^2 fgn \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(b\*(F^((f\*x+e)\*g))^n+a)^2,x)

[Out]  $(d*x+c)/a/f/(b*(F^((f*x+e)*g))^n+a)/g/n/\ln(F)+1/\ln(F)/a^2/f/g/n*c*\ln(F^(f*g*n*x)*F^(-f*g*n*x)*(F^((f*x+e)*g))^n)-1/\ln(F)/a^2/f/g/n*c*\ln(b*F^(f*g*n*x)*$

$$F^{-f*g*n*x} * (F^{((f*x+e)*g)})^{n+a} - 1/\ln(F)^2/a^2/f^2/g^2/n^2*d*polylog(2, -1/a*b*F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{((f*x+e)*g)})^n) - 1/\ln(F)^2/a^2/f^2/g^2/n^2*d * \ln(F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{((f*x+e)*g)})^n) + 1/\ln(F)^2/a^2/f^2/g^2/n^2*d * \ln(b*F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{((f*x+e)*g)})^{n+a}) - 1/\ln(F)^2/a^2/f^2/g^2/n * d * \ln(1/a*b*F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{((f*x+e)*g)})^{n+1}) * \ln(F^{((f*x+e)*g)}) + 1/\ln(F)/a^2/f/g/n*d * \ln(F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{((f*x+e)*g)})^n) * x - 1/\ln(F)^2/a^2/f^2/g^2/n*d * \ln(F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{((f*x+e)*g)})^n) * \ln(F^{((f*x+e)*g)}) - 1/\ln(F)/a^2/f/g/n*d * \ln(b*F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{((f*x+e)*g)})^{n+a}) * x + 1/2/\ln(F)^2/a^2/f^2/g^2*d * \ln(F^{((f*x+e)*g)})^2 + 1/\ln(F)^2/a^2/f^2/g^2/n*d * \ln(b*F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{((f*x+e)*g)})^{n+a}) * \ln(F^{((f*x+e)*g)})$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$d \left( \frac{x}{(F^{fgx})^n (F^{eg})^n abfgn \log(F) + a^2 fgn \log(F)} + \int \frac{fgnx \log(F) - 1}{(F^{fgx})^n (F^{eg})^n abfgn \log(F) + a^2 fgn \log(F)} dx \right) + c \left( \frac{1}{(F^{fgx+eg})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="maxima")

[Out] d\*(x/((F^(f\*g\*x))^n\*(F^(e\*g))^n\*a\*b\*f\*g\*n\*log(F) + a^2\*f\*g\*n\*log(F)) + integrate((f\*g\*n\*x\*log(F) - 1)/((F^(f\*g\*x))^n\*(F^(e\*g))^n\*a\*b\*f\*g\*n\*log(F) + a^2\*f\*g\*n\*log(F)), x)) + c\*(1/(((F^(f\*g\*x + e\*g))^n\*a\*b\*n + a^2\*n)\*f\*g\*log(F) + log(F^(f\*g\*x + e\*g))/(a^2\*f\*g\*log(F)) - log(((F^(f\*g\*x + e\*g))^n\*b + a)/b)/(a^2\*f\*g\*n\*log(F))))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + dx}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n)^2,x

[Out] int((c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c + dx}{a^2 fgn \log(F) + abfgn \left(F^{g(e+fx)}\right)^n \log(F)} + \int \left( -\frac{d}{a+be^{egn \log(F)} e^{fgnx \log(F)}} \right) dx + \int \frac{c fgn \log(F)}{a+be^{egn \log(F)} e^{fgnx \log(F)}} dx + \int \frac{d fgn}{a+be^{egn \log(F)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

```
[Out] (c + d*x)/(a**2*f*g*n*log(F) + a*b*f*g*n*(F**(g*(e + f*x)))**n*log(F)) + (Integral(-d/(a + b*exp(e*g*n*log(F))*exp(f*g*n*x*log(F))), x) + Integral(c*f*g*n*log(F)/(a + b*exp(e*g*n*log(F))*exp(f*g*n*x*log(F))), x) + Integral(d*f*g*n*x*log(F)/(a + b*exp(e*g*n*log(F))*exp(f*g*n*x*log(F))), x))/(a*f*g*n*log(F))
```

$$3.55 \quad \int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)^2} dx$$

**Optimal.** Leaf size=74

$$-\frac{\log\left(a+b\left(Fg^{(e+fx)}\right)^n\right)}{a^2 f g n \log(F)} + \frac{x}{a^2} + \frac{1}{a f g n \log(F) \left(a+b\left(Fg^{(e+fx)}\right)^n\right)}$$

[Out]  $x/a^2+1/a/f/(a+b*(F^{(g*(f*x+e)))^n)/g/n/\ln(F)-\ln(a+b*(F^{(g*(f*x+e)))^n)/a^2/f/g/n/\ln(F)$

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2282, 266, 44}

$$-\frac{\log\left(a+b\left(Fg^{(e+fx)}\right)^n\right)}{a^2 f g n \log(F)} + \frac{x}{a^2} + \frac{1}{a f g n \log(F) \left(a+b\left(Fg^{(e+fx)}\right)^n\right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^(-2), x]

[Out]  $x/a^2 + 1/(a*f*(a + b*(F^{(g*(e + f*x)))^n}*g*n*\text{Log}[F]) - \text{Log}[a + b*(F^{(g*(e + f*x)))^n}]/(a^2*f*g*n*\text{Log}[F])$

**Rule 44**

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

**Rule 266**

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

**Rule 2282**

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi

```

onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, F^{g(e+fx)}\right)}{fg \log(F)} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn \log(F)} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right) dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn \log(F)} \\
&= \frac{x}{a^2} + \frac{1}{af \left(a + b \left(F^{g(e+fx)}\right)^n\right) gn \log(F)} - \frac{\log\left(a + b \left(F^{g(e+fx)}\right)^n\right)}{a^2 fgn \log(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 62, normalized size = 0.84

$$\frac{\frac{a}{a+b\left(F^{g(e+fx)}\right)^n} - \log\left(a + b \left(F^{g(e+fx)}\right)^n\right) + fgnx \log(F)}{a^2 fgn \log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^(-2), x]
```

```
[Out] (a/(a + b*(F^(g*(e + f*x)))^n) + f*g*n*x*Log[F] - Log[a + b*(F^(g*(e + f*x)
))^n])/(a^2*f*g*n*Log[F])
```

**fricas [A]** time = 0.42, size = 100, normalized size = 1.35

$$\frac{F^{fgnx+egn} b fgnx \log(F) + a fgnx \log(F) - (F^{fgnx+egn} b + a) \log(F^{fgnx+egn} b + a) + a}{F^{fgnx+egn} a^2 b fgn \log(F) + a^3 fgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2, x, algorithm="fricas")
```

[Out]  $(F^{(f*g*n*x + e*g*n)}*b*f*g*n*x*\log(F) + a*f*g*n*x*\log(F) - (F^{(f*g*n*x + e*g*n)}*b + a)*\log(F^{(f*g*n*x + e*g*n)}*b + a) + a)/(F^{(f*g*n*x + e*g*n)}*a^2*b*f*g*n*\log(F) + a^3*f*g*n*\log(F))$

**giac** [A] time = 0.41, size = 111, normalized size = 1.50

$$\frac{\log(|F|^{fgnx}|F|^{gne})}{a^2fgn\log(F)} - \frac{\log(|F|^{fgnx}Fg^{ne}b + a|)}{a^2fgn\log(F)} + \frac{1}{(F^{fgnx}Fg^{ne}b + a)afgn\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="giac")

[Out]  $\log(\text{abs}(F)^{(f*g*n*x)}*\text{abs}(F)^{(g*n*e)})/(a^2*f*g*n*\log(F)) - \log(\text{abs}(F^{(f*g*n*x)}*F^{(g*n*e)}*b + a))/(a^2*f*g*n*\log(F)) + 1/((F^{(f*g*n*x)}*F^{(g*n*e)}*b + a)*a*f*g*n*\log(F))$

**maple** [A] time = 0.02, size = 99, normalized size = 1.34

$$\frac{1}{(b(F^{(fx+e)g})^n + a)afgn\ln(F)} + \frac{\ln\left(\left(F^{(fx+e)g}\right)^n\right)}{a^2fgn\ln(F)} - \frac{\ln\left(b\left(F^{(fx+e)g}\right)^n + a\right)}{a^2fgn\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*(F^((f\*x+e)\*g)))^n+a)^2,x)

[Out]  $1/g/f/\ln(F)/n/a^2*\ln((F^{((f*x+e)*g)})^n)-\ln(b*(F^{((f*x+e)*g)})^n+a)/a^2/f/g/n/\ln(F)+1/a/f/(b*(F^{((f*x+e)*g)})^n+a)/g/n/\ln(F)$

**maxima** [A] time = 1.19, size = 100, normalized size = 1.35

$$\frac{1}{\left((F^{fgx+eg})^n abn + a^2n\right)fg\log(F)} + \frac{\log(F^{fgx+eg})}{a^2fg\log(F)} - \frac{\log\left(\frac{(F^{fgx+eg})^n b+a}{b}\right)}{a^2fgn\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="maxima")

[Out]  $1/(((F^{(f*g*x + e*g)})^n*a*b*n + a^2*n)*f*g*\log(F)) + \log(F^{(f*g*x + e*g)})/(a^2*f*g*\log(F)) - \log(((F^{(f*g*x + e*g)})^n*b + a)/b)/(a^2*f*g*n*\log(F))$



**mupad [B]** time = 3.45, size = 80, normalized size = 1.08

$$\frac{x}{a^2} + \frac{1}{a f g n \ln(F) \left( a + b \left( F^{f g x} F^{e g} \right)^n \right)} - \frac{\ln \left( a + b \left( F^{f g x} F^{e g} \right)^n \right)}{a^2 f g n \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*(F^(g\*(e + f\*x)))^n)^2,x)

[Out] x/a^2 + 1/(a\*f\*g\*n\*log(F)\*(a + b\*(F^(f\*g\*x)\*F^(e\*g))^n)) - log(a + b\*(F^(f\*g\*x)\*F^(e\*g))^n)/(a^2\*f\*g\*n\*log(F))

**sympy [A]** time = 0.19, size = 66, normalized size = 0.89

$$\frac{1}{a^2 f g n \log(F) + a b f g n \left( F^{g(e+f x)} \right)^n \log(F)} + \frac{x}{a^2} - \frac{\log \left( \frac{a}{b} + \left( F^{g(e+f x)} \right)^n \right)}{a^2 f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F\*\*(g\*(f\*x+e)))\*\*n)\*\*2,x)

[Out] 1/(a\*\*2\*f\*g\*n\*log(F) + a\*b\*f\*g\*n\*(F\*\*(g\*(e + f\*x)))\*\*n\*log(F)) + x/a\*\*2 - log(a/b + (F\*\*(g\*(e + f\*x)))\*\*n)/(a\*\*2\*f\*g\*n\*log(F))

$$3.56 \quad \int \frac{1}{\left(a+b(Fg^{e+fx})^n\right)^2 (c+dx)} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{1}{(c+dx) \left(a+b(F^{eg+fgx})^n\right)^2}, x \right)$$

[Out] Unintegrable(1/(a+b\*(F^(f\*g\*x+e\*g))^n)^2/(d\*x+c), x)

**Rubi [A]** time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{\left(a+b(Fg^{e+fx})^n\right)^2 (c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x)), x]

[Out] Defer[Int][1/((a + b\*(F^(e\*g + f\*g\*x)))^n)^2\*(c + d\*x)), x]

Rubi steps

$$\int \frac{1}{\left(a+b(Fg^{e+fx})^n\right)^2 (c+dx)} dx = \int \frac{1}{\left(a+b(F^{eg+fgx})^n\right)^2 (c+dx)} dx$$

**Mathematica [A]** time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b(Fg^{e+fx})^n\right)^2 (c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x)), x]

[Out] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x)), x]

**fricas** [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2dx + a^2c + (b^2dx + b^2c)(Ffg^{x+eg})^{2n} + 2(abdx + abc)(Ffg^{x+eg})^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c),x, algorithm="fricas")

[Out] integral(1/(a^2\*d\*x + a^2\*c + (b^2\*d\*x + b^2\*c)\*(F^(f\*g\*x + e\*g))^(2\*n) + 2\*(a\*b\*d\*x + a\*b\*c)\*(F^(f\*g\*x + e\*g))^n), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(F^{(f*x+e)g}\right)^n b + a\right)^2 (dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c),x, algorithm="giac")

[Out] integrate(1/(((F^((f\*x + e)\*g))^n\*b + a)^2\*(d\*x + c)), x)

**maple** [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(F^{(f*x+e)g}\right)^n + a\right)^2 (dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c),x)

[Out] int(1/(b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{a^2dfgnx \log(F) + a^2c fgn \log(F) + \left((F^{eg})^n abdfgnx \log(F) + (F^{eg})^n abc fgn \log(F)\right)(Ffg^x)^n} + \int \frac{1}{a^2d^2fgnx^2 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c),x, algorithm="maxima")

[Out]  $1/(a^2*d*f*g*n*x*\log(F) + a^2*c*f*g*n*\log(F) + ((F^{(e*g)})^n*a*b*d*f*g*n*x*\log(F) + (F^{(e*g)})^n*a*b*c*f*g*n*\log(F))*(F^{(f*g*x)})^n) + \text{integrate}((d*f*g*n*x*\log(F) + c*f*g*n*\log(F) + d)/(a^2*d^2*f*g*n*x^2*\log(F) + 2*a^2*c*d*f*g*n*x*\log(F) + a^2*c^2*f*g*n*\log(F) + ((F^{(e*g)})^n*a*b*d^2*f*g*n*x^2*\log(F) + 2*(F^{(e*g)})^n*a*b*c*d*f*g*n*x*\log(F) + (F^{(e*g)})^n*a*b*c^2*f*g*n*\log(F))*(F^{(f*g*x)})^n), x)$

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2 (c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/((a + b*(F^{(g*(e + f*x))})^n)^2*(c + d*x)), x)$

[Out]  $\text{int}(1/((a + b*(F^{(g*(e + f*x))})^n)^2*(c + d*x)), x)$

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{a^2c f g n \log(F) + a^2d f g n x \log(F) + (a b c f g n \log(F) + a b d f g n x \log(F)) \left(F^{g(e+fx)}\right)^n} + \int \frac{1}{ac^2 + 2acd x + ad^2 x^2 + bc^2 e^{g n \log(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(a+b*(F^{(g*(f*x+e))})^n)^2/(d*x+c), x)$

[Out]  $1/(a**2*c*f*g*n*\log(F) + a**2*d*f*g*n*x*\log(F) + (a*b*c*f*g*n*\log(F) + a*b*d*f*g*n*x*\log(F))*(F^{(g*(e + f*x))})^n) + (\text{Integral}(d/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b*c**2*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F)) + 2*b*c*d*x*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F)) + b*d**2*x**2*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) + \text{Integral}(c*f*g*n*\log(F)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b*c**2*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F)) + 2*b*c*d*x*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F)) + b*d**2*x**2*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x) + \text{Integral}(d*f*g*n*x*\log(F)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b*c**2*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F)) + 2*b*c*d*x*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F)) + b*d**2*x**2*\exp(e*g*n*\log(F))*\exp(f*g*n*x*\log(F))), x))/(a*f*g*n*\log(F))$

$$3.57 \quad \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{1}{(c+dx)^2 \left(a+b\left(F^{eg+fgx}\right)^n\right)^2}, x \right)$$

[Out] Unintegrable(1/(a+b\*(F^(f\*g\*x+e\*g))^n)^2/(d\*x+c)^2,x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x)^2], x]

[Out] Defer[Int][1/((a + b\*(F^(e\*g + f\*g\*x))))^n)^2\*(c + d\*x)^2), x]

Rubi steps

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2} dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2 (c+dx)^2} dx$$

Mathematica [A] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x)^2), x]

[Out] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)^2\*(c + d\*x)^2), x]

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{a^2 d^2 x^2 + 2 a^2 c d x + a^2 c^2 + (b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2) (F^{f g x + e g})^{2n} + 2 (a b d^2 x^2 + 2 a b c d x + a b c^2) (F^{f g x + e g})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c)^2,x, algorithm="fricas")

[Out] integral(1/(a^2\*d^2\*x^2 + 2\*a^2\*c\*d\*x + a^2\*c^2 + (b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*(F^(f\*g\*x + e\*g))^(2\*n) + 2\*(a\*b\*d^2\*x^2 + 2\*a\*b\*c\*d\*x + a\*b\*c^2)\*(F^(f\*g\*x + e\*g))^n), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( \left( F^{(f x + e) g} \right)^n b + a \right)^2 (d x + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(1/(((F^((f\*x + e)\*g))^n\*b + a)^2\*(d\*x + c)^2), x)

**maple** [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( b \left( F^{(f x + e) g} \right)^n + a \right)^2 (d x + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c)^2,x)

[Out] int(1/(b\*(F^((f\*x+e)\*g))^n+a)^2/(d\*x+c)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{a^2 d^2 f g n x^2 \log(F) + 2 a^2 c d f g n x \log(F) + a^2 c^2 f g n \log(F) + \left( (F^{e g})^n a b d^2 f g n x^2 \log(F) + 2 (F^{e g})^n a b c d f g n x \log(F) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^2/(d\*x+c)^2,x, algorithm="maxima")

```
[Out] 1/(a^2*d^2*f*g*n*x^2*log(F) + 2*a^2*c*d*f*g*n*x*log(F) + a^2*c^2*f*g*n*log(F) + ((F^(e*g))^n*a*b*d^2*f*g*n*x^2*log(F) + 2*(F^(e*g))^n*a*b*c*d*f*g*n*x*log(F) + (F^(e*g))^n*a*b*c^2*f*g*n*log(F))*(F^(f*g*x))^n) + integrate((d*f*g*n*x*log(F) + c*f*g*n*log(F) + 2*d)/(a^2*d^3*f*g*n*x^3*log(F) + 3*a^2*c*d^2*f*g*n*x^2*log(F) + 3*a^2*c^2*d*f*g*n*x*log(F) + a^2*c^3*f*g*n*log(F) + ((F^(e*g))^n*a*b*d^3*f*g*n*x^3*log(F) + 3*(F^(e*g))^n*a*b*c*d^2*f*g*n*x^2*log(F) + 3*(F^(e*g))^n*a*b*c^2*d*f*g*n*x*log(F) + (F^(e*g))^n*a*b*c^3*f*g*n*log(F))*(F^(f*g*x))^n), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2 (c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(F^(g*(e + f*x))))^n)^2*(c + d*x)^2), x)
```

```
[Out] int(1/((a + b*(F^(g*(e + f*x))))^n)^2*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

---

1

$$a^2c^2fgn \log(F) + 2a^2cdfgnx \log(F) + a^2d^2fgnx^2 \log(F) + (abc^2fgn \log(F) + 2abcdfgnx \log(F) + abd^2fgnx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(F**(g*(f*x+e))))**n)**2/(d*x+c)**2, x)
```

```
[Out] 1/(a**2*c**2*f*g*n*log(F) + 2*a**2*c*d*f*g*n*x*log(F) + a**2*d**2*f*g*n*x**2*log(F) + (a*b*c**2*f*g*n*log(F) + 2*a*b*c*d*f*g*n*x*log(F) + a*b*d**2*f*g*n*x**2*log(F))*(F**(g*(e + f*x))))**n) + (Integral(2*d/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b*c**3*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + 3*b*c**2*d*x*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + 3*b*c*d**2*x**2*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + b*d**3*x**3*exp(e*g*n*log(F))*exp(f*g*n*x*log(F))), x) + Integral(c*f*g*n*log(F)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b*c**3*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + 3*b*c**2*d*x*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + 3*b*c*d**2*x**2*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + b*d**3*x**3*exp(e*g*n*log(F))*exp(f*g*n*x*log(F))), x) + Integral(d*f*g*n*x*log(F)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b*c**3*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + 3*b*c**2*d*x*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + 3*b*c*d**2*x**2*exp(e*g*n*log(F))*exp(f*g*n*x*log(F)) + b*d**3*x**3*exp(e*g*n*log(F))*exp(f*g*n*x*log(F))), x))/(a*f*g*n*log(F))
```

$$3.58 \quad \int \frac{(c+dx)^3}{\left(a+b(F^g(e+fx))^n\right)^3} dx$$

**Optimal.** Leaf size=594

$$\frac{9d^2(c+dx)\text{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^3 g^3 n^3 \log^3(F)} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^3 g^3 n^3 \log^3(F)} - \frac{3d^2(c+dx) \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^3 f^3 g^3 n^3 \log^3(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^2 g^2 n^2 \log^2(F)}$$

[Out]  $\frac{1}{4}*(d*x+c)^4/a^3/d+3/2*d*(d*x+c)^2/a^3/f^2/g^2/n^2/\ln(F)^2-3/2*d*(d*x+c)^2/a^2/f^2/(a+b*(F^(g*(f*x+e))))^n/g^2/n^2/\ln(F)^2-3/2*(d*x+c)^3/a^3/f/g/n/\ln(F)+1/2*(d*x+c)^3/a/f/(a+b*(F^(g*(f*x+e))))^n^2/g/n/\ln(F)+(d*x+c)^3/a^2/f/(a+b*(F^(g*(f*x+e))))^n/g/n/\ln(F)-3*d^2*(d*x+c)*\ln(1+b*(F^(g*(f*x+e))))^n/a/a^3/f^3/g^3/n^3/\ln(F)^3+9/2*d*(d*x+c)^2*\ln(1+b*(F^(g*(f*x+e))))^n/a/a^3/f^2/g^2/n^2/\ln(F)^2-(d*x+c)^3*\ln(1+b*(F^(g*(f*x+e))))^n/a/a^3/f/g/n/\ln(F)-3*d^3*polylog(2,-b*(F^(g*(f*x+e))))^n/a/a^3/f^4/g^4/n^4/\ln(F)^4+9*d^2*(d*x+c)*polylog(2,-b*(F^(g*(f*x+e))))^n/a/a^3/f^3/g^3/n^3/\ln(F)^3-3*d*(d*x+c)^2*polylog(2,-b*(F^(g*(f*x+e))))^n/a/a^3/f^2/g^2/n^2/\ln(F)^2-9*d^3*polylog(3,-b*(F^(g*(f*x+e))))^n/a/a^3/f^4/g^4/n^4/\ln(F)^4+6*d^2*(d*x+c)*polylog(3,-b*(F^(g*(f*x+e))))^n/a/a^3/f^3/g^3/n^3/\ln(F)^3-6*d^3*polylog(4,-b*(F^(g*(f*x+e))))^n/a/a^3/f^4/g^4/n^4/\ln(F)^4$

**Rubi [A]** time = 1.90, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {2185, 2184, 2190, 2531, 6609, 2282, 6589, 2191, 2279, 2391}

$$\frac{9d^2(c+dx)\text{PolyLog}\left(2, -\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^3 g^3 n^3 \log^3(F)} + \frac{6d^2(c+dx)\text{PolyLog}\left(3, -\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^3 g^3 n^3 \log^3(F)} - \frac{3d(c+dx)^2 \text{PolyLog}\left(2, -\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^2 g^2 n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x))))^n]^3, x]

[Out]  $(c + d*x)^4/(4*a^3*d) + (3*d*(c + d*x)^2)/(2*a^3*f^2*g^2*n^2*\text{Log}[F]^2) - (3*d*(c + d*x)^2)/(2*a^2*f^2*(a + b*(F^(g*(e + f*x))))^n)*g^2*n^2*\text{Log}[F]^2 - (3*(c + d*x)^3)/(2*a^3*f*g*n*\text{Log}[F]) + (c + d*x)^3/(2*a*f*(a + b*(F^(g*(e + f*x))))^n)^2*g*n*\text{Log}[F] + (c + d*x)^3/(a^2*f*(a + b*(F^(g*(e + f*x))))^n)*g*n*\text{Log}[F] - (3*d^2*(c + d*x)*\text{Log}[1 + (b*(F^(g*(e + f*x))))^n/a])/(a^3*f^3*g^3*n^3*\text{Log}[F]^3) + (9*d*(c + d*x)^2*\text{Log}[1 + (b*(F^(g*(e + f*x))))^n/a])/(2$



$$\begin{aligned} & *a^3f^2g^2n^2\text{Log}[F]^2) - ((c + dx)^3\text{Log}[1 + (b(F^{g(e+fx)}))^n/a] \\ & ])/(a^3f^3g^3n^3\text{Log}[F]) - (3d^3\text{PolyLog}[2, -(b(F^{g(e+fx)}))^n/a] \\ & ])/(a^3f^4g^4n^4\text{Log}[F]^4) + (9d^2(c + dx)\text{PolyLog}[2, -(b(F^{g(e+fx)}))^n/a] \\ & ))/(a^3f^3g^3n^3\text{Log}[F]^3) - (3d(c + dx)^2\text{PolyLog}[2, -(b(F^{g(e+fx)}))^n/a] \\ & ))/(a^3f^2g^2n^2\text{Log}[F]^2) - (9d^3\text{PolyLog}[3, -(b(F^{g(e+fx)}))^n/a] \\ & ))/(a^3f^4g^4n^4\text{Log}[F]^4) + (6d^2(c + dx)\text{PolyLog}[3, -(b(F^{g(e+fx)}))^n/a] \\ & ))/(a^3f^3g^3n^3\text{Log}[F]^3) - (6d^3\text{PolyLog}[4, -(b(F^{g(e+fx)}))^n/a] \\ & ))/(a^3f^4g^4n^4\text{Log}[F]^4) \end{aligned}$$
Rule 2184

$$\begin{aligned} & \text{Int}[\{(c_{.}) + (d_{.})(x_{.})\}^{(m_{.})}/\{(a_{.}) + (b_{.})(F_{.})^{((g_{.})(e_{.}) + (f_{.})(x_{.})))} \\ & \}^{(n_{.})}, x_{\text{Symbol}}] \text{:>} \text{Simp}[(c + dx)^{(m + 1)}/(a*d*(m + 1)), x] - \text{Dist}[ \\ & b/a, \text{Int}[\{(c + dx)^m*(F^{g(e+fx)})^n\}/(a + b*(F^{g(e+fx)})^n), x], \\ & x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0] \end{aligned}$$
Rule 2185

$$\begin{aligned} & \text{Int}[\{(a_{.}) + (b_{.})(F_{.})^{((g_{.})(e_{.}) + (f_{.})(x_{.})))} \\ & \}^{(n_{.})}\}^{(p_{.})}\{(c_{.}) + (d_{.})(x_{.})\}^{(m_{.})}, x_{\text{Symbol}}] \text{:>} \text{Dist}[1/a, \text{Int}[(c + dx)^m*(a + b*(F^{g(e+fx)})^n)^{p+1}, x], x] - \text{Dist}[b/a, \text{Int}[(c + dx)^m*(F^{g(e+fx)})^n*(a + b*(F^{g(e+fx)})^n)^p, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{ILtQ}[p, 0] \&\& \text{IGtQ}[m, 0] \end{aligned}$$
Rule 2190

$$\begin{aligned} & \text{Int}[\{(F_{.})^{((g_{.})(e_{.}) + (f_{.})(x_{.})))} \\ & \}^{(n_{.})}\{(c_{.}) + (d_{.})(x_{.})\}^{(m_{.})}/ \\ & \{(a_{.}) + (b_{.})(F_{.})^{((g_{.})(e_{.}) + (f_{.})(x_{.})))} \\ & \}^{(n_{.})}, x_{\text{Symbol}}] \text{:>} \text{Simp} \\ & [((c + dx)^m*\text{Log}[1 + (b*(F^{g(e+fx)})^n)/a] \\ & )/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + dx)^{(m-1)}*\text{Log}[1 + (b*(F^{g(e+fx)})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0] \end{aligned}$$
Rule 2191

$$\begin{aligned} & \text{Int}[\{(F_{.})^{((g_{.})(e_{.}) + (f_{.})(x_{.})))} \\ & \}^{(n_{.})}\{(a_{.}) + (b_{.})(F_{.})^{((g_{.})(e_{.}) + (f_{.})(x_{.})))} \\ & \}^{(n_{.})}\}^{(p_{.})}\{(c_{.}) + (d_{.})(x_{.})\}^{(m_{.})}, x_{\text{Symbol}}] \text{:>} \\ & \text{Simp}[\{(c + dx)^m*(a + b*(F^{g(e+fx)})^n)^{p+1}\}/(b*f*g*n*(p + 1)*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*(p + 1)*\text{Log}[F]), \text{Int}[(c + dx)^{(m-1)}*(a + b*(F^{g(e+fx)})^n)^{p+1}, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, m, n, p\}, x\} \&\& \text{NeQ}[p, -1] \end{aligned}$$
Rule 2279

$$\begin{aligned} & \text{Int}[\text{Log}[\{(a_{.}) + (b_{.})(F_{.})^{((e_{.})(c_{.}) + (d_{.})(x_{.})))} \\ & \}^{(n_{.})}], x_{\text{Symbol}}] \text{:>} \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{e*(c + dx)})] \end{aligned}$$

)<sup>n</sup>], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :=> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :=> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :=> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(p\_.)], x\_Symbol] :=> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p]]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{\left(a+b(Fg^{(e+fx)})^n\right)^3} dx &= \frac{\int \frac{(c+dx)^3}{\left(a+b(Fg^{(e+fx)})^n\right)^2} dx}{a} - \frac{b \int \frac{(Fg^{(e+fx)})^n (c+dx)^3}{\left(a+b(Fg^{(e+fx)})^n\right)^3} dx}{a} \\
&= \frac{(c+dx)^3}{2af \left(a+b(Fg^{(e+fx)})^n\right)^2 gn \log(F)} + \frac{\int \frac{(c+dx)^3}{a+b(Fg^{(e+fx)})^n} dx}{a^2} - \frac{b \int \frac{(Fg^{(e+fx)})^n (c+dx)^3}{\left(a+b(Fg^{(e+fx)})^n\right)^2} dx}{a^2} \quad (1) \\
&= \frac{(c+dx)^4}{4a^3d} + \frac{(c+dx)^3}{2af \left(a+b(Fg^{(e+fx)})^n\right)^2 gn \log(F)} + \frac{(c+dx)^3}{a^2f \left(a+b(Fg^{(e+fx)})^n\right) gn \log(F)} \\
&= \frac{(c+dx)^4}{4a^3d} - \frac{3d(c+dx)^2}{2a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^3}{2a^3fgn \log(F)} + \frac{3(c+dx)^3}{2af \left(a+b(Fg^{(e+fx)})^n\right) gn \log(F)} \\
&= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2 \log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^3}{2a^3fgn} \\
&= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2 \log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^3}{2a^3fgn} \\
&= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2 \log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^3}{2a^3fgn} \\
&= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2 \log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^3}{2a^3fgn}
\end{aligned}$$

**Mathematica** [F] time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^3}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x))))^n]^3,x]

[Out] Integrate[(c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x))))^n]^3, x]

**fricas** [C] time = 0.50, size = 2704, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="fricas")

[Out]  $-1/4*(6*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3) * g^3*n^3*\log(F)^3 + 6*(a^2*d^3*e^2 - 2*a^2*c*d^2*e*f + a^2*c^2*d*f^2)*g^2*n^2*\log(F)^2 - (a^2*d^3*f^4*g^4*n^4*x^4 + 4*a^2*c*d^2*f^4*g^4*n^4*x^3 + 6*a^2*c^2*d*f^4*g^4*n^4*x^2 + 4*a^2*c^3*f^4*g^4*n^4*x - (a^2*d^3*e^4 - 4*a^2*c*d^2*e^3*f + 6*a^2*c^2*d*e^2*f^2 - 4*a^2*c^3*e*f^3)*g^4*n^4)*\log(F)^4 - ((b^2*d^3*f^4*g^4*n^4*x^4 + 4*b^2*c*d^2*f^4*g^4*n^4*x^3 + 6*b^2*c^2*d*f^4*g^4*n^4*x^2 + 4*b^2*c^3*f^4*g^4*n^4*x - (b^2*d^3*e^4 - 4*b^2*c*d^2*e^3*f + 6*b^2*c^2*d*e^2*f^2 - 4*b^2*c^3*e*f^3)*g^4*n^4)*\log(F)^4 - 6*(b^2*d^3*f^3*g^3*n^3*x^3 + 3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*b^2*c^2*d*f^3*g^3*n^3*x + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2)*g^3*n^3)*\log(F)^3 + 6*(b^2*d^3*f^2*g^2*n^2*x^2 + 2*b^2*c*d^2*f^2*g^2*n^2*x - (b^2*d^3*e^2 - 2*b^2*c*d^2*e*f)*g^2*n^2)*\log(F)^2)*F^(2*f*g*n*x + 2*e*g*n) - 2*((a*b*d^3*f^4*g^4*n^4*x^4 + 4*a*b*c*d^2*f^4*g^4*n^4*x^3 + 6*a*b*c^2*d*f^4*g^4*n^4*x^2 + 4*a*b*c^3*f^4*g^4*n^4*x - (a*b*d^3*e^4 - 4*a*b*c*d^2*e^3*f + 6*a*b*c^2*d*e^2*f^2 - 4*a*b*c^3*e*f^3)*g^4*n^4)*\log(F)^4 - 2*(2*a*b*d^3*f^3*g^3*n^3*x^3 + 6*a*b*c*d^2*f^3*g^3*n^3*x^2 + 6*a*b*c^2*d*f^3*g^3*n^3*x + (3*a*b*d^3*e^3 - 9*a*b*c*d^2*e^2*f + 9*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*g^3*n^3)*\log(F)^3 + 3*(a*b*d^3*f^2*g^2*n^2*x^2 + 2*a*b*c*d^2*f^2*g^2*n^2*x - (2*a*b*d^3*e^2 - 4*a*b*c*d^2*e*f + a*b*c^2*d*f^2)*g^2*n^2)*\log(F)^2)*F^(f*g*n*x + e*g*n) + 12*(a^2*d^3 + (a^2*d^3*f^2*g^2*n^2*x^2 + 2*a^2*c*d^2*f^2*g^2*n^2*x + a^2*c^2*d*f^2*g^2*n^2)*\log(F)^2 + (b^2*d^3 + (b^2*d^3*f^2*g^2*n^2*x^2 + 2*b^2*c*d^2*f^2*g^2*n^2*x + b^2*c^2*d*f^2*g^2*n^2)*\log(F)^2 - 3*(b^2*d^3*f*g*n*x + b^2*c*d^2*f*g*n)*\log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*(a*b*d^3 + (a*b*d^3*f^2*g^2*n^2*x^2 + 2*a*b*c*d^2*f^2*g^2*n^2*x + a*b*c^2*d*f^2*g^2*n^2)*\log(F)^2 - 3*(a*b*d^3*f*g*n*x + a*b*c*d^2*f*g*n)*\log(F))*F^(f*g*n*x + e*g*n) - 3*(a^2*d^3*f*g*n*x + a^2*c*d^2*f*g*n)*\log(F))*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1) - 2*($

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*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*g^3*n^
3*log(F)^3 + 9*(a^2*d^3*e^2 - 2*a^2*c*d^2*e*f + a^2*c^2*d*f^2)*g^2*n^2*log(
F)^2 + 6*(a^2*d^3*e - a^2*c*d^2*f)*g*n*log(F) + (2*(b^2*d^3*e^3 - 3*b^2*c*d
^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*g^3*n^3*log(F)^3 + 9*(b^2*d^3*e
^2 - 2*b^2*c*d^2*e*f + b^2*c^2*d*f^2)*g^2*n^2*log(F)^2 + 6*(b^2*d^3*e - b^2
*c*d^2*f)*g*n*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*(2*(a*b*d^3*e^3 - 3*a*b*c
*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*g^3*n^3*log(F)^3 + 9*(a*b*d^3
*e^2 - 2*a*b*c*d^2*e*f + a*b*c^2*d*f^2)*g^2*n^2*log(F)^2 + 6*(a*b*d^3*e - a
*b*c*d^2*f)*g*n*log(F))*F^(f*g*n*x + e*g*n))*log(F^(f*g*n*x + e*g*n)*b + a)
+ 2*(2*(a^2*d^3*f^3*g^3*n^3*x^3 + 3*a^2*c*d^2*f^3*g^3*n^3*x^2 + 3*a^2*c^2*
d*f^3*g^3*n^3*x + (a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2)*g^3
*n^3)*log(F)^3 - 9*(a^2*d^3*f^2*g^2*n^2*x^2 + 2*a^2*c*d^2*f^2*g^2*n^2*x - (
a^2*d^3*e^2 - 2*a^2*c*d^2*e*f)*g^2*n^2)*log(F)^2 + (2*(b^2*d^3*f^3*g^3*n^3*
x^3 + 3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*b^2*c^2*d*f^3*g^3*n^3*x + (b^2*d^3*e^
3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2)*g^3*n^3)*log(F)^3 - 9*(b^2*d^3*f
^2*g^2*n^2*x^2 + 2*b^2*c*d^2*f^2*g^2*n^2*x - (b^2*d^3*e^2 - 2*b^2*c*d^2*e*f
)*g^2*n^2)*log(F)^2 + 6*(b^2*d^3*f*g*n*x + b^2*d^3*e*g*n)*log(F))*F^(2*f*g*
n*x + 2*e*g*n) + 2*(2*(a*b*d^3*f^3*g^3*n^3*x^3 + 3*a*b*c*d^2*f^3*g^3*n^3*x^
2 + 3*a*b*c^2*d*f^3*g^3*n^3*x + (a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^
2*d*e*f^2)*g^3*n^3)*log(F)^3 - 9*(a*b*d^3*f^2*g^2*n^2*x^2 + 2*a*b*c*d^2*f^2
*g^2*n^2*x - (a*b*d^3*e^2 - 2*a*b*c*d^2*e*f)*g^2*n^2)*log(F)^2 + 6*(a*b*d^3
*f*g*n*x + a*b*d^3*e*g*n)*log(F))*F^(f*g*n*x + e*g*n) + 6*(a^2*d^3*f*g*n*x
+ a^2*d^3*e*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)*b + a)/a) + 24*(2*F^(f*g*
n*x + e*g*n)*a*b*d^3 + F^(2*f*g*n*x + 2*e*g*n)*b^2*d^3 + a^2*d^3)*polylog(4
, -F^(f*g*n*x + e*g*n)*b/a) + 12*(3*a^2*d^3 + (3*b^2*d^3 - 2*(b^2*d^3*f*g*n
*x + b^2*c*d^2*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*(3*a*b*d^3 - 2*(a
*b*d^3*f*g*n*x + a*b*c*d^2*f*g*n)*log(F))*F^(f*g*n*x + e*g*n) - 2*(a^2*d^3*
f*g*n*x + a^2*c*d^2*f*g*n)*log(F))*polylog(3, -F^(f*g*n*x + e*g*n)*b/a))/(2
*F^(f*g*n*x + e*g*n)*a^4*b*f^4*g^4*n^4*log(F)^4 + F^(2*f*g*n*x + 2*e*g*n)*a
^3*b^2*f^4*g^4*n^4*log(F)^4 + a^5*f^4*g^4*n^4*log(F)^4)

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**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{\left(\left(F^{(fx+e)g}\right)^n b + a\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e)))^n)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^3/((F^((f\*x + e)\*g))^n\*b + a)^3, x)

**maple [B]** time = 0.10, size = 4005, normalized size = 6.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*x+c)^3/(b*(F((f*x+e)*g))^n+a)^3,x)$

[Out]  $\frac{3}{4} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F((f*x+e)*g))^4 + \frac{3}{2} \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 d^3} \ln(F((f*x+e)*g))^2 x^2 - \frac{2}{f^3} \frac{\ln(F)^3}{a^3 d^3} \ln(F((f*x+e)*g))^3 x^3 + \frac{2}{f^2} \frac{\ln(F)^2}{a^3 d^3} \frac{c^2}{c^2} \ln(F((f*x+e)*g))^2 - \frac{2}{f^3} \frac{\ln(F)^3}{a^3 c^2 d^2} \ln(F((f*x+e)*g))^3 + \frac{3}{2} \frac{\ln(F)^2}{n^2} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F((f*x+e)*g))^2 + \frac{3}{n} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F((f*x+e)*g))^3 - \frac{9}{n^4} \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F((f*x+e)*g))^3 - 9 \frac{n^4}{f^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F((f*x+e)*g))^3 - \text{polylog}(3, -1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) - 6/n^4 \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F((f*x+e)*g))^3 - \text{polylog}(4, -1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) + 1/n \frac{f}{g} \frac{\ln(F)}{a^3 c^3} \ln(F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) - 1/n \frac{f}{g} \frac{\ln(F)}{a^3 c^3} \ln(b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+a) - 3/n^4 \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F((f*x+e)*g))^3 - \text{polylog}(2, -1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) - 6/n \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 c^2 d^2} \ln(F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) * \ln(F((f*x+e)*g))^2 x^2 + 6/n \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 c^2 d^2} \ln(b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+a) * \ln(F((f*x+e)*g))^2 x^2 - 6/n \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 c^2 d^2} \ln(1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+1) * \ln(F((f*x+e)*g))^2 x^2 + 1/2 * (2*\ln(F)*b*d^3*f*g*n*x^3*(F((f*x+e)*g))^n+3*\ln(F)*a*d^3*f*g*n*x^3+6*\ln(F)*b*c*d^2*f*g*n*x^2*(F((f*x+e)*g))^n+9*\ln(F)*a*c*d^2*f*g*n*x^2+6*\ln(F)*b*c^2*d*f*g*n*x*(F((f*x+e)*g))^n+9*\ln(F)*a*c^2*d*f*g*n*x+2*\ln(F)*b*c^3*f*g*n*(F((f*x+e)*g))^n+3*\ln(F)*a*c^3*f*g*n-3*b*d^3*x^2*(F((f*x+e)*g))^n-3*a*d^3*x^2-6*b*c*d^2*x*(F((f*x+e)*g))^n-6*a*c*d^2*x-3*b*c^2*d*(F((f*x+e)*g))^n-3*a*c^2*d)/n^2 \frac{f^2}{g^2} \frac{\ln(F)^2}{a^2} (b*(F((f*x+e)*g))^n+a)^2 - 1/n \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) * \ln(F((f*x+e)*g))^3 - 1/n \frac{f}{g} \frac{\ln(F)}{a^3 d^3} \ln(b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+a) * x^3 + 1/n \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+a) * \ln(F((f*x+e)*g))^3 - 3/n^3 \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+1) * \ln(F((f*x+e)*g))^3 - 3/n^3 \frac{f^3}{g^3} \frac{\ln(F)^3}{a^3 d^3} \ln(b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+a) * x^3 + 3/n^3 \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+a) * \ln(F((f*x+e)*g))^3 + 6/n^3 \frac{f^3}{g^3} \frac{\ln(F)^3}{a^3 c^2 d^2} \text{polylog}(3, -1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) + 9/n^3 \frac{f^3}{g^3} \frac{\ln(F)^3}{a^3 c^2 d^2} \text{polylog}(2, -1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) - 3/n^2 \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 d^3} \text{polylog}(2, -1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) * x^2 + 9/2/n^2 \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 c^2 d^2} \ln(b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+a) - 9/2/n^2 \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 c^2 d^2} \ln(F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) - 3/n^2 \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 c^2 d^2} \text{polylog}(2, -1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) + 3/n^3 \frac{f^3}{g^3} \frac{\ln(F)^3}{a^3 d^3} \ln(F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) * x^3 - 3/n^3 \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n) * \ln(F((f*x+e)*g))^2 - 9/2/n \frac{f^3}{g^3} \frac{\ln(F)^3}{a^3 d^3} \ln(F((f*x+e)*g))^2 x^2 - 9/2/n \frac{f^4}{g^4} \frac{\ln(F)^4}{a^3 d^3} \ln(1/a*b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+1) * \ln(F((f*x+e)*g))^2 + 3/n \frac{f^2}{g^2} \frac{\ln(F)^2}{a^3 d^3} \ln(b*F(f*g*n*x)*F(-f*g*n*x)*(F((f*x+e)*g))^n+a) * \ln(F((f*x+e)*g))^2$

$$\begin{aligned}
& *g)) *x^{2-3/n} / f^3 / g^3 / \ln(F)^3 / a^3 d^3 \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x \\
& + e) * g))^n + a) * \ln(F^{((f * x + e) * g)})^{2 * x - 3/n} / f^2 / g^2 / \ln(F)^2 / a^3 d^3 \ln(1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + 1} * \ln(F^{((f * x + e) * g)}) * x^{2+3/n} / f^3 / g^3 / \ln(F)^3 / a^3 d^3 \ln(1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + 1} * \ln(F^{((f * x + e) * g)})^{2 * x + 3/n} / f / g / \ln(F) / a^3 c^2 d * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * x^{-3/n} / f / g / \ln(F) / a^3 c^2 d * \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * x^{-3/n} / f^2 / g^2 / \ln(F)^2 / a^3 d^3 \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * \ln(F^{((f * x + e) * g)}) * x^{2+3/n} / f / g / \ln(F) / a^3 c * d^2 * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * x^{2+3/n} / f^3 / g^3 / \ln(F)^3 / a^3 c * d^2 * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * \ln(F^{((f * x + e) * g)})^{2-3/n} / f / g / \ln(F) / a^3 c * d^2 * \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * x^{2-3/n} / f^3 / g^3 / \ln(F)^3 / a^3 c * d^2 * \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * \ln(F^{((f * x + e) * g)})^{2-9/2/n} / f^2 / g^2 / \ln(F)^2 / a^3 d^3 \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * x^{2-9/2/n} / f^4 / g^4 / \ln(F)^4 / a^3 d^3 \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * \ln(F^{((f * x + e) * g)})^{2+9/2/n} / f^2 / g^2 / \ln(F)^2 / a^3 d^3 \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * x^{2+9/2/n} / f^4 / g^4 / \ln(F)^4 / a^3 d^3 \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * \ln(F^{((f * x + e) * g)})^{2+9/n} / f^3 / g^3 / \ln(F)^3 / a^3 d^3 \text{polylog}(2, -1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * x^{6/n} / f^3 / g^3 / \ln(F)^3 / a^3 d^3 \text{polylog}(3, -1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * x^{3/n} / f^3 / g^3 / \ln(F)^3 / a^3 c * d^2 * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} - 3/n^3 / f^3 / g^3 / \ln(F)^3 / a^3 c * d^2 * \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) - 1/n / f^4 / g^4 / \ln(F)^4 / a^3 d^3 \ln(1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + 1} * \ln(F^{((f * x + e) * g)})^{3+1/n} / f / g / \ln(F) / a^3 d^3 \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * x^{3+3/n} / f^2 / g^2 / \ln(F)^2 / a^3 c * d^2 * \ln(F^{((f * x + e) * g)})^{2 * x - 3/n} / f^2 / g^2 / \ln(F)^2 / a^3 c^2 d * \ln(1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + 1} * \ln(F^{((f * x + e) * g)})^{-3/n} / f^2 / g^2 / \ln(F)^2 / a^3 c^2 d * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * \ln(F^{((f * x + e) * g)}) + 3/n / f^2 / g^2 / \ln(F)^2 / a^3 c^2 d * \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * \ln(F^{((f * x + e) * g)})^{-6/n} / f^2 / g^2 / \ln(F)^2 / a^3 c * d^2 * \text{polylog}(2, -1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * x^{9/n} / f^3 / g^3 / \ln(F)^3 / a^3 c * d^2 * \ln(1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + 1} * \ln(F^{((f * x + e) * g)}) + 9/n^2 / f^2 / g^2 / \ln(F)^2 / a^3 c * d^2 * \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * x^{-9/n} / f^3 / g^3 / \ln(F)^3 / a^3 c * d^2 * \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * \ln(F^{((f * x + e) * g)})^{-9/n} / f^2 / g^2 / \ln(F)^2 / a^3 c * d^2 * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * x^{9/n} / f^3 / g^3 / \ln(F)^3 / a^3 c * d^2 * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * \ln(F^{((f * x + e) * g)}) + 9/n^2 / f^3 / g^3 / \ln(F)^3 / a^3 d^3 \ln(1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + 1} * \ln(F^{((f * x + e) * g)}) * x^{9/n} / f^3 / g^3 / \ln(F)^3 / a^3 d^3 * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * \ln(F^{((f * x + e) * g)}) * x^{-9/n} / f^3 / g^3 / \ln(F)^3 / a^3 d^3 * \ln(b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + a) * \ln(F^{((f * x + e) * g)}) * x^{3/n} / f^3 / g^3 / \ln(F)^3 / a^3 c * d^2 * \ln(1 / a * b * F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n + 1} * \ln(F^{((f * x + e) * g)})^{2+3/n} / f^3 / g^3 / \ln(F)^3 / a^3 d^3 * \ln(F^{(f * g * n * x)} * F^{-(f * g * n * x)} * (F^{((f * x + e) * g))^n} * \ln(F^{((f * x + e) * g)})^{2 * x}
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} c^3 \left( \frac{2 (F^{fgx+eg})^n b + 3 a}{\left( 2 (F^{fgx+eg})^n a^3 b n + (F^{fgx+eg})^{2n} a^2 b^2 n + a^4 n \right) f g \log(F)} + \frac{2 \log(F^{fgx+eg})}{a^3 f g \log(F)} - \frac{2 \log\left(\frac{(F^{fgx+eg})^n b + a}{b}\right)}{a^3 f g n \log(F)} \right) + \frac{3 a d^3 f g}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="maxima")

[Out] 1/2\*c^3\*((2\*(F^(f\*g\*x + e\*g))^n\*b + 3\*a)/((2\*(F^(f\*g\*x + e\*g))^n\*a^3\*b\*n + (F^(f\*g\*x + e\*g))^(2\*n)\*a^2\*b^2\*n + a^4\*n)\*f\*g\*log(F)) + 2\*log(F^(f\*g\*x + e\*g))/(a^3\*f\*g\*log(F)) - 2\*log(((F^(f\*g\*x + e\*g))^n\*b + a)/b)/(a^3\*f\*g\*n\*log(F))) + 1/2\*(3\*a\*d^3\*f\*g\*n\*x^3\*log(F) - 3\*a\*c^2\*d + 3\*(3\*a\*c\*d^2\*f\*g\*n\*log(F) - a\*d^3)\*x^2 + (2\*(F^(e\*g))^n\*b\*d^3\*f\*g\*n\*x^3\*log(F) - 3\*(F^(e\*g))^n\*b\*c^2\*d + 3\*(2\*(F^(e\*g))^n\*b\*c\*d^2\*f\*g\*n\*log(F) - (F^(e\*g))^n\*b\*d^3)\*x^2 + 6\*(F^(e\*g))^n\*b\*c^2\*d\*f\*g\*n\*log(F) - (F^(e\*g))^n\*b\*c\*d^2)\*x\*(F^(f\*g\*x))^n + 3\*(3\*a\*c^2\*d\*f\*g\*n\*log(F) - 2\*a\*c\*d^2)\*x)/(2\*(F^(f\*g\*x))^n\*(F^(e\*g))^n\*a^3\*b\*f^2\*g^2\*n^2\*log(F)^2 + (F^(f\*g\*x))^(2\*n)\*(F^(e\*g))^(2\*n)\*a^2\*b^2\*f^2\*g^2\*n^2\*log(F)^2 + a^4\*f^2\*g^2\*n^2\*log(F)^2) + integrate(1/2\*(2\*d^3\*f^2\*g^2\*n^2\*x^3\*log(F)^2 - 9\*c^2\*d\*f\*g\*n\*log(F) + 6\*c\*d^2 + 3\*(2\*c\*d^2\*f^2\*g^2\*n^2\*log(F)^2 - 3\*d^3\*f\*g\*n\*log(F))\*x^2 + 6\*(c^2\*d\*f^2\*g^2\*n^2\*log(F)^2 - 3\*c\*d^2\*f\*g\*n\*log(F) + d^3)\*x)/((F^(f\*g\*x))^n\*(F^(e\*g))^n\*a^2\*b\*f^2\*g^2\*n^2\*log(F)^2 + a^3\*f^2\*g^2\*n^2\*log(F)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^3}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x))))^n)^3,x

[Out] int((c + d\*x)^3/(a + b\*(F^(g\*(e + f\*x))))^n)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3ac^3fgn \log(F) + 9ac^2dfgnx \log(F) - 3ac^2d + 9acd^2fgnx^2 \log(F) - 6acd^2x + 3ad^3fgnx^3 \log(F) - 3ad^3x^2 + (2a^4f^2g^2n^2 \log(F)^2 + 4a^3bf^2g^2n^2 (F^{g(e+fx)}))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*x+c)\*\*3/(a+b\*(F\*\*(g\*(f\*x+e)))\*\*n)\*\*3,x)

[Out] (3\*a\*c\*\*3\*f\*g\*n\*log(F) + 9\*a\*c\*\*2\*d\*f\*g\*n\*x\*log(F) - 3\*a\*c\*\*2\*d + 9\*a\*c\*d\*\*2\*f\*g\*n\*x\*\*2\*log(F) - 6\*a\*c\*d\*\*2\*x + 3\*a\*d\*\*3\*f\*g\*n\*x\*\*3\*log(F) - 3\*a\*d\*\*3\*x\*\*2 + (2\*b\*c\*\*3\*f\*g\*n\*log(F) + 6\*b\*c\*\*2\*d\*f\*g\*n\*x\*log(F) - 3\*b\*c\*\*2\*d + 6\*b\*c\*d\*\*2\*f\*g\*n\*x\*\*2\*log(F) - 6\*b\*c\*d\*\*2\*x + 2\*b\*d\*\*3\*f\*g\*n\*x\*\*3\*log(F) - 3\*b\*d\*\*3\*x\*\*2)\*(F\*\*(g\*(e + f\*x)))\*\*n)/(2\*a\*\*4\*f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2 + 4\*a\*\*3\*b\*f\*\*2\*g\*\*2\*n\*\*2\*(F\*\*(g\*(e + f\*x)))\*\*n\*log(F)\*\*2 + 2\*a\*\*2\*b\*\*2\*f\*\*2\*g\*\*2\*n\*\*2\*(F\*\*(g\*(e + f\*x)))\*\*2\*n\*log(F)\*\*2) + (Integral(6\*c\*d\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(6\*d\*\*3\*x/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(2\*c\*\*3\*f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(-9\*c\*\*2\*d\*f\*g\*n\*log(F)/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(-9\*d\*\*3\*f\*g\*n\*x\*\*2\*log(F)/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(2\*d\*\*3\*f\*\*2\*g\*\*2\*n\*\*2\*x\*\*3\*log(F)\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(-18\*c\*d\*\*2\*f\*g\*n\*x\*log(F)/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(6\*c\*d\*\*2\*f\*\*2\*g\*\*2\*n\*\*2\*x\*\*2\*log(F)\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(6\*c\*\*2\*d\*f\*\*2\*g\*\*2\*n\*\*2\*x\*log(F)\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x))/(2\*a\*\*2\*f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2)

$$3.59 \quad \int \frac{(c+dx)^2}{\left(a+b(F^g(e+fx))^n\right)^3} dx$$

**Optimal.** Leaf size=439

$$\frac{2d(c+dx)\text{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^2 g^2 n^2 \log^2(F)} + \frac{3d(c+dx) \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^3 f^2 g^2 n^2 \log^2(F)} - \frac{(c+dx)^2 \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^3 f g n \log(F)} + \frac{3d^2 \text{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^3 g^3 n^3 \log^3(F)}$$

[Out]  $\frac{1}{3}*(d*x+c)^3/a^3/d+d^2*x/a^3/f^2/g^2/n^2/\ln(F)^2-d*(d*x+c)/a^2/f^2/(a+b*(F^{g*(f*x+e)}))^n/g^2/n^2/\ln(F)^2-3/2*(d*x+c)^2/a^3/f/g/n/\ln(F)+1/2*(d*x+c)^2/a/f/(a+b*(F^{g*(f*x+e)}))^n^2/g/n/\ln(F)+(d*x+c)^2/a^2/f/(a+b*(F^{g*(f*x+e)}))^n/g/n/\ln(F)-d^2*\ln(a+b*(F^{g*(f*x+e)}))^n/a^3/f^3/g^3/n^3/\ln(F)^3+3*d*(d*x+c)*\ln(1+b*(F^{g*(f*x+e)}))^n/a/a^3/f^2/g^2/n^2/\ln(F)^2-(d*x+c)^2*\ln(1+b*(F^{g*(f*x+e)}))^n/a/a^3/f/g/n/\ln(F)+3*d^2*\text{polylog}(2,-b*(F^{g*(f*x+e)}))^n/a/a^3/f^3/g^3/n^3/\ln(F)^3-2*d*(d*x+c)*\text{polylog}(2,-b*(F^{g*(f*x+e)}))^n/a/a^3/f^2/g^2/n^2/\ln(F)^2+2*d^2*\text{polylog}(3,-b*(F^{g*(f*x+e)}))^n/a/a^3/f^3/g^3/n^3/\ln(F)^3$

**Rubi [A]** time = 1.32, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {2185, 2184, 2190, 2531, 2282, 6589, 2191, 2279, 2391, 266, 36, 29, 31}

$$\frac{2d(c+dx)\text{PolyLog}\left(2,-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^2 g^2 n^2 \log^2(F)} + \frac{3d^2 \text{PolyLog}\left(2,-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^3 g^3 n^3 \log^3(F)} + \frac{2d^2 \text{PolyLog}\left(3,-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^3 g^3 n^3 \log^3(F)} + \frac{3d(c+dx) \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^3 f^2 g^2 n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x))))^n]^3, x]

[Out]  $(c+d*x)^3/(3*a^3*d) + (d^2*x)/(a^3*f^2*g^2*n^2*\text{Log}[F]^2) - (d*(c+d*x))/(a^2*f^2*(a+b*(F^{g*(e+f*x)}))^n)*g^2*n^2*\text{Log}[F]^2 - (3*(c+d*x)^2)/(2*a^3*f*g*n*\text{Log}[F]) + (c+d*x)^2/(2*a*f*(a+b*(F^{g*(e+f*x)}))^n)^2*g*n*\text{Log}[F] + (c+d*x)^2/(a^2*f*(a+b*(F^{g*(e+f*x)}))^n)*g*n*\text{Log}[F] - (d^2*\text{Log}[a+b*(F^{g*(e+f*x)}))^n]/(a^3*f^3*g^3*n^3*\text{Log}[F]^3) + (3*d*(c+d*x)*\text{Log}[1+(b*(F^{g*(e+f*x)}))^n/a])/a^3*f^2*g^2*n^2*\text{Log}[F]^2 - ((c+d*x)^2*\text{Log}[1+(b*(F^{g*(e+f*x)}))^n/a])/a^3*f*g*n*\text{Log}[F] + (3*d^2*\text{PolyLog}[2,-((b*(F^{g*(e+f*x)}))^n/a])/a^3*f^3*g^3*n^3*\text{Log}[F]^3) - (2*d*(c+d*x)*\text{PolyLog}[2,-((b*(F^{g*(e+f*x)}))^n/a])/a^3*f^2*g^2*n^2*\text{Log}[F]^2) + (2*d^2*\text{PolyLog}[3,-((b*(F^{g*(e+f*x)}))^n/a])/a^3*f^3*g^3*n^3*\text{Log}[F]^3)$

Rule 29

$\text{Int}[(x\_)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

Rule 31

$\text{Int}[(a\_ + (b\_)*(x\_))^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 36

$\text{Int}[1/(((a\_.) + (b\_)*(x\_))*((c\_.) + (d\_)*(x\_))), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] \text{ ; FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

Rule 266

$\text{Int}[(x\_)^{(m\_)}*((a\_ + (b\_)*(x\_)^{(n\_))}^{(p\_)}), x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] \text{ ; FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 2184

$\text{Int}[(c\_ + d\_*(x\_))^{(m\_)} / ((a\_ + (b\_)*(F\_)^{(g\_)*((e\_ + (f\_)*(x\_)))}^{(n\_)}))^{(n\_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)} / (a*d*(m + 1)), x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m * (F^{(g*(e + f*x))})^n / (a + b*(F^{(g*(e + f*x))})^n), x], x] \text{ ; FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

Rule 2185

$\text{Int}[(a\_ + (b\_)*(F\_)^{(g\_)*((e\_ + (f\_)*(x\_)))}^{(n\_)}))^{(p\_)*((c\_ + d\_*(x\_))^{(m\_)}), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(c + d*x)^m * (a + b*(F^{(g*(e + f*x))})^n)^{(p + 1)}, x], x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m * (F^{(g*(e + f*x))})^n * (a + b*(F^{(g*(e + f*x))})^n)^p, x], x] \text{ ; FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{ILtQ}[p, 0] \&\& \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(F\_)^{(g\_)*((e\_ + (f\_)*(x\_)))}^{(n\_)*((c\_ + d\_*(x\_))^{(m\_)} / ((a\_ + (b\_)*(F\_)^{(g\_)*((e\_ + (f\_)*(x\_)))}^{(n\_)}))^{(n\_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a] / (b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m) / (b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)} * \text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] \text{ ; FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

Rule 2191

```
Int[((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((a_) + (b_)*((F_)^((g_)*
(e_) + (f_)*(x_))))^(n_))^(p_)*((c_) + (d_)*(x_))^(m_), x_Symbol] :>
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{\left(a+b(Fg^{(e+fx)})^n\right)^3} dx &= \frac{\int \frac{(c+dx)^2}{\left(a+b(Fg^{(e+fx)})^n\right)^2} dx}{a} - \frac{b \int \frac{(Fg^{(e+fx)})^n (c+dx)^2}{\left(a+b(Fg^{(e+fx)})^n\right)^3} dx}{a} \\
&= \frac{(c+dx)^2}{2af \left(a+b(Fg^{(e+fx)})^n\right)^2 gn \log(F)} + \frac{\int \frac{(c+dx)^2}{a+b(Fg^{(e+fx)})^n} dx}{a^2} - \frac{b \int \frac{(Fg^{(e+fx)})^n (c+dx)^2}{\left(a+b(Fg^{(e+fx)})^n\right)^2} dx}{a^2} \\
&= \frac{(c+dx)^3}{3a^3d} + \frac{(c+dx)^2}{2af \left(a+b(Fg^{(e+fx)})^n\right)^2 gn \log(F)} + \frac{(c+dx)^2}{a^2f \left(a+b(Fg^{(e+fx)})^n\right) gn \log(F)} \\
&= \frac{(c+dx)^3}{3a^3d} - \frac{d(c+dx)}{a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn \log(F)} + \frac{(c+dx)^2}{2af \left(a+b(Fg^{(e+fx)})^n\right) gn \log(F)} \\
&= \frac{(c+dx)^3}{3a^3d} - \frac{d(c+dx)}{a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn \log(F)} + \frac{(c+dx)^2}{2af \left(a+b(Fg^{(e+fx)})^n\right) gn \log(F)} \\
&= \frac{(c+dx)^3}{3a^3d} - \frac{d(c+dx)}{a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn \log(F)} + \frac{(c+dx)^2}{2af \left(a+b(Fg^{(e+fx)})^n\right) gn \log(F)} \\
&= \frac{(c+dx)^3}{3a^3d} + \frac{d^2x}{a^3f^2g^2n^2 \log^2(F)} - \frac{d(c+dx)}{a^2f^2 \left(a+b(Fg^{(e+fx)})^n\right) g^2n^2 \log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn \log(F)}
\end{aligned}$$

**Mathematica** [F] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^2}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x))))^n]^3,x]

[Out] Integrate[(c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x))))^n]^3, x]

**fricas** [C] time = 0.48, size = 1518, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="fricas")

[Out] 1/6\*(9\*(a^2\*d^2\*e^2 - 2\*a^2\*c\*d\*e\*f + a^2\*c^2\*f^2)\*g^2\*n^2\*log(F)^2 + 6\*(a^2\*d^2\*e - a^2\*c\*d\*f)\*g\*n\*log(F) + 2\*(a^2\*d^2\*f^3\*g^3\*n^3\*x^3 + 3\*a^2\*c\*d\*f^3\*g^3\*n^3\*x^2 + 3\*a^2\*c^2\*f^3\*g^3\*n^3\*x + (a^2\*d^2\*e^3 - 3\*a^2\*c\*d\*e^2\*f + 3\*a^2\*c^2\*e\*f^2)\*g^3\*n^3)\*log(F)^3 + (2\*(b^2\*d^2\*f^3\*g^3\*n^3\*x^3 + 3\*b^2\*c\*d\*f^3\*g^3\*n^3\*x^2 + 3\*b^2\*c^2\*f^3\*g^3\*n^3\*x + (b^2\*d^2\*e^3 - 3\*b^2\*c\*d\*e^2\*f + 3\*b^2\*c^2\*e\*f^2)\*g^3\*n^3)\*log(F)^3 - 9\*(b^2\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*b^2\*c\*d\*f^2\*g^2\*n^2\*x - (b^2\*d^2\*e^2 - 2\*b^2\*c\*d\*e\*f)\*g^2\*n^2)\*log(F)^2 + 6\*(b^2\*d^2\*f\*g\*n\*x + b^2\*d^2\*e\*g\*n)\*log(F))\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n) + 2\*(2\*(a\*b\*d^2\*f^3\*g^3\*n^3\*x^3 + 3\*a\*b\*c\*d\*f^3\*g^3\*n^3\*x^2 + 3\*a\*b\*c^2\*f^3\*g^3\*n^3\*x + (a\*b\*d^2\*e^3 - 3\*a\*b\*c\*d\*e^2\*f + 3\*a\*b\*c^2\*e\*f^2)\*g^3\*n^3)\*log(F)^3 - 3\*(2\*a\*b\*d^2\*f^2\*g^2\*n^2\*x^2 + 4\*a\*b\*c\*d\*f^2\*g^2\*n^2\*x - (3\*a\*b\*d^2\*e^2 - 6\*a\*b\*c\*d\*e\*f + a\*b\*c^2\*f^2)\*g^2\*n^2)\*log(F)^2 + 3\*(a\*b\*d^2\*f\*g\*n\*x + (2\*a\*b\*d^2\*e - a\*b\*c\*d\*f)\*g\*n)\*log(F))\*F^(f\*g\*n\*x + e\*g\*n) + 6\*(3\*a^2\*d^2 + (3\*b^2\*d^2 - 2\*(b^2\*d^2\*f\*g\*n\*x + b^2\*c\*d\*f\*g\*n)\*log(F))\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n) + 2\*(3\*a\*b\*d^2 - 2\*(a\*b\*d^2\*f\*g\*n\*x + a\*b\*c\*d\*f\*g\*n)\*log(F))\*F^(f\*g\*n\*x + e\*g\*n) - 2\*(a^2\*d^2\*f\*g\*n\*x + a^2\*c\*d\*f\*g\*n)\*log(F))\*dilog(-(F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a + 1) - 6\*((a^2\*d^2\*e^2 - 2\*a^2\*c\*d\*e\*f + a^2\*c^2\*f^2)\*g^2\*n^2\*log(F)^2 + a^2\*d^2 + 3\*(a^2\*d^2\*e - a^2\*c\*d\*f)\*g\*n\*log(F) + ((b^2\*d^2\*e^2 - 2\*b^2\*c\*d\*e\*f + b^2\*c^2\*f^2)\*g^2\*n^2\*log(F)^2 + b^2\*d^2 + 3\*(b^2\*d^2\*e - b^2\*c\*d\*f)\*g\*n\*log(F))\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n) + 2\*((a\*b\*d^2\*e^2 - 2\*a\*b\*c\*d\*e\*f + a\*b\*c^2\*f^2)\*g^2\*n^2\*log(F)^2 + a\*b\*d^2 + 3\*(a\*b\*d^2\*e - a\*b\*c\*d\*f)\*g\*n\*log(F))\*F^(f\*g\*n\*x + e\*g\*n)\*log(F^(f\*g\*n\*x + e\*g\*n)\*b + a) - 6\*((a^2\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*a^2\*c\*d\*f^2\*g^2\*n^2\*x - (a^2\*d^2\*e^2 - 2\*a^2\*c\*d\*e\*f)\*g^2\*n^2)\*log(F)^2 + ((b^2\*d^2\*f^2\*g^2\*n^2\*x^2 + 2\*b^2\*c\*d\*f^2\*g^2\*n^2\*x - (b^2\*d^2\*e^2 - 2\*b^2\*c\*d\*e\*f)\*g^2\*n^2)\*log(F)^2 - 3\*(b^2\*d^2\*f\*g\*n\*x + b^2\*d^2\*e\*g\*n)\*log(F))\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n) + 2\*((a\*b\*d^2\*f^2\*g^2\*n^2\*x^2 +

$2*a*b*c*d*f^2*g^2*n^2*x - (a*b*d^2*e^2 - 2*a*b*c*d*e*f)*g^2*n^2*\log(F)^2 - 3*(a*b*d^2*f*g*n*x + a*b*d^2*e*g*n)*\log(F))*F^{(f*g*n*x + e*g*n)} - 3*(a^2*d^2*f*g*n*x + a^2*d^2*e*g*n)*\log(F))*\log((F^{(f*g*n*x + e*g*n)}*b + a)/a) + 12*(2*F^{(f*g*n*x + e*g*n)}*a*b*d^2 + F^{(2*f*g*n*x + 2*e*g*n)}*b^2*d^2 + a^2*d^2)*\text{polylog}(3, -F^{(f*g*n*x + e*g*n)}*b/a)/(2*F^{(f*g*n*x + e*g*n)}*a^4*b*f^3*g^3*n^3*\log(F)^3 + F^{(2*f*g*n*x + 2*e*g*n)}*a^3*b^2*f^3*g^3*n^3*\log(F)^3 + a^5*f^3*g^3*n^3*\log(F)^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{\left(\left(F^{(fx+e)g}\right)^n b + a\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^2/((F^((f\*x + e)\*g))^n\*b + a)^3, x)

**maple** [B] time = 0.08, size = 1887, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2/(b\*(F^((f\*x+e)\*g))^n+a)^3,x)

[Out]  $\frac{1}{2}*(2*\ln(F)*b*d^2*f*g*n*x^2*(F^((f*x+e)*g))^n+3*\ln(F)*a*d^2*f*g*n*x^2+4*\ln(F)*b*c*d*f*g*n*x*(F^((f*x+e)*g))^n+6*\ln(F)*a*c*d*f*g*n*x+2*\ln(F)*b*c^2*f*g*n*(F^((f*x+e)*g))^n+3*\ln(F)*a*c^2*f*g*n-2*b*d^2*x*(F^((f*x+e)*g))^n-2*a*d^2*x-2*b*c*d*(F^((f*x+e)*g))^n-2*a*c*d)/n^2/f^2/g^2/\ln(F)^2/a^2/(b*(F^((f*x+e)*g))^n+a)^2-2/3/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(F^((f*x+e)*g))^3-2/n/f^2/g^2/1/n(F)^2/a^3*d^2*\ln(1/a*b*F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n+1)*\ln(F^((f*x+e)*g))^x+2/n/f^2/g^2/\ln(F)^2/a^3*c*d*\ln(b*F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n+a)*\ln(F^((f*x+e)*g))^2/n/f^2/g^2/\ln(F)^2/a^3*d^2*\ln(b*F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n+a)*\ln(F^((f*x+e)*g))^x+2/n/f/g/\ln(F)/a^3*c*d*\ln(F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n)*x-2/n/f/g/\ln(F)/a^3*c*d*\ln(b*F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n+a)*x-2/n/f^2/g^2/\ln(F)^2/a^3*d^2*\ln(F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n)*\ln(F^((f*x+e)*g))^x-2/n/f^2/g^2/\ln(F)^2/a^3*c*d*\ln(1/a*b*F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n+1)*\ln(F^((f*x+e)*g))-2/n/f^2/g^2/\ln(F)^2/a^3*c*d*\ln(F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n)*\ln(F^((f*x+e)*g))-2/n^2/f^2/g^2/\ln(F)^2/a^3*c*d*\text{polylog}(2, -1/a*b*F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n)-3/n^2/f^2/g^2/\ln(F)^2/a^3*c*d*\ln(F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n)+3/n^2/f^2/g^2/\ln(F)^2/a^3*c*d*\ln(b*F^{(f*g*n*x)}*F^{(-f*g*n*x)}*(F^((f*x+e)*g))^n+a)-2/n^2$

$$\begin{aligned} & /f^2/g^2/\ln(F)^2/a^3*d^2*polylog(2,-1/a*b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n)*x+1/n/f/g/\ln(F)/a^3*d^2*\ln(F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n)*x^2+1/n/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n)*\ln(F^{((f*x+e)*g)})^2+1/n/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(1/a*b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^{n+1})*\ln(F^{((f*x+e)*g)})^2-1/n/f/g/\ln(F)/a^3*d^2*\ln(b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^{n+a})*x^2-1/n/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^{n+a})*\ln(F^{((f*x+e)*g)})^2-3/n^2/f^2/g^2/\ln(F)^2/a^3*d^2*\ln(F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n)*x+3/n^2/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n)*\ln(F^{((f*x+e)*g)})+3/n^2/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(1/a*b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^{n+1})*\ln(F^{((f*x+e)*g)})+3/n^2/f^2/g^2/\ln(F)^2/a^3*d^2*\ln(b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^{n+a})*x-3/n^2/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^{n+a})*\ln(F^{((f*x+e)*g)})+1/f^2/g^2/\ln(F)^2/a^3*d*c*\ln(F^{((f*x+e)*g)})^2+1/f^2/g^2/\ln(F)^2/a^3*d^2*\ln(F^{((f*x+e)*g)})^2*x+1/n^3/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n)+2/n^3/f^3/g^3/\ln(F)^3/a^3*d^2*polylog(3,-1/a*b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n)-1/n/f/g/\ln(F)/a^3*c^2*\ln(b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^{n+a})-3/2/n/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(F^{((f*x+e)*g)})^2+1/n/f/g/\ln(F)/a^3*c^2*\ln(F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n)-1/n^3/f^3/g^3/\ln(F)^3/a^3*d^2*\ln(b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^{n+a})+3/n^3/f^3/g^3/\ln(F)^3/a^3*d^2*polylog(2,-1/a*b*F^{(f*g*n*x)}*F^{-(f*g*n*x)}*(F^{((f*x+e)*g)})^n) \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}c^2 \left( \frac{2(Ffg^{x+eg})^n b + 3a}{(2(Ffg^{x+eg})^n a^3 b n + (Ffg^{x+eg})^{2n} a^2 b^2 n + a^4 n)fg \log(F)} + \frac{2 \log(Ffg^{x+eg})}{a^3 fg \log(F)} - \frac{2 \log\left(\frac{(Ffg^{x+eg})^{n+b+a}}{b}\right)}{a^3 fg n \log(F)} \right) + \frac{3ad^2 fg}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 1/2*c^2*((2*(F^{(f*g*x + e*g)})^n*b + 3*a)/((2*(F^{(f*g*x + e*g)})^n*a^3*b*n + \\ & (F^{(f*g*x + e*g)})^{(2*n)}*a^2*b^2*n + a^4*n)*f*g*\log(F)) + 2*\log(F^{(f*g*x + e \\ & *g)})/(a^3*f*g*\log(F)) - 2*\log(((F^{(f*g*x + e*g)})^n*b + a)/b)/(a^3*f*g*n*\log \\ & (F)) + 1/2*(3*a*d^2*f*g*n*x^2*\log(F) - 2*a*c*d + 2*((F^{(e*g)})^n*b*d^2*f*g* \\ & n*x^2*\log(F) - (F^{(e*g)})^n*b*c*d + (2*(F^{(e*g)})^n*b*c*d*f*g*n*\log(F) - (F^{( \\ & e*g)})^n*b*d^2)*x)*(F^{(f*g*x)})^n + 2*(3*a*c*d*f*g*n*\log(F) - a*d^2)*x)/(2*(F \\ & ^{(f*g*x)})^n*(F^{(e*g)})^n*a^3*b*f^2*g^2*n^2*\log(F)^2 + (F^{(f*g*x)})^{(2*n)}*(F^{( \\ & e*g)})^{(2*n)}*a^2*b^2*f^2*g^2*n^2*\log(F)^2 + a^4*f^2*g^2*n^2*\log(F)^2) + inte \\ & grate((d^2*f^2*g^2*n^2*x^2*\log(F)^2 - 3*c*d*f*g*n*\log(F) + d^2 + (2*c*d*f^2 \\ & *g^2*n^2*\log(F)^2 - 3*d^2*f*g*n*\log(F))*x)/((F^{(f*g*x)})^n*(F^{(e*g)})^n*a^2*b \\ & *f^2*g^2*n^2*\log(F)^2 + a^3*f^2*g^2*n^2*\log(F)^2), x) \end{aligned}$$



mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^2}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x)))^n)^3,x)

[Out] int((c + d\*x)^2/(a + b\*(F^(g\*(e + f\*x)))^n)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3ac^2fgn \log(F) + 6acdfgnx \log(F) - 2acd + 3ad^2fgnx^2 \log(F) - 2ad^2x + (2bc^2fgn \log(F) + 4bcdfgnx \log(F) + 2a^4f^2g^2n^2 \log(F)^2 + 4a^3bf^2g^2n^2 \left(F^{g(e+fx)}\right)^n \log(F)^2 + 2a^2b^2f^2g^2n^2 \left(F^{g(e+fx)}\right)^{2n} \log(F)^2)}{2a^4f^2g^2n^2 \log(F)^2 + 4a^3bf^2g^2n^2 \left(F^{g(e+fx)}\right)^n \log(F)^2 + 2a^2b^2f^2g^2n^2 \left(F^{g(e+fx)}\right)^{2n} \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3,x)

[Out] (3\*a\*c\*\*2\*f\*g\*n\*log(F) + 6\*a\*c\*d\*f\*g\*n\*x\*log(F) - 2\*a\*c\*d + 3\*a\*d\*\*2\*f\*g\*n\*x\*\*2\*log(F) - 2\*a\*d\*\*2\*x + (2\*b\*c\*\*2\*f\*g\*n\*log(F) + 4\*b\*c\*d\*f\*g\*n\*x\*log(F) - 2\*b\*c\*d + 2\*b\*d\*\*2\*f\*g\*n\*x\*\*2\*log(F) - 2\*b\*d\*\*2\*x)\*(F\*\*(g\*(e + f\*x))))\*\*n)/(2\*a\*\*4\*f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2 + 4\*a\*\*3\*b\*f\*\*2\*g\*\*2\*n\*\*2\*(F\*\*(g\*(e + f\*x))))\*\*n\*log(F)\*\*2 + 2\*a\*\*2\*b\*\*2\*f\*\*2\*g\*\*2\*n\*\*2\*(F\*\*(g\*(e + f\*x)))\*\*2\*log(F)\*\*2) + (Integral(d\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(c\*\*2\*f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(-3\*c\*d\*f\*g\*n\*log(F)/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(-3\*d\*\*2\*f\*g\*n\*x\*log(F)/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(d\*\*2\*f\*\*2\*g\*\*2\*n\*\*2\*x\*\*2\*log(F)\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(2\*c\*d\*f\*\*2\*g\*\*2\*n\*\*2\*x\*log(F)\*\*2/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x))/(a\*\*2\*f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2)

$$3.60 \quad \int \frac{c+dx}{\left(a+b(F^g(e+fx))^n\right)^3} dx$$

**Optimal.** Leaf size=276

$$\frac{(c+dx) \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^3 f g n \log(F)} - \frac{d \operatorname{Li}_2\left(-\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^2 g^2 n^2 \log^2(F)} + \frac{3d \log\left(a + b(F^g(e+fx))^n\right)}{2a^3 f^2 g^2 n^2 \log^2(F)} + \frac{(c+dx)^2}{2a^3 d} - \frac{3dx}{2a^3 f g n \log(F)} + \frac{1}{a^2 f g n}$$

[Out]  $\frac{1}{2} \frac{(d*x+c)^2}{a^3 d} - \frac{1}{2} \frac{d}{a^2 f^2} \frac{1}{(a+b*(F^{g*(f*x+e)}))^n} \frac{1}{g^2 n^2 \ln(F)^2} - \frac{3}{2} \frac{d*x}{a^3 f g n \ln(F)} + \frac{1}{2} \frac{(d*x+c)}{a f} \frac{1}{(a+b*(F^{g*(f*x+e)}))^n} \frac{1}{g n \ln(F)} + \frac{(d*x+c)}{a^2 f} \frac{1}{(a+b*(F^{g*(f*x+e)}))^n} \frac{1}{g n \ln(F)} + \frac{3}{2} \frac{d * \ln(a+b*(F^{g*(f*x+e)}))^n}{a^3 f^2 g^2 n^2 \ln(F)^2} - \frac{(d*x+c) * \ln(1+b*(F^{g*(f*x+e)}))^n / a}{a^3 f g n \ln(F)} - d * \operatorname{polylog}(2, -b*(F^{g*(f*x+e)}))^n / a / a^3 f^2 g^2 n^2 \ln(F)^2$

**Rubi [A]** time = 0.56, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {2185, 2184, 2190, 2279, 2391, 2191, 2282, 266, 36, 29, 31, 44}

$$\frac{d \operatorname{PolyLog}\left(2, -\frac{b(F^g(e+fx))^n}{a}\right)}{a^3 f^2 g^2 n^2 \log^2(F)} - \frac{(c+dx) \log\left(\frac{b(F^g(e+fx))^n}{a} + 1\right)}{a^3 f g n \log(F)} + \frac{c+dx}{a^2 f g n \log(F) \left(a + b(F^g(e+fx))^n\right)} + \frac{3d \log\left(a + b(F^g(e+fx))^n\right)}{2a^3 f^2 g^2 n^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)/(a + b*(F^{g*(e + f*x)}))^n]^3, x]$

[Out]  $(c + d*x)^2 / (2*a^3*d) - d / (2*a^2*f^2*(a + b*(F^{g*(e + f*x)}))^n) * g^2*n^2 * \operatorname{Log}[F]^2 - (3*d*x) / (2*a^3*f*g*n * \operatorname{Log}[F]) + (c + d*x) / (2*a*f*(a + b*(F^{g*(e + f*x)}))^n) * g^2*n^2 * \operatorname{Log}[F] + (c + d*x) / (a^2*f*(a + b*(F^{g*(e + f*x)}))^n) * g*n * \operatorname{Log}[F] + (3*d * \operatorname{Log}[a + b*(F^{g*(e + f*x)}))^n] / (2*a^3*f^2*g^2*n^2 * \operatorname{Log}[F]^2) - ((c + d*x) * \operatorname{Log}[1 + (b*(F^{g*(e + f*x)}))^n / a]) / (a^3*f*g*n * \operatorname{Log}[F]) - (d * \operatorname{PolyLog}[2, -((b*(F^{g*(e + f*x)}))^n / a)]) / (a^3*f^2*g^2*n^2 * \operatorname{Log}[F]^2)$

**Rule 29**

$\operatorname{Int}[(x_)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[x], x]$

**Rule 31**

$\operatorname{Int}[(a_) + (b_)*(x_)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 36

```
Int[1/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n]/(a + b*(F^(g*(e + f*x)))^n), x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2185

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
(d_.)*(x_.))^(m_.), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*
```

```
(e_.) + (f_.)*(x_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^3} dx &= \frac{\int \frac{c+dx}{\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^2} dx}{a} - \frac{b \int \frac{\left(Fg^{(e+fx)}\right)^n (c+dx)}{\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^3} dx}{a} \\
&= \frac{c+dx}{2af\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^2 gn \log(F)} + \frac{\int \frac{c+dx}{a+b\left(Fg^{(e+fx)}\right)^n} dx}{a^2} - \frac{b \int \frac{\left(Fg^{(e+fx)}\right)^n (c+dx)}{\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^2} dx}{a^2} \\
&= \frac{(c+dx)^2}{2a^3d} + \frac{c+dx}{2af\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^2 gn \log(F)} + \frac{c+dx}{a^2f\left(a+b\left(Fg^{(e+fx)}\right)^n\right) gn \log(F)} \\
&= \frac{(c+dx)^2}{2a^3d} + \frac{c+dx}{2af\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^2 gn \log(F)} + \frac{c+dx}{a^2f\left(a+b\left(Fg^{(e+fx)}\right)^n\right) gn \log(F)} \\
&= \frac{(c+dx)^2}{2a^3d} + \frac{c+dx}{2af\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^2 gn \log(F)} + \frac{c+dx}{a^2f\left(a+b\left(Fg^{(e+fx)}\right)^n\right) gn \log(F)} \\
&= \frac{(c+dx)^2}{2a^3d} - \frac{d}{2a^2f^2\left(a+b\left(Fg^{(e+fx)}\right)^n\right)g^2n^2 \log^2(F)} - \frac{dx}{2a^3fgn \log(F)} + \frac{dx}{2af\left(a+b\left(Fg^{(e+fx)}\right)^n\right)} \\
&= \frac{(c+dx)^2}{2a^3d} - \frac{d}{2a^2f^2\left(a+b\left(Fg^{(e+fx)}\right)^n\right)g^2n^2 \log^2(F)} - \frac{3dx}{2a^3fgn \log(F)} + \frac{dx}{2af\left(a+b\left(Fg^{(e+fx)}\right)^n\right)}
\end{aligned}$$

**Mathematica [F]** time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a+b\left(Fg^{(e+fx)}\right)^n\right)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n]^3,x]

[Out] Integrate[(c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n]^3, x]

**fricas** [B] time = 0.43, size = 696, normalized size = 2.52

$$\frac{3(a^2de - a^2cf)gn \log(F) + a^2d - (a^2df^2g^2n^2x^2 + 2a^2cf^2g^2n^2x - (a^2de^2 - 2a^2cef)g^2n^2) \log(F)^2 - ((b^2df^2g^2n^2x^2 + 2b^2dfe^2g^2n^2x - (b^2de^2 - 2b^2cef)g^2n^2) \log(F)^2 - (b^2df^2g^2n^2x^2 + 2b^2dfe^2g^2n^2x - (b^2de^2 - 2b^2cef)g^2n^2) \log(F)^2 - 3(b^2dfe^2g^2n^2x + b^2dfe^2g^2n^2) \log(F)) F^{2fgn^2x + 2egn} + (ab^2d - 2(ab^2dfe^2g^2n^2x^2 + 2a^2b^2cfe^2g^2n^2x - (ab^2de^2 - 2a^2b^2cfe^2g^2n^2) \log(F)^2 + 2(2a^2b^2dfe^2g^2n^2x + (3a^2b^2de - ab^2cfe^2g^2n^2) \log(F)) F^{fgn^2x + egn} + 2(2F^{fgn^2x + egn} ab^2d + F^{2fgn^2x + 2egn} b^2d + a^2d) \operatorname{dilog}(-F^{fgn^2x + egn} b + a)/a + 1) - (2(a^2dfe^2g^2n^2x - a^2cfe^2g^2n^2) \log(F) + 3a^2d + (2(b^2dfe^2g^2n^2x - b^2cfe^2g^2n^2) \log(F) + 3b^2d) F^{2fgn^2x + 2egn} + 2(2(ab^2dfe^2g^2n^2x - ab^2cfe^2g^2n^2) \log(F) + 3a^2b^2d) F^{fgn^2x + egn} \log(F^{fgn^2x + egn} b + a) + 2((b^2dfe^2g^2n^2x + b^2dfe^2g^2n^2) F^{2fgn^2x + 2egn} \log(F) + 2(ab^2dfe^2g^2n^2x + ab^2dfe^2g^2n^2) F^{fgn^2x + egn} \log(F) + (a^2dfe^2g^2n^2x + a^2dfe^2g^2n^2) \log(F)) \log((F^{fgn^2x + egn} b + a)/a)) / (2F^{fgn^2x + egn} a^4 b^2 f^2 g^2 n^2 \log(F)^2 + F^{2fgn^2x + 2egn} a^3 b^2 f^2 g^2 n^2 \log(F)^2 + a^5 f^2 g^2 n^2 \log(F)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="fricas")

[Out] -1/2\*(3\*(a^2\*d\*e - a^2\*c\*f)\*g\*n\*log(F) + a^2\*d - (a^2\*d\*f^2\*g^2\*n^2\*x^2 + 2\*a^2\*c\*f^2\*g^2\*n^2\*x - (a^2\*d\*e^2 - 2\*a^2\*c\*e\*f)\*g^2\*n^2)\*log(F)^2 - ((b^2\*d\*f^2\*g^2\*n^2\*x^2 + 2\*b^2\*c\*f^2\*g^2\*n^2\*x - (b^2\*d\*e^2 - 2\*b^2\*c\*e\*f)\*g^2\*n^2)\*log(F)^2 - 3\*(b^2\*d\*f\*g\*n\*x + b^2\*d\*e\*g\*n)\*log(F))\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n) + (a\*b\*d - 2\*(a\*b\*d\*f^2\*g^2\*n^2\*x^2 + 2\*a\*b\*c\*f^2\*g^2\*n^2\*x - (a\*b\*d\*e^2 - 2\*a\*b\*c\*e\*f)\*g^2\*n^2)\*log(F)^2 + 2\*(2\*a\*b\*d\*f\*g\*n\*x + (3\*a\*b\*d\*e - a\*b\*c\*f)\*g\*n)\*log(F))\*F^(f\*g\*n\*x + e\*g\*n) + 2\*(2\*F^(f\*g\*n\*x + e\*g\*n)\*a\*b\*d + F^(2\*f\*g\*n\*x + 2\*e\*g\*n)\*b^2\*d + a^2\*d)\*dilog(-(F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a + 1) - (2\*(a^2\*d\*e - a^2\*c\*f)\*g\*n\*log(F) + 3\*a^2\*d + (2\*(b^2\*d\*e - b^2\*c\*f)\*g\*n\*log(F) + 3\*b^2\*d)\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n) + 2\*(2\*(a\*b\*d\*e - a\*b\*c\*f)\*g\*n\*log(F) + 3\*a\*b\*d)\*F^(f\*g\*n\*x + e\*g\*n))\*log(F^(f\*g\*n\*x + e\*g\*n)\*b + a) + 2\*((b^2\*d\*f\*g\*n\*x + b^2\*d\*e\*g\*n)\*F^(2\*f\*g\*n\*x + 2\*e\*g\*n)\*log(F) + 2\*(a\*b\*d\*f\*g\*n\*x + a\*b\*d\*e\*g\*n)\*F^(f\*g\*n\*x + e\*g\*n)\*log(F) + (a^2\*d\*f\*g\*n\*x + a^2\*d\*e\*g\*n)\*log(F))\*log((F^(f\*g\*n\*x + e\*g\*n)\*b + a)/a))/(2\*F^(f\*g\*n\*x + e\*g\*n)\*a^4\*b^2\*f^2\*g^2\*n^2\*log(F)^2 + F^(2\*f\*g\*n\*x + 2\*e\*g\*n)\*a^3\*b^2\*f^2\*g^2\*n^2\*log(F)^2 + a^5\*f^2\*g^2\*n^2\*log(F)^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left( \left( F^{(fx+e)g} \right)^n b + a \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)/((F^((f\*x + e)\*g))^n\*b + a)^3, x)

**maple [B]** time = 0.07, size = 668, normalized size = 2.42

$$\frac{dx \ln\left(\frac{Ffg^{nx}F-fg^{nx}\left(F^{(fx+e)g}\right)^n}{a^3fgn \ln(F)}\right)}{a^3fgn \ln(F)} - \frac{dx \ln\left(\frac{bFfg^{nx}F-fg^{nx}\left(F^{(fx+e)g}\right)^n + a}{a^3fgn \ln(F)}\right)}{a^3fgn \ln(F)} + \frac{c \ln\left(\frac{Ffg^{nx}F-fg^{nx}\left(F^{(fx+e)g}\right)^n}{a^3fgn \ln(F)}\right)}{a^3fgn \ln(F)} - \frac{c \ln\left(\frac{bFfg^{nx}F-fg^{nx}\left(F^{(fx+e)g}\right)^n + a}{a^3fgn \ln(F)}\right)}{a^3fgn \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(b\*(F^((f\*x+e)\*g))^n+a)^3,x)

[Out]  $\frac{1}{2} * (2 * \ln(F) * (F^{(f*x+e)*g})^n * b * d * f * g * n * x + 3 * \ln(F) * a * d * f * g * n * x + 2 * \ln(F) * (F^{(f*x+e)*g})^n * b * c * f * g * n + 3 * c * \ln(F) * a * f * g * n - (F^{(f*x+e)*g})^n * b * d - a * d) / n^2 / f^2 / g^2 / \ln(F)^2 / a^2 / (b * (F^{(f*x+e)*g})^n + a)^2 + 1 / a^3 / n / f / g / \ln(F) * c * \ln(F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n) - 1 / a^3 / n / f / g / \ln(F) * c * \ln(b * F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n + a) - 3 / 2 / a^3 / n^2 / f^2 / g^2 / \ln(F)^2 * d * \ln(F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n) + 3 / 2 / a^3 / n^2 / f^2 / g^2 / \ln(F)^2 * d * \ln(b * F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n + a) - 1 / a^3 / n^2 / f^2 / g^2 / \ln(F)^2 * d * \text{polylog}(2, -1 / a * b * F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n) - 1 / a^3 / n / f / g / \ln(F) * d * \ln(b * F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n + a) * x + 1 / a^3 / n / f^2 / g^2 / \ln(F)^2 * d * \ln(b * F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n + a) * \ln(F^{(f*x+e)*g}) - 1 / a^3 / n / f^2 / g^2 / \ln(F)^2 * d * \ln(1 / a * b * F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n + 1) * \ln(F^{(f*x+e)*g}) + 1 / a^3 / n / f / g / \ln(F) * d * \ln(F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n) * x - 1 / a^3 / n / f^2 / g^2 / \ln(F)^2 * d * \ln(F^{(f*g*n*x)} * F^{-f*g*n*x} * (F^{(f*x+e)*g})^n) * \ln(F^{(f*x+e)*g}) + 1 / 2 / a^3 / f^2 / g^2 / \ln(F)^2 * d * \ln(F^{(f*x+e)*g})^2$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} d \left( \frac{3 a f g n x \log(F) + (2 (F^{e g})^n b f g n x \log(F) - (F^{e g})^n b) (F^{f g x})^n - a}{2 (F^{f g x})^n (F^{e g})^n a^3 b f^2 g^2 n^2 \log(F)^2 + (F^{f g x})^{2 n} (F^{e g})^{2 n} a^2 b^2 f^2 g^2 n^2 \log(F)^2 + a^4 f^2 g^2 n^2 \log(F)^2} \right) + 2 \int \frac{1}{2 \left( (F^{f g x})^n (F^{e g})^n a^3 b f^2 g^2 n^2 \log(F)^2 + (F^{f g x})^{2 n} (F^{e g})^{2 n} a^2 b^2 f^2 g^2 n^2 \log(F)^2 + a^4 f^2 g^2 n^2 \log(F)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2} * d * ((3 * a * f * g * n * x * \log(F) + (2 * (F^{(e * g)})^n * b * f * g * n * x * \log(F) - (F^{(e * g)})^n * b) * (F^{(f * g * x)})^n - a) / (2 * (F^{(f * g * x)})^n * (F^{(e * g)})^n * a^3 * b * f^2 * g^2 * n^2 * \log(F)^2 + (F^{(f * g * x)})^{(2 * n)} * (F^{(e * g)})^{(2 * n)} * a^2 * b^2 * f^2 * g^2 * n^2 * \log(F)^2 + a^4 * f^2 * g^2 * n^2 * \log(F)^2) + 2 * \text{integrate}(1 / 2 * (2 * f * g * n * x * \log(F) - 3) / ((F^{(f * g * x)})^n * (F^{(e * g)})^n * a^2 * b * f * g * n * \log(F) + a^3 * f * g * n * \log(F)), x)) + 1 / 2 * c * ((2 * (F^{(f * g * x + e * g)})^n * b + 3 * a) / ((2 * (F^{(f * g * x + e * g)})^n * a^3 * b * n + (F^{(f * g * x + e * g)})^n))$

$(2n)a^{2b^{2n} + a^{4n}}f^g \log(F) + 2 \log(F^{(f^g x + e^g)}) / (a^{3f^g \log(F)} - 2 \log((F^{(f^g x + e^g)})^n b + a) / b) / (a^{3f^g n \log(F)})$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + dx}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n)^3, x)

[Out] int((c + d\*x)/(a + b\*(F^(g\*(e + f\*x))))^n)^3, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{3acfgn \log(F) + 3adfgnx \log(F) - ad + (2bcfgn \log(F) + 2bdfgnx \log(F) - bd) \left(F^{g(e+fx)}\right)^n}{2a^4 f^2 g^2 n^2 \log(F)^2 + 4a^3 b f^2 g^2 n^2 \left(F^{g(e+fx)}\right)^n \log(F)^2 + 2a^2 b^2 f^2 g^2 n^2 \left(F^{g(e+fx)}\right)^{2n} \log(F)^2} + \int \left(-\frac{3d}{a + b e^{g n \log(F) e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3, x)

[Out] (3\*a\*c\*f\*g\*n\*log(F) + 3\*a\*d\*f\*g\*n\*x\*log(F) - a\*d + (2\*b\*c\*f\*g\*n\*log(F) + 2\*b\*d\*f\*g\*n\*x\*log(F) - b\*d)\*(F\*\*(g\*(e + f\*x))))\*\*n)/(2\*a\*\*4\*f\*\*2\*g\*\*2\*n\*\*2\*log(F)\*\*2 + 4\*a\*\*3\*b\*f\*\*2\*g\*\*2\*n\*\*2\*(F\*\*(g\*(e + f\*x))))\*\*n\*log(F)\*\*2 + 2\*a\*\*2\*b\*\*2\*f\*\*2\*g\*\*2\*n\*\*2\*(F\*\*(g\*(e + f\*x))))\*\*2\*n\*log(F)\*\*2) + Integral(-3\*d/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(2\*c\*f\*g\*n\*log(F)/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x) + Integral(2\*d\*f\*g\*n\*x\*log(F)/(a + b\*exp(e\*g\*n\*log(F))\*exp(f\*g\*n\*x\*log(F))), x)/(2\*a\*\*2\*f\*g\*n\*log(F))



$$3.61 \quad \int \frac{1}{\left(a+b(F^{g(e+fx)})^n\right)^3} dx$$

**Optimal.** Leaf size=111

$$\frac{\log\left(a+b(F^{g(e+fx)})^n\right)}{a^3 f g n \log(F)} + \frac{x}{a^3} + \frac{1}{a^2 f g n \log(F) \left(a+b(F^{g(e+fx)})^n\right)} + \frac{1}{2 a f g n \log(F) \left(a+b(F^{g(e+fx)})^n\right)^2}$$

[Out]  $x/a^3 + 1/2/a/f/(a+b*(F^{(g*(f*x+e))})^n)^2/g/n/\ln(F) + 1/a^2/f/(a+b*(F^{(g*(f*x+e))})^n)/g/n/\ln(F) - \ln(a+b*(F^{(g*(f*x+e))})^n)/a^3/f/g/n/\ln(F)$

**Rubi [A]** time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2282, 266, 44}

$$\frac{\log\left(a+b(F^{g(e+fx)})^n\right)}{a^3 f g n \log(F)} + \frac{1}{a^2 f g n \log(F) \left(a+b(F^{g(e+fx)})^n\right)} + \frac{x}{a^3} + \frac{1}{2 a f g n \log(F) \left(a+b(F^{g(e+fx)})^n\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^(-3), x]

[Out]  $x/a^3 + 1/(2*a*f*(a + b*(F^{(g*(e + f*x))})^n)^2*g*n*Log[F]) + 1/(a^2*f*(a + b*(F^{(g*(e + f*x))})^n)*g*n*Log[F]) - Log[a + b*(F^{(g*(e + f*x))})^n]/(a^3*f*g*n*Log[F])$

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3} dx, x, F^{g(e+fx)}\right)}{fg \log(F)} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3} dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn \log(F)} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right) dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn \log(F)} \\ &= \frac{x}{a^3} + \frac{1}{2af \left(a + b \left(F^{g(e+fx)}\right)^n\right)^2 gn \log(F)} + \frac{1}{a^2 f \left(a + b \left(F^{g(e+fx)}\right)^n\right) gn \log(F)} - \frac{\log(a)}{a^3} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 84, normalized size = 0.76

$$\frac{\frac{a(3a+2b(F^{g(e+fx)})^n)}{(a+b(F^{g(e+fx)})^n)^2} - 2 \log\left(a + b \left(F^{g(e+fx)}\right)^n\right) + 2fgnx \log(F)}{2a^3 fgn \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^(-3), x]

[Out] ((a\*(3\*a + 2\*b\*(F^(g\*(e + f\*x)))^n))/(a + b\*(F^(g\*(e + f\*x)))^n)^2 + 2\*f\*g\*n\*x\*Log[F] - 2\*Log[a + b\*(F^(g\*(e + f\*x)))^n])/(2\*a^3\*f\*g\*n\*Log[F])

**fricas [A]** time = 0.42, size = 190, normalized size = 1.71

$$\frac{2F^2fgnx+2egn b^2fgnx \log(F) + 2a^2fgnx \log(F) + 2(2abfgnx \log(F) + ab)Ffgnx+egn + 3a^2 - 2(2Ffgnx+egn ab + F}{2(2Ffgnx+egn a^4bfgn \log(F) + F^2fgnx+2egn a^3b^2fgn \log(F) + a^5fgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="fricas")

[Out]  $\frac{1}{2} \cdot (2 \cdot F^{(2 \cdot f \cdot g \cdot n \cdot x + 2 \cdot e \cdot g \cdot n)} \cdot b^{2 \cdot f \cdot g \cdot n \cdot x} \cdot \log(F) + 2 \cdot a^{2 \cdot f \cdot g \cdot n \cdot x} \cdot \log(F) + 2 \cdot (2 \cdot a \cdot b \cdot f \cdot g \cdot n \cdot x \cdot \log(F) + a \cdot b) \cdot F^{(f \cdot g \cdot n \cdot x + e \cdot g \cdot n)} + 3 \cdot a^2 - 2 \cdot (2 \cdot F^{(f \cdot g \cdot n \cdot x + e \cdot g \cdot n)} \cdot a \cdot b + F^{(2 \cdot f \cdot g \cdot n \cdot x + 2 \cdot e \cdot g \cdot n)} \cdot b^2 + a^2) \cdot \log(F^{(f \cdot g \cdot n \cdot x + e \cdot g \cdot n)} \cdot b + a)) / (2 \cdot F^{(f \cdot g \cdot n \cdot x + e \cdot g \cdot n)} \cdot a^4 \cdot b \cdot f \cdot g \cdot n \cdot \log(F) + F^{(2 \cdot f \cdot g \cdot n \cdot x + 2 \cdot e \cdot g \cdot n)} \cdot a^3 \cdot b^2 \cdot f \cdot g \cdot n \cdot \log(F) + a^5 \cdot f \cdot g \cdot n \cdot \log(F))$

**giac** [A] time = 0.38, size = 136, normalized size = 1.23

$$\frac{\log(|F|^{fgnx}|F|^{gne})}{a^3 fgn \log(F)} - \frac{\log(|F^{fgnx} F^{gne} b + a|)}{a^3 fgn \log(F)} + \frac{2 F^{fgnx} F^{gne} a b + 3 a^2}{2 (F^{fgnx} F^{gne} b + a)^2 a^3 fgn \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="giac")

[Out]  $\log(\text{abs}(F)^{(f \cdot g \cdot n \cdot x)} \cdot \text{abs}(F)^{(g \cdot n \cdot e)}) / (a^3 \cdot f \cdot g \cdot n \cdot \log(F)) - \log(\text{abs}(F^{(f \cdot g \cdot n \cdot x)} \cdot F^{(g \cdot n \cdot e)} \cdot b + a)) / (a^3 \cdot f \cdot g \cdot n \cdot \log(F)) + 1/2 \cdot (2 \cdot F^{(f \cdot g \cdot n \cdot x)} \cdot F^{(g \cdot n \cdot e)} \cdot a \cdot b + 3 \cdot a^2) / ((F^{(f \cdot g \cdot n \cdot x)} \cdot F^{(g \cdot n \cdot e)} \cdot b + a)^2 \cdot a^3 \cdot f \cdot g \cdot n \cdot \log(F))$

**maple** [A] time = 0.02, size = 134, normalized size = 1.21

$$\frac{1}{2 \left( b \left( F^{(fx+e)g} \right)^n + a \right)^2 a fgn \ln(F)} + \frac{1}{\left( b \left( F^{(fx+e)g} \right)^n + a \right) a^2 fgn \ln(F)} + \frac{\ln \left( \left( F^{(fx+e)g} \right)^n \right)}{a^3 fgn \ln(F)} - \frac{\ln \left( b \left( F^{(fx+e)g} \right)^n + a \right)}{a^3 fgn \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*(F^((f\*x+e)\*g)))^n+a)^3,x)

[Out]  $\frac{1}{g/f/\ln(F)/n/a^3 \ln((F^((f*x+e)*g)))^n} - \frac{\ln(b*(F^((f*x+e)*g)))^n+a}{a^3/f/g/n/\ln(F)+1/a^2/f/(b*(F^((f*x+e)*g)))^n+a}/g/n/\ln(F) + 1/2/a/f/(b*(F^((f*x+e)*g)))^n+a)^2/g/n/\ln(F)$

**maxima** [A] time = 1.22, size = 145, normalized size = 1.31

$$\frac{2 (F^{gx+eg})^n b + 3 a}{2 \left( 2 (F^{gx+eg})^n a^3 b n + (F^{gx+eg})^{2n} a^2 b^2 n + a^4 n \right) f g \log(F)} + \frac{\log(F^{gx+eg})}{a^3 f g \log(F)} - \frac{\log \left( \frac{(F^{gx+eg})^{n+b+a}}{b} \right)}{a^3 f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2} \cdot \frac{2 \cdot (F^{(f \cdot g \cdot x + e \cdot g)})^n \cdot b + 3 \cdot a}{((2 \cdot (F^{(f \cdot g \cdot x + e \cdot g)})^n \cdot a^{3 \cdot b \cdot n} + (F^{(f \cdot g \cdot x + e \cdot g)})^{(2 \cdot n)} \cdot a^{2 \cdot b \cdot 2 \cdot n} + a^{4 \cdot n}) \cdot f \cdot g \cdot \log(F)) + \log(F^{(f \cdot g \cdot x + e \cdot g)}) / (a^{3 \cdot f \cdot g \cdot \log(F)}) - \log(((F^{(f \cdot g \cdot x + e \cdot g)})^n \cdot b + a) / b) / (a^{3 \cdot f \cdot g \cdot n \cdot \log(F)})}$

mupad [B] time = 3.50, size = 142, normalized size = 1.28

$$\frac{x}{a^3} + \frac{1}{2 a f g n \ln(F) \left( a^2 + b^2 \left( F^{f g x} F^{e g} \right)^{2 n} + 2 a b \left( F^{f g x} F^{e g} \right)^n \right)} + \frac{1}{a^2 f g n \ln(F) \left( a + b \left( F^{f g x} F^{e g} \right)^n \right)} - \frac{\ln \left( a + b \left( F^{f g x} F^{e g} \right)^n \right)}{a^3 f g n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*(F^(g\*(e + f\*x))))^n)^3,x)

[Out]  $\frac{x}{a^3} + \frac{1}{(2 \cdot a \cdot f \cdot g \cdot n \cdot \log(F) \cdot (a^2 + b^2 \cdot (F^{(f \cdot g \cdot x)} \cdot F^{(e \cdot g)})^{(2 \cdot n)} + 2 \cdot a \cdot b \cdot (F^{(f \cdot g \cdot x)} \cdot F^{(e \cdot g)})^n)) + 1 / (a^2 \cdot f \cdot g \cdot n \cdot \log(F) \cdot (a + b \cdot (F^{(f \cdot g \cdot x)} \cdot F^{(e \cdot g)})^n)) - \log(a + b \cdot (F^{(f \cdot g \cdot x)} \cdot F^{(e \cdot g)})^n) / (a^3 \cdot f \cdot g \cdot n \cdot \log(F))}$

sympy [A] time = 0.24, size = 116, normalized size = 1.05

$$\frac{3a + 2b \left( F^{g(e+fx)} \right)^n}{2a^4 f g n \log(F) + 4a^3 b f g n \left( F^{g(e+fx)} \right)^n \log(F) + 2a^2 b^2 f g n \left( F^{g(e+fx)} \right)^{2n} \log(F)} + \frac{x}{a^3} - \frac{\log \left( \frac{a}{b} + \left( F^{g(e+fx)} \right)^n \right)}{a^3 f g n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3,x)

[Out]  $\frac{(3 \cdot a + 2 \cdot b \cdot (F^{(g \cdot (e + f \cdot x))})^n) \cdot n}{(2 \cdot a^{**4} \cdot f \cdot g \cdot n \cdot \log(F) + 4 \cdot a^{**3} \cdot b \cdot f \cdot g \cdot n \cdot (F^{(g \cdot (e + f \cdot x))})^n \cdot \log(F) + 2 \cdot a^{**2} \cdot b^{**2} \cdot f \cdot g \cdot n \cdot (F^{(g \cdot (e + f \cdot x))})^{(2 \cdot n)} \cdot \log(F)) + x / a^{**3} - \log(a / b + (F^{(g \cdot (e + f \cdot x))})^n) / (a^{**3} \cdot f \cdot g \cdot n \cdot \log(F))}$

$$3.62 \quad \int \frac{1}{\left(a+b(F^{g(e+fx)})^n\right)^3 (c+dx)} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{1}{(c+dx) \left(a+b(F^{eg+fgx})^n\right)^3}, x \right)$$

[Out] Unintegrable(1/(a+b\*(F^(f\*g\*x+e\*g))^n)^3/(d\*x+c), x)

**Rubi** [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{\left(a+b(F^{g(e+fx)})^n\right)^3 (c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)), x]

[Out] Defer[Int][1/((a + b\*(F^(e\*g + f\*g\*x))))^n)^3\*(c + d\*x)), x]

Rubi steps

$$\int \frac{1}{\left(a+b(F^{g(e+fx)})^n\right)^3 (c+dx)} dx = \int \frac{1}{\left(a+b(F^{eg+fgx})^n\right)^3 (c+dx)} dx$$

**Mathematica** [A] time = 1.87, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b(F^{g(e+fx)})^n\right)^3 (c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)), x]

[Out] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)), x]

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{a^3 dx + a^3 c + (b^3 dx + b^3 c)(F^{fgx+eg})^{3n} + 3(ab^2 dx + ab^2 c)(F^{fgx+eg})^{2n} + 3(a^2 b dx + a^2 bc)(F^{fgx+eg})^n}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c),x, algorithm="fricas")

[Out] integral(1/(a^3\*d\*x + a^3\*c + (b^3\*d\*x + b^3\*c)\*(F^(f\*g\*x + e\*g))^(3\*n) + 3\*(a\*b^2\*d\*x + a\*b^2\*c)\*(F^(f\*g\*x + e\*g))^(2\*n) + 3\*(a^2\*b\*d\*x + a^2\*b\*c)\*(F^(f\*g\*x + e\*g))^n), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( \left( F^{(f x + e) g} \right)^n b + a \right)^3 (dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c),x, algorithm="giac")

[Out] integrate(1/(((F^((f\*x + e)\*g))^n\*b + a)^3\*(d\*x + c)), x)

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( b \left( F^{(f x + e) g} \right)^n + a \right)^3 (dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*(F^((f\*x+e)\*g))^n+a)^3/(d\*x+c),x)

[Out] int(1/(b\*(F^((f\*x+e)\*g))^n+a)^3/(d\*x+c),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3 a d f g n x \log(F) + 3 a c}{2 \left( a^4 d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 2 a^4 c d f^2 g^2 n^2 x \log(F)^2 + a^4 c^2 f^2 g^2 n^2 \log(F)^2 + \left( F^{e g} \right)^{2 n} a^2 b^2 d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 2 \left( F^{e g} \right)^{2 n} a^2 b^2 d^2 f^2 g^2 n^2 x \log(F)^2 + \left( F^{e g} \right)^{2 n} a^2 b^2 d^2 f^2 g^2 n^2 \log(F)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c),x, algorithm="maxima")

[Out]  $\frac{1}{2} \cdot (3adfgn \log(F) + 3acfgn \log(F) + (2(F^{eg}))^n bdfgn \log(F) + 2(F^{eg})^n bcd(F^{fgx})^n + ad) / (a^4 d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 2a^4 c d f^2 g^2 n^2 x \log(F)^2 + a^4 c^2 f^2 g^2 n^2 \log(F)^2 + ((F^{eg})^{2n}) a^2 b^2 d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 2(F^{eg})^{2n} a^2 b^2 c d f^2 g^2 n^2 x \log(F)^2 + (F^{eg})^{2n} a^2 b^2 c^2 f^2 g^2 n^2 \log(F)^2) \cdot (F^{fgx})^{2n} + 2((F^{eg})^n a^3 b d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 2(F^{eg})^n a^3 b c d f^2 g^2 n^2 x \log(F)^2 + (F^{eg})^n a^3 b c^2 f^2 g^2 n^2 \log(F)^2) \cdot (F^{fgx})^n + \int \frac{1}{2} (2d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 2c^2 f^2 g^2 n^2 \log(F)^2 + 3c d f g n \log(F) + 2d^2 + (4c d f^2 g^2 n^2 \log(F)^2 + 3d^2 f g n \log(F)) x) / (a^3 d^3 f^2 g^2 n^2 x^3 \log(F)^2 + 3a^3 c d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 3a^3 c^2 d f^2 g^2 n^2 x \log(F)^2 + a^3 c^3 f^2 g^2 n^2 \log(F)^2 + ((F^{eg})^n a^2 b d^3 f^2 g^2 n^2 x^3 \log(F)^2 + 3(F^{eg})^n a^2 b c d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 3(F^{eg})^n a^2 b c^2 d f^2 g^2 n^2 x \log(F)^2 + (F^{eg})^n a^2 b c^3 f^2 g^2 n^2 \log(F)^2) \cdot (F^{fgx})^n, x$

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3 (c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x),x)

[Out] int(1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3/(d\*x+c),x)

[Out] Timed out

$$3.63 \quad \int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)^3 (c+dx)^2} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{1}{(c+dx)^2 \left(a+b(F^{eg+fgx})^n\right)^3}, x \right)$$

[Out] Unintegrable(1/(a+b\*(F^(f\*g\*x+e\*g))^n)^3/(d\*x+c)^2,x)

**Rubi [A]** time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)^3 (c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)^2], x]

[Out] Defer[Int][1/((a + b\*(F^(e\*g + f\*g\*x))))^n)^3\*(c + d\*x)^2), x]

Rubi steps

$$\int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)^3 (c+dx)^2} dx = \int \frac{1}{\left(a+b(F^{eg+fgx})^n\right)^3 (c+dx)^2} dx$$

**Mathematica [A]** time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b(Fg^{(e+fx)})^n\right)^3 (c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)^2], x]

[Out] Integrate[1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)^2), x]



**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{a^3 d^2 x^2 + 2 a^3 c d x + a^3 c^2 + (b^3 d^2 x^2 + 2 b^3 c d x + b^3 c^2) (F^{f g x + e g})^{3n} + 3 (a b^2 d^2 x^2 + 2 a b^2 c d x + a b^2 c^2) (F^{f g x + e g})^{2n} + 3 (a^2 b d^2 x^2 + 2 a^2 b c d x + a^2 b c^2) (F^{f g x + e g})^n}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e)))^n)^3/(d\*x+c)^2,x, algorithm="fricas")

[Out] integral(1/(a^3\*d^2\*x^2 + 2\*a^3\*c\*d\*x + a^3\*c^2 + (b^3\*d^2\*x^2 + 2\*b^3\*c\*d\*x + b^3\*c^2)\*(F^(f\*g\*x + e\*g))^(3\*n) + 3\*(a\*b^2\*d^2\*x^2 + 2\*a\*b^2\*c\*d\*x + a\*b^2\*c^2)\*(F^(f\*g\*x + e\*g))^(2\*n) + 3\*(a^2\*b\*d^2\*x^2 + 2\*a^2\*b\*c\*d\*x + a^2\*b\*c^2)\*(F^(f\*g\*x + e\*g))^n), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( \left( F^{(f x + e) g} \right)^n b + a \right)^3 (d x + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e)))^n)^3/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(1/(((F^((f\*x + e)\*g))^n\*b + a)^3\*(d\*x + c)^2), x)

**maple** [A] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\left( b \left( F^{(f x + e) g} \right)^n + a \right)^3 (d x + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*(F^((f\*x+e)\*g))^n+a)^3/(d\*x+c)^2,x)

[Out] int(1/(b\*(F^((f\*x+e)\*g))^n+a)^3/(d\*x+c)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

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$$2 \left( a^4 d^3 f^2 g^2 n^2 x^3 \log(F)^2 + 3 a^4 c d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 3 a^4 c^2 d f^2 g^2 n^2 x \log(F)^2 + a^4 c^3 f^2 g^2 n^2 \log(F)^2 + (F^{e g})^{2n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F^(g\*(f\*x+e))))^n)^3/(d\*x+c)^2,x, algorithm="maxima")

[Out]  $\frac{1}{2} \cdot (3adfgn \log(F) + 3acfgn \log(F) + 2((F^{eg})^n bdfg \log(F) + (F^{eg})^n bcd)(F^{fgx})^n + 2ad) / (a^4 d^3 f^2 g^2 n^2 x^3 \log(F)^2 + 3a^4 c d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 3a^4 c^2 d f^2 g^2 n^2 x \log(F)^2 + a^4 c^3 f^2 g^2 n^2 \log(F)^2 + (F^{eg})^{2n} a^2 b^2 d^3 f^2 g^2 n^2 x^3 \log(F)^2 + 3(F^{eg})^{2n} a^2 b^2 c d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 3(F^{eg})^{2n} a^2 b^2 c^2 d f^2 g^2 n^2 x \log(F)^2 + (F^{eg})^{2n} a^2 b^2 c^3 f^2 g^2 n^2 \log(F)^2) \cdot (F^{fgx})^{2n} + 2((F^{eg})^n a^3 b d^3 f^2 g^2 n^2 x^3 \log(F)^2 + 3(F^{eg})^n a^3 b c d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 3(F^{eg})^n a^3 b c^2 d f^2 g^2 n^2 x \log(F)^2 + (F^{eg})^n a^3 b c^3 f^2 g^2 n^2 \log(F)^2) \cdot (F^{fgx})^n + \int (d^2 f^2 g^2 n^2 x^2 \log(F)^2 + c^2 f^2 g^2 n^2 \log(F)^2 + 3c d f g n \log(F) + 3d^2 + (2c d f^2 g^2 n^2 \log(F)^2 + 3d^2 f g n \log(F))) x / (a^3 d^4 f^2 g^2 n^2 x^4 \log(F)^2 + 4a^3 c d^3 f^2 g^2 n^2 x^3 \log(F)^2 + 6a^3 c^2 d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 4a^3 c^3 d f^2 g^2 n^2 x \log(F)^2 + a^3 c^4 f^2 g^2 n^2 \log(F)^2 + ((F^{eg})^n a^2 b d^4 f^2 g^2 n^2 x^4 \log(F)^2 + 4(F^{eg})^n a^2 b c d^3 f^2 g^2 n^2 x^3 \log(F)^2 + 6(F^{eg})^n a^2 b c^2 d^2 f^2 g^2 n^2 x^2 \log(F)^2 + 4(F^{eg})^n a^2 b c^3 d f^2 g^2 n^2 x \log(F)^2 + (F^{eg})^n a^2 b c^4 f^2 g^2 n^2 \log(F)^2) \cdot (F^{fgx})^n, x)$

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^3 (c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)^2), x)

[Out] int(1/((a + b\*(F^(g\*(e + f\*x))))^n)^3\*(c + d\*x)^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3/(d\*x+c)\*\*2,x)

[Out] Timed out

### 3.64 $\int (a + be^x) \sqrt{c + dx} dx$

Optimal. Leaf size=71

$$\frac{2a(c + dx)^{3/2}}{3d} - \frac{1}{2}\sqrt{\pi}b\sqrt{d}e^{-\frac{c}{d}}\operatorname{erfi}\left(\frac{\sqrt{c + dx}}{\sqrt{d}}\right) + be^x\sqrt{c + dx}$$

[Out]  $2/3*a*(d*x+c)^{(3/2)}/d-1/2*b*\operatorname{erfi}((d*x+c)^{(1/2)}/d^{(1/2)})*d^{(1/2)}*\operatorname{Pi}^{(1/2)}/\exp(c/d)+b*\exp(x)*(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {2183, 2176, 2180, 2204}

$$\frac{2a(c + dx)^{3/2}}{3d} - \frac{1}{2}\sqrt{\pi}b\sqrt{d}e^{-\frac{c}{d}}\operatorname{Erfi}\left(\frac{\sqrt{c + dx}}{\sqrt{d}}\right) + be^x\sqrt{c + dx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^x)\*Sqrt[c + d\*x], x]

[Out]  $b*E^x*\operatorname{Sqrt}[c + d*x] + (2*a*(c + d*x)^{(3/2)})/(3*d) - (b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[c + d*x]/\operatorname{Sqrt}[d]])/(2*E^{(c/d)})$

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2183

Int[((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rubi steps

$$\begin{aligned}
 \int (a + be^x) \sqrt{c + dx} \, dx &= \int \left( a\sqrt{c + dx} + be^x \sqrt{c + dx} \right) dx \\
 &= \frac{2a(c + dx)^{3/2}}{3d} + b \int e^x \sqrt{c + dx} \, dx \\
 &= be^x \sqrt{c + dx} + \frac{2a(c + dx)^{3/2}}{3d} - \frac{1}{2}(bd) \int \frac{e^x}{\sqrt{c + dx}} \, dx \\
 &= be^x \sqrt{c + dx} + \frac{2a(c + dx)^{3/2}}{3d} - b \operatorname{Subst} \left( \int e^{-\frac{c}{d} + \frac{x^2}{d}} \, dx, x, \sqrt{c + dx} \right) \\
 &= be^x \sqrt{c + dx} + \frac{2a(c + dx)^{3/2}}{3d} - \frac{1}{2} b \sqrt{d} e^{-\frac{c}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{c + dx}}{\sqrt{d}} \right)
 \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 71, normalized size = 1.00

$$\frac{2a(c + dx)^{3/2}}{3d} - \frac{1}{2} \sqrt{\pi} b \sqrt{d} e^{-\frac{c}{d}} \operatorname{erfi} \left( \frac{\sqrt{c + dx}}{\sqrt{d}} \right) + be^x \sqrt{c + dx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^x)\*Sqrt[c + d\*x], x]

[Out] b\*E^x\*Sqrt[c + d\*x] + (2\*a\*(c + d\*x)^(3/2))/(3\*d) - (b\*Sqrt[d]\*Sqrt[Pi]\*Erfi[Sqrt[c + d\*x]/Sqrt[d]])/(2\*E^(c/d))

**fricas** [A] time = 0.43, size = 70, normalized size = 0.99

$$\frac{3 \sqrt{\pi} b d^2 \sqrt{-\frac{1}{d}} \operatorname{erf} \left( \sqrt{dx + c} \sqrt{-\frac{1}{d}} \right) e^{\left(-\frac{c}{d}\right)} + 2(2 adx + 3 bde^x + 2 ac) \sqrt{dx + c}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))\*(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/6\*(3\*sqrt(pi)\*b\*d^2\*sqrt(-1/d)\*erf(sqrt(d\*x + c)\*sqrt(-1/d))\*e^(-c/d) + 2\*(2\*a\*d\*x + 3\*b\*d\*e^x + 2\*a\*c)\*sqrt(d\*x + c))/d

**giac** [B] time = 0.34, size = 132, normalized size = 1.86

$$\frac{6\sqrt{\pi}bcd \operatorname{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)} - 12\sqrt{dx+c}ac - 4\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}c\right)a - 3\left(\frac{\sqrt{\pi}(2c+d)d \operatorname{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}} + 2\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out]  $-1/6*(6*\sqrt{\pi}*b*c*d*\operatorname{erf}(-\sqrt{d*x+c}*\sqrt{-d}/d)*e^{(-c/d)}/\sqrt{-d} - 12*\sqrt{d*x+c}*a*c - 4*((d*x+c)^{3/2} - 3*\sqrt{d*x+c}*c)*a - 3*(\sqrt{\pi}*(2*c+d)*d*\operatorname{erf}(-\sqrt{d*x+c}*\sqrt{-d}/d)*e^{(-c/d)}/\sqrt{-d} + 2*\sqrt{d*x+c}*d*e^x)*b)/d$

**maple** [A] time = 0.02, size = 77, normalized size = 1.08

$$\frac{2\left(-\frac{\sqrt{\pi}d \operatorname{erf}\left(\sqrt{-\frac{1}{d}}\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}} + \frac{\sqrt{dx+c}de^{\frac{dx+c}{d}}}{2}\right)be^{-\frac{c}{d}} + \frac{2(dx+c)^{\frac{3}{2}}a}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*exp(x))\*(d\*x+c)^(1/2),x)

[Out]  $2/d*(1/3*a*(d*x+c)^{3/2}+b/\exp(c/d)*(1/2*(d*x+c)^{1/2}*\exp((d*x+c)/d)*d-1/4*d*\operatorname{Pi}^{1/2}/(-1/d)^{1/2}*\operatorname{erf}((-1/d)^{1/2}*(d*x+c)^{1/2}))$

**maxima** [A] time = 1.17, size = 82, normalized size = 1.15

$$\frac{4(dx+c)^{\frac{3}{2}}a - 3\left(\frac{\sqrt{\pi}d \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 2\sqrt{dx+c}de^{\left(\frac{dx+c}{d}-\frac{c}{d}\right)}\right)b}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))\*(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $1/6*(4*(d*x+c)^{3/2}*a - 3*(\sqrt{\pi}*d*\operatorname{erf}(\sqrt{d*x+c}*\sqrt{-1/d}))*e^{(-c/d)}/\sqrt{-1/d} - 2*\sqrt{d*x+c}*d*e^{((d*x+c)/d - c/d)})*b)/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b e^x) \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*exp(x))*(c + d*x)^(1/2), x)`

[Out] `int((a + b*exp(x))*(c + d*x)^(1/2), x)`

sympy [A] time = 2.14, size = 85, normalized size = 1.20

$$\frac{2a(c + dx)^{\frac{3}{2}}}{3d} - \frac{\sqrt{\pi} b \sqrt{d} e^{-\frac{c}{d}} \operatorname{erfi}\left(\frac{\sqrt{c+dx}}{d\sqrt{\frac{1}{d}}}\right)}{2} + \frac{b\sqrt{c+dx} e^{-\frac{c}{d}} e^{\frac{c}{d}+x}}{\sqrt{d} \sqrt{\frac{1}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*exp(x))*(d*x+c)**(1/2), x)`

[Out] `2*a*(c + d*x)**(3/2)/(3*d) - sqrt(pi)*b*sqrt(d)*exp(-c/d)*erfi(sqrt(c + d*x)/(d*sqrt(1/d)))/2 + b*sqrt(c + d*x)*exp(-c/d)*exp(c/d + x)/(sqrt(d)*sqrt(1/d))`

### 3.65 $\int (a + be^x)^2 \sqrt{c + dx} dx$

**Optimal.** Leaf size=145

$$\frac{2a^2(c + dx)^{3/2}}{3d} - \sqrt{\pi} ab\sqrt{d} e^{-\frac{c}{d}} \operatorname{erfi}\left(\frac{\sqrt{c + dx}}{\sqrt{d}}\right) + 2abe^x \sqrt{c + dx} - \frac{1}{4} \sqrt{\frac{\pi}{2}} b^2 \sqrt{d} e^{-\frac{2c}{d}} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{c + dx}}{\sqrt{d}}\right) + \frac{1}{2} b^2 e^{2x} \sqrt{c + dx}$$

[Out]  $\frac{2}{3} a^2 (d x + c)^{3/2} / d - 1/8 b^2 \operatorname{erfi}(2^{1/2} (d x + c)^{1/2} / d^{1/2}) d^{1/2} 2^{1/2} \pi^{1/2} / \exp(2c/d) - a b \operatorname{erfi}((d x + c)^{1/2} / d^{1/2}) d^{1/2} \pi^{1/2} / \exp(c/d) + 2 a b \exp(x) (d x + c)^{1/2} + 1/2 b^2 \exp(2x) (d x + c)^{1/2}$

**Rubi [A]** time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {2183, 2176, 2180, 2204}

$$\frac{2a^2(c + dx)^{3/2}}{3d} - \sqrt{\pi} ab\sqrt{d} e^{-\frac{c}{d}} \operatorname{Erfi}\left(\frac{\sqrt{c + dx}}{\sqrt{d}}\right) + 2abe^x \sqrt{c + dx} - \frac{1}{4} \sqrt{\frac{\pi}{2}} b^2 \sqrt{d} e^{-\frac{2c}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{c + dx}}{\sqrt{d}}\right) + \frac{1}{2} b^2 e^{2x} \sqrt{c + dx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*E^x)^2\*Sqrt[c + d\*x], x]

[Out]  $2 a b E^x \sqrt{c + d x} + (b^2 E^{2 x} \sqrt{c + d x}) / 2 + (2 a^2 (c + d x)^{3/2}) / (3 d) - (a b \sqrt{d} \sqrt{\pi} \operatorname{Erfi}[\sqrt{c + d x} / \sqrt{d}]) / E^{c/d} - (b^2 \sqrt{d} \sqrt{\pi/2} \operatorname{Erfi}[(\sqrt{2} \sqrt{c + d x}) / \sqrt{d}]) / (4 E^{(2 c / d)})$

#### Rule 2176

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*(b\*F^(g\*(e + f\*x)))^n)/(f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !\$UseGamma == True

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2183

Int[((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F

$\int (g*(e + f*x))^n dx, x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, m, n\}, x] \&\& \text{IGtQ}[p, 0]$

### Rule 2204

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2), x\_Symbol] :> \text{Simp}[(F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(c + d*x)*\text{Rt}[b*\text{Log}[F], 2]])/(2*d*\text{Rt}[b*\text{Log}[F], 2]), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

### Rubi steps

$$\begin{aligned} \int (a + be^x)^2 \sqrt{c + dx} dx &= \int (a^2\sqrt{c + dx} + 2abe^x\sqrt{c + dx} + b^2e^{2x}\sqrt{c + dx}) dx \\ &= \frac{2a^2(c + dx)^{3/2}}{3d} + (2ab) \int e^x\sqrt{c + dx} dx + b^2 \int e^{2x}\sqrt{c + dx} dx \\ &= 2abe^x\sqrt{c + dx} + \frac{1}{2}b^2e^{2x}\sqrt{c + dx} + \frac{2a^2(c + dx)^{3/2}}{3d} - (abd) \int \frac{e^x}{\sqrt{c + dx}} dx - \frac{1}{4}(b^2d) \int \frac{e^{2x}}{\sqrt{c + dx}} dx \\ &= 2abe^x\sqrt{c + dx} + \frac{1}{2}b^2e^{2x}\sqrt{c + dx} + \frac{2a^2(c + dx)^{3/2}}{3d} - (2ab) \text{Subst}\left(\int e^{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx}\right) \\ &= 2abe^x\sqrt{c + dx} + \frac{1}{2}b^2e^{2x}\sqrt{c + dx} + \frac{2a^2(c + dx)^{3/2}}{3d} - ab\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi} \text{erfi}\left(\frac{\sqrt{c + dx}}{\sqrt{d}}\right) - \frac{1}{4}b^2d \int \frac{e^{2x}}{\sqrt{c + dx}} dx \end{aligned}$$

**Mathematica** [A] time = 0.32, size = 134, normalized size = 0.92

$$\frac{4\sqrt{c + dx} (4a^2(c + dx) + 12abde^x + 3b^2de^{2x}) - 3\sqrt{2\pi} b^2d^{3/2}e^{-\frac{2c}{d}} \text{erfi}\left(\frac{\sqrt{2}\sqrt{c + dx}}{\sqrt{d}}\right) - \sqrt{\pi} ab\sqrt{d}e^{-\frac{c}{d}} \text{erfi}\left(\frac{\sqrt{c + dx}}{\sqrt{d}}\right)}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*E^x)^2\*Sqrt[c + d\*x], x]

[Out] -((a\*b\*Sqrt[d]\*Sqrt[Pi]\*Erfi[Sqrt[c + d\*x]/Sqrt[d]])/E^(c/d)) + (4\*Sqrt[c + d\*x]\*(12\*a\*b\*d\*E^x + 3\*b^2\*d\*E^(2\*x) + 4\*a^2\*(c + d\*x)) - (3\*b^2\*d^(3/2)\*Sqrt[2\*Pi]\*Erfi[(Sqrt[2]\*Sqrt[c + d\*x])/Sqrt[d]])/E^((2\*c)/d))/(24\*d)

**fricas** [A] time = 0.42, size = 133, normalized size = 0.92

$$\frac{3\sqrt{2}\sqrt{\pi}b^2d^2\sqrt{-\frac{1}{d}} \text{erf}\left(\sqrt{2}\sqrt{dx + c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{2c}{d}\right)} + 24\sqrt{\pi}abd^2\sqrt{-\frac{1}{d}} \text{erf}\left(\sqrt{dx + c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)} + 4(4a^2dx + 3b^2d)\sqrt{c + dx}}{24d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))^2\*(d\*x+c)^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{24}*(3*\sqrt{2}*\sqrt{\pi})*b^2*d^2*\sqrt{-1/d}*\operatorname{erf}(\sqrt{2}*\sqrt{d*x+c})*\sqrt{-1/d})*e^{(-2*c/d)} + 24*\sqrt{\pi}*a*b*d^2*\sqrt{-1/d}*\operatorname{erf}(\sqrt{d*x+c})*\sqrt{-1/d})*e^{(-c/d)} + 4*(4*a^2*d*x + 3*b^2*d*e^{(2*x)} + 12*a*b*d*e^x + 4*a^2*c)*\sqrt{d*x+c})/d$

**giac** [B] time = 0.42, size = 250, normalized size = 1.72

$$\frac{12\sqrt{2}\sqrt{\pi}b^2cd\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{2c}{d}\right)}}{\sqrt{-d}} + \frac{48\sqrt{\pi}abcd\operatorname{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}} - 48\sqrt{dx+c}a^2c - 16\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))^2\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out]  $-1/24*(12*\sqrt{2}*\sqrt{\pi})*b^2*c*d*\operatorname{erf}(-\sqrt{2}*\sqrt{d*x+c})*\sqrt{-d}/d)*e^{(-2*c/d)/\sqrt{-d}} + 48*\sqrt{\pi}*a*b*c*d*\operatorname{erf}(-\sqrt{d*x+c})*\sqrt{-d}/d)*e^{(-c/d)/\sqrt{-d}} - 48*\sqrt{d*x+c}*a^2*c - 16*((d*x+c)^{(3/2)} - 3*\sqrt{d*x+c}*c)*a^2 - 24*(\sqrt{\pi}*(2*c+d)*d*\operatorname{erf}(-\sqrt{d*x+c})*\sqrt{-d}/d)*e^{(-c/d)/\sqrt{-d}} + 2*\sqrt{d*x+c}*d*e^x)*a*b - 3*(\sqrt{2}*\sqrt{\pi}*(4*c+d)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{d*x+c})*\sqrt{-d}/d)*e^{(-2*c/d)/\sqrt{-d}} + 4*\sqrt{d*x+c}*d*e^{(2*x)})*b^2)/d$

**maple** [A] time = 0.02, size = 144, normalized size = 0.99

$$\frac{4\left(-\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{\frac{-1}{d}}\sqrt{dx+c}\right)}{4\sqrt{\frac{-1}{d}}} + \frac{\sqrt{dx+c}de^{\frac{dx+c}{d}}}{2}\right)abe^{-\frac{c}{d}} + 2\left(-\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{\frac{-2}{d}}\sqrt{dx+c}\right)}{8\sqrt{\frac{-2}{d}}} + \frac{\sqrt{dx+c}de^{\frac{2dx+2c}{d}}}{4}\right)b^2e^{-\frac{2c}{d}} + \frac{2(dx+c)^{\frac{3}{2}}a^2}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*exp(x))^2\*(d\*x+c)^(1/2),x)

[Out]  $\frac{2}{d}*(1/3*a^2*(d*x+c)^{(3/2)}+b^2/\exp(c/d)^2*(1/4*d*(d*x+c)^{(1/2)}*\exp(2*(d*x+c)/d)-1/8*d*\Pi^{(1/2)}/(-2/d)^{(1/2)}*\operatorname{erf}((-2/d)^{(1/2)}*(d*x+c)^{(1/2)}))+2*a*b/\exp(c/d)*(1/2*(d*x+c)^{(1/2)}*d*\exp((d*x+c)/d)-1/4*d*\Pi^{(1/2)}/(-1/d)^{(1/2)}*\operatorname{erf}((-1/d)^{(1/2)}*(d*x+c)^{(1/2)}))$

**maxima** [A] time = 2.49, size = 160, normalized size = 1.10

$$\frac{16(dx+c)^{\frac{3}{2}}a^2 - 24 \left( \frac{\sqrt{\pi}d \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{\frac{1}{d}}} - 2\sqrt{dx+c}de^{\left(\frac{dx+c}{d}-\frac{c}{d}\right)} \right) ab - 3 \left( \frac{\sqrt{2}\sqrt{\pi}d \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{1}{d}}\right)e^{\left(-\frac{2c}{d}\right)}}{\sqrt{\frac{1}{d}}} - 4\sqrt{dx+c}de^{\left(\frac{2(dx+c)}{d}-\frac{2c}{d}\right)} \right) b^2}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))^2\*(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/24\*(16\*(d\*x + c)^(3/2)\*a^2 - 24\*(sqrt(pi)\*d\*erf(sqrt(d\*x + c)\*sqrt(-1/d))\*e^(-c/d)/sqrt(-1/d) - 2\*sqrt(d\*x + c)\*d\*e^((d\*x + c)/d - c/d))\*a\*b - 3\*(sqrt(2)\*sqrt(pi)\*d\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-1/d))\*e^(-2\*c/d)/sqrt(-1/d) - 4\*sqrt(d\*x + c)\*d\*e^(2\*(d\*x + c)/d - 2\*c/d))\*b^2/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b e^x)^2 \sqrt{c + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*exp(x))^2\*(c + d\*x)^(1/2),x)

[Out] int((a + b\*exp(x))^2\*(c + d\*x)^(1/2), x)

**sympy** [A] time = 3.21, size = 184, normalized size = 1.27

$$\frac{2a^2(c+dx)^{\frac{3}{2}}}{3d} - \sqrt{\pi}ab\sqrt{d}e^{-\frac{c}{d}}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{d\sqrt{\frac{1}{d}}}\right) + \frac{2ab\sqrt{c+dx}e^{-\frac{c}{d}}e^{\frac{c}{d}+x}}{\sqrt{d}\sqrt{\frac{1}{d}}} - \frac{\sqrt{2}\sqrt{\pi}b^2\sqrt{d}e^{-\frac{2c}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{d\sqrt{\frac{1}{d}}}\right)}{8} + \frac{b^2\sqrt{c+dx}}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))\*\*2\*(d\*x+c)\*\*(1/2),x)

[Out] 2\*a\*\*2\*(c + d\*x)\*\*(3/2)/(3\*d) - sqrt(pi)\*a\*b\*sqrt(d)\*exp(-c/d)\*erfi(sqrt(c + d\*x)/(d\*sqrt(1/d))) + 2\*a\*b\*sqrt(c + d\*x)\*exp(-c/d)\*exp(c/d + x)/(sqrt(d)\*sqrt(1/d)) - sqrt(2)\*sqrt(pi)\*b\*\*2\*sqrt(d)\*exp(-2\*c/d)\*erfi(sqrt(2)\*sqrt(c + d\*x)/(d\*sqrt(1/d)))/8 + b\*\*2\*sqrt(c + d\*x)\*exp(-2\*c/d)\*exp(2\*c/d + 2\*x)/(2\*sqrt(d)\*sqrt(1/d))

### 3.66 $\int (a + be^x)^3 \sqrt{c + dx} dx$

**Optimal.** Leaf size=224

$$\frac{2a^3(c+dx)^{3/2}}{3d} - \frac{3}{2}\sqrt{\pi}a^2b\sqrt{d}e^{-\frac{c}{d}}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) + 3a^2be^x\sqrt{c+dx} - \frac{3}{4}\sqrt{\frac{\pi}{2}}ab^2\sqrt{d}e^{-\frac{2c}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right) + \frac{3}{2}ab^2e^{2x}$$

[Out]  $2/3*a^3*(d*x+c)^{(3/2)}/d-1/18*b^3*\operatorname{erfi}(3^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*d^{(1/2)}$   
 $)^3^{(1/2)}*\operatorname{Pi}^{(1/2)}/\exp(3*c/d)-3/8*a^2*b^2*\operatorname{erfi}(2^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})$   
 $*d^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/\exp(2*c/d)-3/2*a^2*b*\operatorname{erfi}((d*x+c)^{(1/2)}/d^{(1/2)})*$   
 $d^{(1/2)}*\operatorname{Pi}^{(1/2)}/\exp(c/d)+3*a^2*b*\exp(x)*(d*x+c)^{(1/2)}+3/2*a*b^2*\exp(2*x)*$   
 $(d*x+c)^{(1/2)}+1/3*b^3*\exp(3*x)*(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.26, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {2183, 2176, 2180, 2204}

$$-\frac{3}{2}\sqrt{\pi}a^2b\sqrt{d}e^{-\frac{c}{d}}\operatorname{Erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) + 3a^2be^x\sqrt{c+dx} + \frac{2a^3(c+dx)^{3/2}}{3d} - \frac{3}{4}\sqrt{\frac{\pi}{2}}ab^2\sqrt{d}e^{-\frac{2c}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right) + \frac{3}{2}ab^2e^{2x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + bE^x)^3 \operatorname{Sqrt}[c + dx], x]$

[Out]  $3*a^2*b*E^x*\operatorname{Sqrt}[c + dx] + (3*a*b^2*E^{(2*x)}*\operatorname{Sqrt}[c + dx])/2 + (b^3*E^{(3*x)})*\operatorname{Sqrt}[c + dx])/3 + (2*a^3*(c + dx)^{(3/2)})/(3*d) - (3*a^2*b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[c + dx]/\operatorname{Sqrt}[d]])/(2*E^{(c/d)}) - (3*a*b^2*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c + dx])/ \operatorname{Sqrt}[d]])/(4*E^{(2*c/d)}) - (b^3*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[c + dx])/ \operatorname{Sqrt}[d]])/(6*E^{(3*c/d)})$

#### Rule 2176

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}}, x\_Symbol] :> \operatorname{Simp}[(c + dx)^m*(b*F^{(g*(e + f*x)))^n)/(f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + dx)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /;$   $\operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x \ \&\& \operatorname{GtQ}[m, 0] \ \&\& \operatorname{IntegerQ}[2*m] \ \&\& \operatorname{!UseGamma} == \operatorname{True}$

#### Rule 2180

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}/\operatorname{Sqrt}[(c_*) + (d_*)*(x_*)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + dx]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \operatorname{!UseGamma} == \operatorname{True}$

Rule 2183

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int (a + be^x)^3 \sqrt{c + dx} \, dx &= \int \left( a^3 \sqrt{c + dx} + 3a^2 be^x \sqrt{c + dx} + 3ab^2 e^{2x} \sqrt{c + dx} + b^3 e^{3x} \sqrt{c + dx} \right) dx \\
&= \frac{2a^3(c + dx)^{3/2}}{3d} + (3a^2b) \int e^x \sqrt{c + dx} \, dx + (3ab^2) \int e^{2x} \sqrt{c + dx} \, dx + b^3 \int e^{3x} \sqrt{c + dx} \, dx \\
&= 3a^2 be^x \sqrt{c + dx} + \frac{3}{2} ab^2 e^{2x} \sqrt{c + dx} + \frac{1}{3} b^3 e^{3x} \sqrt{c + dx} + \frac{2a^3(c + dx)^{3/2}}{3d} - \frac{1}{2} (3a^2bd) \int \frac{e^x}{\sqrt{c + dx}} \, dx \\
&= 3a^2 be^x \sqrt{c + dx} + \frac{3}{2} ab^2 e^{2x} \sqrt{c + dx} + \frac{1}{3} b^3 e^{3x} \sqrt{c + dx} + \frac{2a^3(c + dx)^{3/2}}{3d} - (3a^2b) \operatorname{Subst} \int \frac{e^u}{\sqrt{c + du}} \, du \\
&= 3a^2 be^x \sqrt{c + dx} + \frac{3}{2} ab^2 e^{2x} \sqrt{c + dx} + \frac{1}{3} b^3 e^{3x} \sqrt{c + dx} + \frac{2a^3(c + dx)^{3/2}}{3d} - \frac{3}{2} a^2 b \sqrt{d} e^{-\frac{c}{d}} \operatorname{Erfi} \left( \frac{\sqrt{c + dx}}{\sqrt{d}} \right)
\end{aligned}$$

**Mathematica** [A] time = 0.80, size = 196, normalized size = 0.88

$$\frac{108\sqrt{\pi} a^2 b d^{3/2} e^{-\frac{c}{d}} \operatorname{erfi} \left( \frac{\sqrt{c + dx}}{\sqrt{d}} \right) - 12\sqrt{c + dx} \left( 4a^3(c + dx) + 18a^2 b d e^x + 9a^2 b d e^{2x} + 2b^3 d e^{3x} \right) + 27\sqrt{2\pi} a b^2 d^{3/2} e^{-\frac{c}{d}}}{72d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^x)^3*Sqrt[c + d*x], x]
```

```
[Out] -1/72*(-12*Sqrt[c + d*x]*(18*a^2*b*d*E^x + 9*a*b^2*d*E^(2*x) + 2*b^3*d*E^(3
*x) + 4*a^3*(c + d*x)) + (108*a^2*b*d^(3/2)*Sqrt[Pi]*Erfi[Sqrt[c + d*x]/Sqr
t[d]])/E^(c/d) + (27*a*b^2*d^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[c + d*x])/
Sqrt[d]])/E^((2*c)/d) + (4*b^3*d^(3/2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[c + d*
x])/Sqrt[d]])/E^((3*c)/d)/d
```

**fricas [A]** time = 0.42, size = 196, normalized size = 0.88

$$\frac{27\sqrt{2}\sqrt{\pi}ab^2d^2\sqrt{-\frac{1}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{2c}{d}\right)} + 4\sqrt{3}\sqrt{\pi}b^3d^2\sqrt{-\frac{1}{d}}\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{3c}{d}\right)} + 108\sqrt{\pi}a^2b^2d^2\sqrt{-\frac{1}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)} + 12(4a^3d^2x + 2b^3d^2e^{3x} + 9a^2b^2d^2e^{2x} + 18a^2b^2d^2e^x + 4a^3c)\sqrt{dx+c}}{72d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))^3\*(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/72\*(27\*sqrt(2)\*sqrt(pi)\*a\*b^2\*d^2\*sqrt(-1/d)\*erf(sqrt(2)\*sqrt(dx+c)\*sqrt(-1/d))\*e^(-2\*c/d) + 4\*sqrt(3)\*sqrt(pi)\*b^3\*d^2\*sqrt(-1/d)\*erf(sqrt(3)\*sqrt(dx+c)\*sqrt(-1/d))\*e^(-3\*c/d) + 108\*sqrt(pi)\*a^2\*b\*d^2\*sqrt(-1/d)\*erf(sqrt(dx+c)\*sqrt(-1/d))\*e^(-c/d) + 12\*(4\*a^3\*d\*x + 2\*b^3\*d\*e^(3\*x) + 9\*a\*b^2\*d\*e^(2\*x) + 18\*a^2\*b\*d\*e^x + 4\*a^3\*c)\*sqrt(dx+c))/d

**giac [B]** time = 0.53, size = 368, normalized size = 1.64

$$\frac{108\sqrt{2}\sqrt{\pi}ab^2cd\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{2c}{d}\right)}}{\sqrt{-d}} + \frac{24\sqrt{3}\sqrt{\pi}b^3cd\operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{3c}{d}\right)}}{\sqrt{-d}} + \frac{216\sqrt{\pi}a^2bcd\operatorname{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}} - 144\sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))^3\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out] -1/72\*(108\*sqrt(2)\*sqrt(pi)\*a\*b^2\*c\*d\*erf(-sqrt(2)\*sqrt(dx+c)\*sqrt(-d)/d)\*e^(-2\*c/d)/sqrt(-d) + 24\*sqrt(3)\*sqrt(pi)\*b^3\*c\*d\*erf(-sqrt(3)\*sqrt(dx+c)\*sqrt(-d)/d)\*e^(-3\*c/d)/sqrt(-d) + 216\*sqrt(pi)\*a^2\*b\*c\*d\*erf(-sqrt(dx+c)\*sqrt(-d)/d)\*e^(-c/d)/sqrt(-d) - 144\*sqrt(dx+c)\*a^3\*c - 48\*((dx+c)^(3/2) - 3\*sqrt(dx+c)\*c)\*a^3 - 108\*(sqrt(pi)\*(2\*c+d)\*d\*erf(-sqrt(dx+c)\*sqrt(-d)/d)\*e^(-c/d)/sqrt(-d) + 2\*sqrt(dx+c)\*d\*e^x)\*a^2\*b - 27\*(sqrt(2)\*sqrt(pi)\*(4\*c+d)\*d\*erf(-sqrt(2)\*sqrt(dx+c)\*sqrt(-d)/d)\*e^(-2\*c/d)/sqrt(-d) + 4\*sqrt(dx+c)\*d\*e^(2\*x))\*a\*b^2 - 4\*(sqrt(3)\*sqrt(pi)\*(6\*c+d)\*d\*erf(-sqrt(3)\*sqrt(dx+c)\*sqrt(-d)/d)\*e^(-3\*c/d)/sqrt(-d) + 6\*sqrt(dx+c)\*d\*e^(3\*x))\*b^3)/d

**maple [A]** time = 0.02, size = 211, normalized size = 0.94

$$\frac{6\left(-\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{-\frac{1}{d}}\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}} + \frac{\sqrt{dx+c}de^{\frac{dx+c}{d}}}{2}\right)a^2be^{-\frac{c}{d}} + 6\left(-\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{-\frac{2}{d}}\sqrt{dx+c}\right)}{8\sqrt{-\frac{2}{d}}} + \frac{\sqrt{dx+c}de^{\frac{2dx+2c}{d}}}{4}\right)ab^2e^{-\frac{2c}{d}} + 2\left(-\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{-\frac{1}{d}}\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}} + \frac{\sqrt{dx+c}de^{\frac{dx+c}{d}}}{2}\right)a^2b^2e^{-\frac{c}{d}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*exp(x))^3\*(d\*x+c)^(1/2),x)

[Out]  $\frac{2}{d} \cdot \frac{1}{3} \cdot (d \cdot x + c)^{3/2} \cdot a^3 + b^3 / \exp(c/d)^3 \cdot \frac{1}{6} \cdot d \cdot (d \cdot x + c)^{1/2} \cdot \exp(3 \cdot (d \cdot x + c) / d) - \frac{1}{12} \cdot d \cdot \pi^{1/2} / (-3/d)^{1/2} \cdot \operatorname{erf}((-3/d)^{1/2} \cdot (d \cdot x + c)^{1/2}) + 3 \cdot a \cdot b^2 / \exp(c/d)^2 \cdot \frac{1}{4} \cdot (d \cdot x + c)^{1/2} \cdot d \cdot \exp(2 \cdot (d \cdot x + c) / d) - \frac{1}{8} \cdot d \cdot \pi^{1/2} / (-2/d)^{1/2} \cdot \operatorname{erf}((-2/d)^{1/2} \cdot (d \cdot x + c)^{1/2}) + 3 \cdot a^2 \cdot b / \exp(c/d) \cdot \frac{1}{2} \cdot (d \cdot x + c)^{1/2} \cdot d \cdot \exp((d \cdot x + c) / d) - \frac{1}{4} \cdot d \cdot \pi^{1/2} / (-1/d)^{1/2} \cdot \operatorname{erf}((-1/d)^{1/2} \cdot (d \cdot x + c)^{1/2})$

**maxima** [A] time = 2.42, size = 238, normalized size = 1.06

$$\frac{48(dx+c)^{\frac{3}{2}}a^3 - 108 \left( \frac{\sqrt{\pi} d \operatorname{erf}\left(\sqrt{dx+c} \sqrt{-\frac{1}{d}}\right) e^{\left(-\frac{c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 2\sqrt{dx+c} d e^{\left(\frac{dx+c}{d} - \frac{c}{d}\right)} \right) a^2 b - 27 \left( \frac{\sqrt{2} \sqrt{\pi} d \operatorname{erf}\left(\sqrt{2} \sqrt{dx+c} \sqrt{-\frac{1}{d}}\right) e^{\left(-\frac{2c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 4\sqrt{dx+c} d e^{\left(\frac{2dx+2c}{d} - \frac{2c}{d}\right)} \right) a b^2 - 4 \left( \frac{\sqrt{3} \sqrt{\pi} d \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{-\frac{1}{d}}\right) e^{\left(-\frac{3c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 6\sqrt{dx+c} d e^{\left(\frac{3dx+3c}{d} - \frac{3c}{d}\right)} \right) b^3}{72d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))^3\*(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{72} \cdot (48 \cdot (d \cdot x + c)^{3/2} \cdot a^3 - 108 \cdot (\sqrt{\pi} \cdot d \cdot \operatorname{erf}(\sqrt{d \cdot x + c} \cdot \sqrt{-1/d}) \cdot e^{-c/d} / \sqrt{-1/d} - 2 \cdot \sqrt{d \cdot x + c} \cdot d \cdot e^{((d \cdot x + c)/d - c/d)}) \cdot a^2 \cdot b - 27 \cdot (\sqrt{2} \cdot \sqrt{\pi} \cdot d \cdot \operatorname{erf}(\sqrt{2} \cdot \sqrt{d \cdot x + c} \cdot \sqrt{-1/d}) \cdot e^{-2c/d} / \sqrt{-1/d} - 4 \cdot \sqrt{d \cdot x + c} \cdot d \cdot e^{(2 \cdot (d \cdot x + c)/d - 2c/d)}) \cdot a \cdot b^2 - 4 \cdot (\sqrt{3} \cdot \sqrt{\pi} \cdot d \cdot \operatorname{erf}(\sqrt{3} \cdot \sqrt{d \cdot x + c} \cdot \sqrt{-1/d}) \cdot e^{-3c/d} / \sqrt{-1/d} - 6 \cdot \sqrt{d \cdot x + c} \cdot d \cdot e^{(3 \cdot (d \cdot x + c)/d - 3c/d)}) \cdot b^3) / d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b e^x)^3 \sqrt{c + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*exp(x))^3\*(c + d\*x)^(1/2),x)

[Out] int((a + b\*exp(x))^3\*(c + d\*x)^(1/2), x)

**sympy** [A] time = 4.36, size = 291, normalized size = 1.30

$$\frac{2a^3(c+dx)^{\frac{3}{2}}}{3d} - \frac{3\sqrt{\pi}a^2b\sqrt{d}e^{-\frac{c}{d}}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{d\sqrt{\frac{1}{d}}}\right)}{2} + \frac{3a^2b\sqrt{c+dx}e^{-\frac{c}{d}}e^{\frac{c}{d}+x}}{\sqrt{d}\sqrt{\frac{1}{d}}} - \frac{3\sqrt{2}\sqrt{\pi}ab^2\sqrt{d}e^{-\frac{2c}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{d\sqrt{\frac{1}{d}}}\right)}{8} + \frac{3ab^2\sqrt{c+dx}e^{-\frac{3c}{d}}e^{\frac{3c}{d}+3x}}{6\sqrt{d}\sqrt{\frac{1}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*exp(x))\*\*3\*(d\*x+c)\*\*(1/2),x)

```
[Out] 2*a**3*(c + d*x)**(3/2)/(3*d) - 3*sqrt(pi)*a**2*b*sqrt(d)*exp(-c/d)*erfi(sqrt(c + d*x)/(d*sqrt(1/d)))/2 + 3*a**2*b*sqrt(c + d*x)*exp(-c/d)*exp(c/d + x)/(sqrt(d)*sqrt(1/d)) - 3*sqrt(2)*sqrt(pi)*a*b**2*sqrt(d)*exp(-2*c/d)*erfi(sqrt(2)*sqrt(c + d*x)/(d*sqrt(1/d)))/8 + 3*a*b**2*sqrt(c + d*x)*exp(-2*c/d)*exp(2*c/d + 2*x)/(2*sqrt(d)*sqrt(1/d)) - sqrt(3)*sqrt(pi)*b**3*sqrt(d)*exp(-3*c/d)*erfi(sqrt(3)*sqrt(c + d*x)/(d*sqrt(1/d)))/18 + b**3*sqrt(c + d*x)*exp(-3*c/d)*exp(3*c/d + 3*x)/(3*sqrt(d)*sqrt(1/d))
```

$$3.67 \quad \int \frac{\sqrt{c+dx}}{a+be^x} dx$$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\sqrt{c+dx}}{a+be^x}, x\right)$$

[Out] Unintegrable((d\*x+c)^(1/2)/(a+b\*exp(x)), x)

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{c+dx}}{a+be^x} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[c + d\*x]/(a + b\*E^x), x]

[Out] Defer[Int][Sqrt[c + d\*x]/(a + b\*E^x), x]

Rubi steps

$$\int \frac{\sqrt{c+dx}}{a+be^x} dx = \int \frac{\sqrt{c+dx}}{a+be^x} dx$$

**Mathematica** [A] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{a+be^x} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[c + d\*x]/(a + b\*E^x), x]

[Out] Integrate[Sqrt[c + d\*x]/(a + b\*E^x), x]

**fricas** [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx+c}}{be^x+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x + c)/(b\*e^x + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{be^x + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x)),x, algorithm="giac")

[Out] integrate(sqrt(d\*x + c)/(b\*e^x + a), x)

**maple** [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{be^x + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(1/2)/(a+b\*exp(x)),x)

[Out] int((d\*x+c)^(1/2)/(a+b\*exp(x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{be^x + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x)),x, algorithm="maxima")

[Out] integrate(sqrt(d\*x + c)/(b\*e^x + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c + dx}}{a + be^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^(1/2)/(a + b\*exp(x)),x)

[Out] int((c + d\*x)^(1/2)/(a + b\*exp(x)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx}}{a + be^x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(a+b*exp(x)),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*exp(x)), x)
```

$$3.68 \quad \int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx$$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\sqrt{c+dx}}{(a+be^x)^2}, x\right)$$

[Out] Unintegrable((d\*x+c)^(1/2)/(a+b\*exp(x))^2, x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[c + d\*x]/(a + b\*E^x)^2, x]

[Out] Defer[Int][Sqrt[c + d\*x]/(a + b\*E^x)^2, x]

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx$$

Mathematica [A] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[c + d\*x]/(a + b\*E^x)^2, x]

[Out] Integrate[Sqrt[c + d\*x]/(a + b\*E^x)^2, x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx+c}}{b^2e^{(2x)}+2abe^x+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x + c)/(b^2\*e^(2\*x) + 2\*a\*b\*e^x + a^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{(be^x + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x))^2,x, algorithm="giac")

[Out] integrate(sqrt(d\*x + c)/(b\*e^x + a)^2, x)

**maple** [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{(be^x + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(1/2)/(b\*exp(x)+a)^2,x)

[Out] int((d\*x+c)^(1/2)/(b\*exp(x)+a)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{(be^x + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x))^2,x, algorithm="maxima")

[Out] integrate(sqrt(d\*x + c)/(b\*e^x + a)^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c + dx}}{(a + be^x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^(1/2)/(a + b\*exp(x))^2,x)

```
[Out] int((c + d*x)^(1/2)/(a + b*exp(x))^2, x)
```

```
sympy [A] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{c + dx}}{(a + be^x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(a+b*exp(x))**2,x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*exp(x))**2, x)
```

$$3.69 \quad \int \frac{\sqrt{c+dx}}{(a+be^x)^3} dx$$

Optimal. Leaf size=22

$$\text{Int} \left( \frac{\sqrt{c+dx}}{(a+be^x)^3}, x \right)$$

[Out] Unintegrable((d\*x+c)^(1/2)/(a+b\*exp(x))^3, x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[c + d\*x]/(a + b\*E^x)^3, x]

[Out] Defer[Int][Sqrt[c + d\*x]/(a + b\*E^x)^3, x]

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3} dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^3} dx$$

Mathematica [A] time = 2.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[c + d\*x]/(a + b\*E^x)^3, x]

[Out] Integrate[Sqrt[c + d\*x]/(a + b\*E^x)^3, x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx+c}}{b^3 e^{(3x)} + 3 ab^2 e^{(2x)} + 3 a^2 b e^x + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x))^3,x, algorithm="fricas")

[Out] integral(sqrt(d\*x + c)/(b^3\*e^(3\*x) + 3\*a\*b^2\*e^(2\*x) + 3\*a^2\*b\*e^x + a^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{(be^x + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x))^3,x, algorithm="giac")

[Out] integrate(sqrt(d\*x + c)/(b\*e^x + a)^3, x)

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{(be^x + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(1/2)/(b\*exp(x)+a)^3,x)

[Out] int((d\*x+c)^(1/2)/(b\*exp(x)+a)^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx + c}}{(be^x + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(1/2)/(a+b\*exp(x))^3,x, algorithm="maxima")

[Out] integrate(sqrt(d\*x + c)/(b\*e^x + a)^3, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c + dx}}{(a + be^x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*exp(x))^3,x)
```

```
[Out] int((c + d*x)^(1/2)/(a + b*exp(x))^3, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx}}{(a + be^x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(a+b*exp(x))**3,x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*exp(x))**3, x)
```



$$3.70 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^m dx$$

**Optimal.** Leaf size=340

$$\frac{a^3(c+dx)^{m+1}}{d(m+1)} + \frac{3a^2b(c+dx)^m \left( F^{eg+fgx} \right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left( -\frac{fgn \log(F)(c+dx)}{d} \right)^{-m} \Gamma\left(m+1, -\frac{fgn(c+dx) \log(F)}{d}\right)}{fgn \log(F)} + \frac{3ab^2}{d(m+1)}$$

[Out]  $a^3(d*x+c)^{(1+m)/d/(1+m)+3^{(-1-m)*b^3F^{(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))}*(F^{(f*g*x+e*g)})^{(3*n)*(d*x+c)^m \text{GAMMA}(1+m, -3*f*g*n*(d*x+c)*\ln(F)/d)/f/g/n/\ln(F)/((-f*g*n*(d*x+c)*\ln(F)/d)^m)+3*2^{(-1-m)*a*b^2F^{(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))}*(F^{(f*g*x+e*g)})^{(2*n)*(d*x+c)^m \text{GAMMA}(1+m, -2*f*g*n*(d*x+c)*\ln(F)/d)/f/g/n/\ln(F)/((-f*g*n*(d*x+c)*\ln(F)/d)^m)+3*a^2*bF^{((e-c*f/d)*g*n-g*n*(f*x+e))}*(F^{(f*g*x+e*g)})^n*(d*x+c)^m \text{GAMMA}(1+m, -f*g*n*(d*x+c)*\ln(F)/d)/f/g/n/\ln(F)/((-f*g*n*(d*x+c)*\ln(F)/d)^m}$

**Rubi [A]** time = 0.47, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2183, 2182, 2181}

$$\frac{3a^2b(c+dx)^m \left( F^{eg+fgx} \right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left( -\frac{fgn \log(F)(c+dx)}{d} \right)^{-m} \text{Gamma}\left(m+1, -\frac{fgn \log(F)(c+dx)}{d}\right)}{fgn \log(F)} + \frac{3ab^2 2^{-m-1} c}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]^3\*(c + d\*x)^m, x]

[Out]  $(a^3(c+dx)^{(1+m)/d/(1+m)})/(d*(1+m)) + (3^{(-1-m)*b^3F^{(3*(e-(c*f)/d)*g*n-3*g*n*(e+f*x))}*(F^{(e*g+f*g*x)})^{(3*n)*(c+dx)^m \text{Gamma}[1+m, (-3*f*g*n*(c+dx)*\text{Log}[F])/d]}/(f*g*n*\text{Log}[F]*(-((f*g*n*(c+dx)*\text{Log}[F])/d))^m) + (3*2^{(-1-m)*a*b^2F^{(2*(e-(c*f)/d)*g*n-2*g*n*(e+f*x))}*(F^{(e*g+f*g*x)})^{(2*n)*(c+dx)^m \text{Gamma}[1+m, (-2*f*g*n*(c+dx)*\text{Log}[F])/d]}/(f*g*n*\text{Log}[F]*(-((f*g*n*(c+dx)*\text{Log}[F])/d))^m) + (3*a^2*bF^{((e-(c*f)/d)*g*n-g*n*(e+f*x))}*(F^{(e*g+f*g*x)})^n*(c+dx)^m \text{Gamma}[1+m, -((f*g*n*(c+dx)*\text{Log}[F])/d]}/(f*g*n*\text{Log}[F]*(-((f*g*n*(c+dx)*\text{Log}[F])/d))^m)$

**Rule 2181**

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol]  
 :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rule 2182

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(b\*F^(g\*(e + f\*x)))^n/F^(g\*n\*(e + f\*x)), Int[(c + d\*x)^m\*F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

Rule 2183

Int[((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^m dx &= \int \left( a^3 (c + dx)^m + 3a^2 b \left( F^{eg+fgx} \right)^n (c + dx)^m + 3ab^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)^m + \right. \\ &= \frac{a^3 (c + dx)^{1+m}}{d(1+m)} + (3a^2 b) \int \left( F^{eg+fgx} \right)^n (c + dx)^m dx + (3ab^2) \int \left( F^{eg+fgx} \right)^{2n} (c + dx)^m dx \\ &= \frac{a^3 (c + dx)^{1+m}}{d(1+m)} + \left( 3a^2 b F^{-n(eg+fgx)} \left( F^{eg+fgx} \right)^n \right) \int F^{n(eg+fgx)} (c + dx)^m dx + \left( 3ab^2 F^{-2n(eg+fgx)} \left( F^{eg+fgx} \right)^{2n} \right) \int F^{2n(eg+fgx)} (c + dx)^m dx \\ &= \frac{a^3 (c + dx)^{1+m}}{d(1+m)} + \frac{3^{-1-m} b^3 F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)} \left( F^{eg+fgx} \right)^{3n} (c + dx)^m \Gamma(1+m, \log(F))}{fgn \log(F)} \end{aligned}$$

**Mathematica** [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^m dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^3\*(c + d\*x)^m, x]

[Out] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^3\*(c + d\*x)^m, x]

**fricas** [A] time = 0.45, size = 270, normalized size = 0.79

$$18 \left( a^2 b d m + a^2 b d \right) e^{\left( \frac{(de-cf)gn \log(F) - dm \log\left(-\frac{fgn \log(F)}{d}\right)}{d} \right)} \Gamma\left(m + 1, -\frac{(dfgnx + c fgn) \log(F)}{d}\right) + 9 \left( a b^2 d m + a b^2 d \right) e^{\left( \frac{2(de-cf)gn \log(F) - dm \log\left(-\frac{fgn \log(F)}{d}\right)}{d} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^m,x, algorithm="fricas")

[Out]  $\frac{1}{6} * (18 * (a^2 * b * d * m + a^2 * b * d) * e^{((d * e - c * f) * g * n * \log(F) - d * m * \log(-f * g * n * \log(F) / d)) / d} * \text{gamma}(m + 1, -(d * f * g * n * x + c * f * g * n) * \log(F) / d) + 9 * (a * b^2 * d * m + a * b^2 * d) * e^{((2 * (d * e - c * f) * g * n * \log(F) - d * m * \log(-2 * f * g * n * \log(F) / d)) / d) * \text{gamma}(m + 1, -2 * (d * f * g * n * x + c * f * g * n) * \log(F) / d) + 2 * (b^3 * d * m + b^3 * d) * e^{((3 * (d * e - c * f) * g * n * \log(F) - d * m * \log(-3 * f * g * n * \log(F) / d)) / d) * \text{gamma}(m + 1, -3 * (d * f * g * n * x + c * f * g * n) * \log(F) / d) + 6 * (a^3 * d * f * g * n * x + a^3 * c * f * g * n) * (d * x + c)^m * \log(F)) / ((d * f * g * m + d * f * g) * n * \log(F))$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( \left( F^{(f x + e)g} \right)^n b + a \right)^3 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^m,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^3\*(d\*x + c)^m, x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(f x + e)g} \right)^n + a \right)^3 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^3\*(d\*x+c)^m,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^3\*(d\*x+c)^m,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^3 n b^3 \int e^{(m \log(dx+c) + 3n \log(F^{fgx}))} dx + 3 (F^{eg})^2 n a b^2 \int e^{(m \log(dx+c) + 2n \log(F^{fgx}))} dx + 3 (F^{eg})^n a^2 b \int e^{(m \log(dx+c) + n \log(F^{fgx}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^3\*(d\*x+c)^m,x, algorithm="maxima")

[Out]  $(F^{(e * g)})^{(3 * n)} * b^3 * \text{integrate}(e^{(m * \log(d * x + c) + 3 * n * \log(F^{(f * g * x)}))}, x) + 3 * (F^{(e * g)})^{(2 * n)} * a * b^2 * \text{integrate}(e^{(m * \log(d * x + c) + 2 * n * \log(F^{(f * g * x)}))}, x) + 3 * (F^{(e * g)})^n * a^2 * b * \text{integrate}(e^{(m * \log(d * x + c) + n * \log(F^{(f * g * x)}))}, x) + (d * x + c)^{(m + 1)} * a^3 / (d * (m + 1))$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x)))^n)^3\*(c + d\*x)^m, x)

[Out] int((a + b\*(F^(g\*(e + f\*x)))^n)^3\*(c + d\*x)^m, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e))))\*\*n)\*\*3\*(d\*x+c)\*\*m, x)

[Out] Timed out

$$3.71 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx)^m dx$$

**Optimal.** Leaf size=228

$$\frac{a^2(c+dx)^{m+1}}{d(m+1)} + \frac{2ab(c+dx)^m (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left(\frac{-fgn \log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{fgn(c+dx) \log(F)}{d}\right)}{fgn \log(F)} + \frac{b^2 2^{-m}}{d(m+1)}$$

[Out]  $a^2(d*x+c)^{(1+m)/d/(1+m)+2^{-1-m}} * b^2 * F^{(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))} * (F^{(f*g*x+e*g)})^{(2*n)} * (d*x+c)^m * \text{GAMMA}(1+m, -2*f*g*n*(d*x+c)*\ln(F)/d) / f/g/n/\ln(F) / ((-f*g*n*(d*x+c)*\ln(F)/d)^m + 2*a*b * F^{((e-c*f/d)*g*n-g*n*(f*x+e))} * (F^{(f*g*x+e*g)})^n * (d*x+c)^m * \text{GAMMA}(1+m, -f*g*n*(d*x+c)*\ln(F)/d) / f/g/n/\ln(F) / ((-f*g*n*(d*x+c)*\ln(F)/d)^m)$

**Rubi [A]** time = 0.28, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2183, 2182, 2181}

$$\frac{2ab(c+dx)^m (F^{eg+fgx})^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left(\frac{-fgn \log(F)(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{fgn \log(F)(c+dx)}{d}\right)}{fgn \log(F)} + \frac{b^2 2^{-m-1}(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)^2\*(c + d\*x)^m, x]

[Out]  $(a^2*(c+d*x)^{(1+m)/(d*(1+m))} + (2^{-1-m}*b^2 * F^{(2*(e-(c*f)/d)*g*n-2*g*n*(e+f*x))} * (F^{(e*g+f*g*x)})^{(2*n)} * (c+d*x)^m * \text{Gamma}[1+m, (-2*f*g*n*(c+d*x)*\text{Log}[F])/d]) / (f*g*n*\text{Log}[F] * (-((f*g*n*(c+d*x)*\text{Log}[F])/d))^m + (2*a*b * F^{((e-(c*f)/d)*g*n-g*n*(e+f*x))} * (F^{(e*g+f*g*x)})^n * (c+d*x)^m * \text{Gamma}[1+m, -(f*g*n*(c+d*x)*\text{Log}[F])/d]) / (f*g*n*\text{Log}[F] * (-((f*g*n*(c+d*x)*\text{Log}[F])/d))^m)$

**Rule 2181**

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x]])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

**Rule 2182**

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_
.), x_Symbol] :> Dist[(b * F^(g*(e + f*x)))^n / F^(g*n*(e + f*x)), Int[(c + d*x
```

)^m \* F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

### Rule 2183

Int[((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx)^m dx &= \int \left( a^2 (c + dx)^m + 2ab \left( F^{eg+fgx} \right)^n (c + dx)^m + b^2 \left( F^{eg+fgx} \right)^{2n} (c + dx)^m \right) dx \\ &= \frac{a^2 (c + dx)^{1+m}}{d(1+m)} + (2ab) \int \left( F^{eg+fgx} \right)^n (c + dx)^m dx + b^2 \int \left( F^{eg+fgx} \right)^{2n} (c + dx)^m dx \\ &= \frac{a^2 (c + dx)^{1+m}}{d(1+m)} + \left( 2ab F^{-n(eg+fgx)} \left( F^{eg+fgx} \right)^n \right) \int F^{n(eg+fgx)} (c + dx)^m dx + \left( b^2 F^{-2n(eg+fgx)} \left( F^{eg+fgx} \right)^{2n} \right) \int (c + dx)^m dx \\ &= \frac{a^2 (c + dx)^{1+m}}{d(1+m)} + \frac{2^{-1-m} b^2 F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)} \left( F^{eg+fgx} \right)^{2n} (c + dx)^m \Gamma(1+m)}{fgn \log(F)} \end{aligned}$$

**Mathematica** [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx)^m dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^2\*(c + d\*x)^m, x]

[Out] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)^2\*(c + d\*x)^m, x]

**fricas** [A] time = 0.45, size = 191, normalized size = 0.84

$$\frac{4(abdm + abd)e^{\left( \frac{(de-cf)gn \log(F) - dm \log\left(-\frac{fgn \log(F)}{d}\right)}{d} \right)} \Gamma\left(m + 1, -\frac{(dfgnx + cfdgn) \log(F)}{d}\right) + (b^2 dm + b^2 d)e^{\left( \frac{2(de-cf)gn \log(F) - dm \log\left(-\frac{2fgn \log(F)}{d}\right)}{d} \right)}}{2(dfgm + dfg)n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^m,x, algorithm="fricas")

[Out] 1/2\*(4\*(a\*b\*d\*m + a\*b\*d)\*e^(((d\*e - c\*f)\*g\*n\*log(F) - d\*m\*log(-f\*g\*n\*log(F)/d))/d)\*gamma(m + 1, -(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + (b^2\*d\*m + b^2\*d)\*e^(((2\*(d\*e - c\*f)\*g\*n\*log(F) - d\*m\*log(-2\*f\*g\*n\*log(F)/d))/d)\*gamma(m + 1, -2\*(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + 2\*(a^2\*d\*f\*g\*n\*x + a^2\*c\*f\*g\*n)\*(d\*x + c)^m\*log(F))/((d\*f\*g\*m + d\*f\*g)\*n\*log(F))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( \left( F^{(f x + e)g} \right)^n b + a \right)^2 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^m,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^2\*(d\*x + c)^m, x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(f x + e)g} \right)^n + a \right)^2 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^2\*(d\*x+c)^m,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^2\*(d\*x+c)^m,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^{2n} b^2 \int e^{(m \log(dx+c) + 2n \log(F^{fgx}))} dx + 2 (F^{eg})^n ab \int e^{(m \log(dx+c) + n \log(F^{fgx}))} dx + \frac{(dx + c)^{m+1} a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^2\*(d\*x+c)^m,x, algorithm="maxima")

[Out] (F^(e\*g))^(2\*n)\*b^2\*integrate(e^(m\*log(d\*x + c) + 2\*n\*log(F^(f\*g\*x))), x) + 2\*(F^(e\*g))^n\*a\*b\*integrate(e^(m\*log(d\*x + c) + n\*log(F^(f\*g\*x))), x) + (d\*x + c)^(m + 1)\*a^2/(d\*(m + 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m, x)
```

```
[Out] int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c)**m, x)
```

```
[Out] Timed out
```



$$3.72 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx)^m dx$$

**Optimal.** Leaf size=116

$$\frac{a(c + dx)^{m+1}}{d(m + 1)} + \frac{b(c + dx)^m \left( F^{eg+fgx} \right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left( -\frac{fgn \log(F)(c+dx)}{d} \right)^{-m} \Gamma\left(m + 1, -\frac{fgn(c+dx) \log(F)}{d}\right)}{fgn \log(F)}$$

[Out] a\*(d\*x+c)^(1+m)/d/(1+m)+b\*F^((e-c\*f/d)\*g\*n-g\*n\*(f\*x+e))\*(F^(f\*g\*x+e\*g))^n\*(d\*x+c)^m\*GAMMA(1+m,-f\*g\*n\*(d\*x+c)\*ln(F)/d)/f/g/n/ln(F)/((-f\*g\*n\*(d\*x+c)\*ln(F)/d)^m)

**Rubi [A]** time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {2183, 2182, 2181}

$$\frac{b(c + dx)^m \left( F^{eg+fgx} \right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left( -\frac{fgn \log(F)(c+dx)}{d} \right)^{-m} \text{Gamma}\left(m + 1, -\frac{fgn \log(F)(c+dx)}{d}\right)}{fgn \log(F)} + \frac{a(c + dx)^{m+1}}{d(m + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*(F^(g\*(e + f\*x)))^n)\*(c + d\*x)^m,x]

[Out] (a\*(c + d\*x)^(1 + m))/(d\*(1 + m)) + (b\*F^((e - (c\*f)/d)\*g\*n - g\*n\*(e + f\*x))\*(F^(e\*g + f\*g\*x))^n\*(c + d\*x)^m\*Gamma[1 + m, -((f\*g\*n\*(c + d\*x)\*Log[F])/d)])/((f\*g\*n\*Log[F]\*(-(f\*g\*n\*(c + d\*x)\*Log[F])/d))^m)

**Rule 2181**

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-(f*g*Log[F]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

**Rule 2182**

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

**Rule 2183**

```
Int[((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))^(p_)*((c_) +
(d_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned} \int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx)^m dx &= \int \left( a(c + dx)^m + b \left( F^{eg+fgx} \right)^n (c + dx)^m \right) dx \\ &= \frac{a(c + dx)^{1+m}}{d(1+m)} + b \int \left( F^{eg+fgx} \right)^n (c + dx)^m dx \\ &= \frac{a(c + dx)^{1+m}}{d(1+m)} + \left( b F^{-n(eg+fgx)} \left( F^{eg+fgx} \right)^n \right) \int F^{n(eg+fgx)} (c + dx)^m dx \\ &= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{b F^{\left( e - \frac{cf}{d} \right) gn - gn(e+fx)} \left( F^{eg+fgx} \right)^n (c + dx)^m \Gamma \left( 1 + m, -\frac{fgn(c+dx) \log(F)}{d} \right)}{fgn \log(F)} \end{aligned}$$

**Mathematica** [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right) (c + dx)^m dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)\*(c + d\*x)^m,x]

[Out] Integrate[(a + b\*(F^(g\*(e + f\*x)))^n)\*(c + d\*x)^m, x]

**fricas** [A] time = 0.44, size = 111, normalized size = 0.96

$$\frac{(bdm + bd)e^{\left( \frac{(de-cf)gn \log(F) - dm \log\left( -\frac{fgn \log(F)}{d} \right)}{d} \right)} \Gamma \left( m + 1, -\frac{(dfgnx + cfgn) \log(F)}{d} \right) + (adfgnx + acfgn)(dx + c)^m \log(F)}{(dfgm + dfg)n \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e)))^n)\*(d\*x+c)^m,x, algorithm="fricas")

[Out] ((b\*d\*m + b\*d)\*e^(((d\*e - c\*f)\*g\*n\*log(F) - d\*m\*log(-f\*g\*n\*log(F)/d))/d)\*gamma(m + 1, -(d\*f\*g\*n\*x + c\*f\*g\*n)\*log(F)/d) + (a\*d\*f\*g\*n\*x + a\*c\*f\*g\*n)\*(d\*x + c)^m\*log(F))/((d\*f\*g\*m + d\*f\*g)\*n\*log(F))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( \left( F^{(f^{x+e})g} \right)^n b + a \right) (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)\*(d\*x+c)^m,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)\*(d\*x + c)^m, x)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(f^{x+e})g} \right)^n + a \right) (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)\*(d\*x+c)^m,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)\*(d\*x+c)^m,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(F^{eg})^n b \int e^{(m \log(dx+c) + n \log(F^{fgx}))} dx + \frac{(dx+c)^{m+1} a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)\*(d\*x+c)^m,x, algorithm="maxima")

[Out] (F^(e\*g))^n\*b\*integrate(e^(m\*log(d\*x + c) + n\*log(F^(f\*g\*x))), x) + (d\*x + c)^(m + 1)\*a/(d\*(m + 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + b \left( F^{g(e+f^x)} \right)^n \right) (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)\*(c + d\*x)^m,x)

[Out] int((a + b\*(F^(g\*(e + f\*x))))^n)\*(c + d\*x)^m, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c)**m,x)
```

```
[Out] Timed out
```

$$3.73 \quad \int \frac{(c+dx)^m}{a+b(Fg^{e+fx})^n} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{(c+dx)^m}{a+b(F^{eg+fgx})^n}, x \right)$$

[Out] Unintegrable((d\*x+c)^m/(a+b\*(F^(f\*g\*x+e\*g))^n), x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(c+dx)^m}{a+b(Fg^{e+fx})^n} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + b\*(F^(g\*(e + f\*x))))^n], x]

[Out] Defer[Int] [(c + d\*x)^m/(a + b\*(F^(e\*g + f\*g\*x))^n), x]

Rubi steps

$$\int \frac{(c+dx)^m}{a+b(Fg^{e+fx})^n} dx = \int \frac{(c+dx)^m}{a+b(F^{eg+fgx})^n} dx$$

Mathematica [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+b(Fg^{e+fx})^n} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + b\*(F^(g\*(e + f\*x))))^n], x]

[Out] Integrate[(c + d\*x)^m/(a + b\*(F^(g\*(e + f\*x))))^n], x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(dx+c)^m}{(F^{fgx+eg})^n b+a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*(F^(g\*(f\*x+e))))^n),x, algorithm="fricas")

[Out] integral((d\*x + c)^m/((F^(f\*g\*x + e\*g))^n\*b + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{\left(F^{(f^{x+e})g}\right)^n b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*(F^(g\*(f\*x+e))))^n),x, algorithm="giac")

[Out] integrate((d\*x + c)^m/((F^((f\*x + e)\*g))^n\*b + a), x)

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{b \left(F^{(f^{x+e})g}\right)^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(b\*(F^((f\*x+e)\*g))^n+a),x)

[Out] int((d\*x+c)^m/(b\*(F^((f\*x+e)\*g))^n+a),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{\left(F^{(f^{x+e})g}\right)^n b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*(F^(g\*(f\*x+e))))^n),x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/((F^((f\*x + e)\*g))^n\*b + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c + dx)^m}{a + b \left(F^{g(e+fx)}\right)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n),x)
```

```
[Out] int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*(F**(g*(f*x+e)))**n),x)
```

```
[Out] Timed out
```

$$3.74 \quad \int \frac{(c+dx)^m}{\left(a+b(F^g(e+fx))^n\right)^2} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{(c+dx)^m}{\left(a+b(F^{eg+fgx})^n\right)^2}, x \right)$$

[Out] Unintegrable((d\*x+c)^m/(a+b\*(F^(f\*g\*x+e\*g))^n)^2,x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(c+dx)^m}{\left(a+b(F^g(e+fx))^n\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + b\*(F^(g\*(e + f\*x))))^n]^2,x]

[Out] Defer[Int][(c + d\*x)^m/(a + b\*(F^(e\*g + f\*g\*x))^n)^2, x]

Rubi steps

$$\int \frac{(c+dx)^m}{\left(a+b(F^g(e+fx))^n\right)^2} dx = \int \frac{(c+dx)^m}{\left(a+b(F^{eg+fgx})^n\right)^2} dx$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{\left(a+b(F^g(e+fx))^n\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + b\*(F^(g\*(e + f\*x))))^n]^2,x]

[Out] Integrate[(c + d\*x)^m/(a + b\*(F^(g\*(e + f\*x))))^n]^2, x]



**fricas** [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(dx + c)^m}{2 (F^{fgx+eg})^n ab + (F^{fgx+eg})^{2n} b^2 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="fricas")

[Out] integral((d\*x + c)^m/(2\*(F^(f\*g\*x + e\*g))^n\*a\*b + (F^(f\*g\*x + e\*g))^(2\*n)\*b^2 + a^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{\left( \left( F^{(fx+e)g} \right)^n b + a \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^m/((F^((f\*x + e)\*g))^n\*b + a)^2, x)

**maple** [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{\left( b \left( F^{(fx+e)g} \right)^n + a \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(b\*(F^((f\*x+e)\*g))^n+a)^2,x)

[Out] int((d\*x+c)^m/(b\*(F^((f\*x+e)\*g))^n+a)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx + c)^m}{(F^{fgx})^n (F^{eg})^n abfgn \log(F) + a^2fgn \log(F)} + \int \frac{(dfgnx \log(F) + cfgn \log(F) - dm)(dx + c)^m}{a^2dfgnx \log(F) + a^2cfgn \log(F) + ((F^{eg})^n abdfgnx \log(F) + (F^{fgx})^n abdfgnx \log(F))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*(F^(g\*(f\*x+e))))^n)^2,x, algorithm="maxima")

```
[Out] (d*x + c)^m/((F^(f*g*x))^n*(F^(e*g))^n*a*b*f*g*n*log(F) + a^2*f*g*n*log(F))
+ integrate((d*f*g*n*x*log(F) + c*f*g*n*log(F) - d*m)*(d*x + c)^m/(a^2*d*f
*g*n*x*log(F) + a^2*c*f*g*n*log(F) + ((F^(e*g))^n*a*b*d*f*g*n*x*log(F) + (F
^(e*g))^n*a*b*c*f*g*n*log(F))*(F^(f*g*x))^n), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c + dx)^m}{\left(a + b \left(F^{g(e+fx)}\right)^n\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + b*(F^(g*(e + f*x))))^n)^2,x)
```

```
[Out] int((c + d*x)^m/(a + b*(F^(g*(e + f*x))))^n)^2, x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*(F**(g*(f*x+e))))**n)**2,x)
```

```
[Out] Timed out
```

$$3.75 \quad \int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^p (c + dx)^m dx$$

Optimal. Leaf size=29

$$\text{Int} \left( (c + dx)^m \left( a + b \left( F^{eg+fgx} \right)^n \right)^p, x \right)$$

[Out] Unintegrable((a+b\*(F^(f\*g\*x+e\*g)))^n)^p\*(d\*x+c)^m,x)

**Rubi** [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^p (c + dx)^m dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*(F^(g\*(e + f\*x))))^n]^p\*(c + d\*x)^m,x]

[Out] Defer[Int][(a + b\*(F^(e\*g + f\*g\*x)))^n]^p\*(c + d\*x)^m, x]

Rubi steps

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^p (c + dx)^m dx = \int \left( a + b \left( F^{eg+fgx} \right)^n \right)^p (c + dx)^m dx$$

**Mathematica** [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^p (c + dx)^m dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n]^p\*(c + d\*x)^m,x]

[Out] Integrate[(a + b\*(F^(g\*(e + f\*x))))^n]^p\*(c + d\*x)^m, x]

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left( \left( \left( F^{fgx+eg} \right)^n b + a \right)^p (dx + c)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^p\*(d\*x+c)^m,x, algorithm="fricas")

[Out] integral(((F^(f\*g\*x + e\*g))^n\*b + a)^p\*(d\*x + c)^m, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( \left( F^{(fx+e)g} \right)^n b + a \right)^p (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^p\*(d\*x+c)^m,x, algorithm="giac")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^p\*(d\*x + c)^m, x)

**maple** [A] time = 0.21, size = 0, normalized size = 0.00

$$\int \left( b \left( F^{(fx+e)g} \right)^n + a \right)^p (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*(F^((f\*x+e)\*g))^n+a)^p\*(d\*x+c)^m,x)

[Out] int((b\*(F^((f\*x+e)\*g))^n+a)^p\*(d\*x+c)^m,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( \left( F^{(fx+e)g} \right)^n b + a \right)^p (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F^(g\*(f\*x+e))))^n)^p\*(d\*x+c)^m,x, algorithm="maxima")

[Out] integrate(((F^((f\*x + e)\*g))^n\*b + a)^p\*(d\*x + c)^m, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^p (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*(F^(g\*(e + f\*x))))^n)^p\*(c + d\*x)^m,x)

[Out] int((a + b\*(F^(g\*(e + f\*x))))^n)^p\*(c + d\*x)^m, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*(F\*\*(g\*(f\*x+e)))\*\*n)\*\*p\*(d\*x+c)\*\*m,x)

[Out] Timed out

$$3.76 \quad \int \frac{F^{c+dx} x^3}{a+bF^{c+dx}} dx$$

**Optimal.** Leaf size=115

$$\frac{6\text{Li}_4\left(-\frac{bF^{c+dx}}{a}\right)}{bd^4 \log^4(F)} - \frac{6x\text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{3x^2\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} + \frac{x^3 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

[Out]  $x^3 \ln(1+bF^{(d*x+c)}/a)/b/d/\ln(F)+3*x^2*\text{polylog}(2,-bF^{(d*x+c)}/a)/b/d^2/\ln(F)^2-6*x*\text{polylog}(3,-bF^{(d*x+c)}/a)/b/d^3/\ln(F)^3+6*\text{polylog}(4,-bF^{(d*x+c)}/a)/b/d^4/\ln(F)^4$

**Rubi [A]** time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{6x\text{PolyLog}\left(3, -\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{6\text{PolyLog}\left(4, -\frac{bF^{c+dx}}{a}\right)}{bd^4 \log^4(F)} + \frac{x^3 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] `Int[(F^(c + d*x)*x^3)/(a + bF^(c + d*x)),x]`

[Out]  $(x^3*\text{Log}[1 + (bF^{(c + d*x)})/a])/(b*d*\text{Log}[F]) + (3*x^2*\text{PolyLog}[2, -((bF^{(c + d*x)})/a)])/(b*d^2*\text{Log}[F]^2) - (6*x*\text{PolyLog}[3, -((bF^{(c + d*x)})/a)])/(b*d^3*\text{Log}[F]^3) + (6*\text{PolyLog}[4, -((bF^{(c + d*x)})/a)])/(b*d^4*\text{Log}[F]^4)$

#### Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*(f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{F^{c+dx} x^3}{a + bF^{c+dx}} dx &= \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} - \frac{3 \int x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right) dx}{bd \log(F)} \\
&= \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} + \frac{3x^2 \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{6 \int x \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right) dx}{bd^2 \log^2(F)} \\
&= \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} + \frac{3x^2 \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{6x \text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{6 \int \text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right) dx}{bd^3 \log^3(F)} \\
&= \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} + \frac{3x^2 \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{6x \text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{6 \text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{bx}{a}\right)}{x} dx, x, F^{c+dx}\right)}{bd^4 \log^4(F)} \\
&= \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} + \frac{3x^2 \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{6x \text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{6 \text{Li}_4\left(-\frac{bF^{c+dx}}{a}\right)}{bd^4 \log^4(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 115, normalized size = 1.00

$$\frac{6\text{Li}_4\left(-\frac{bF^{c+dx}}{a}\right)}{bd^4 \log^4(F)} - \frac{6x\text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{3x^2\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} + \frac{x^3 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c + d\*x)\*x^3)/(a + b\*F^(c + d\*x)), x]

[Out] (x^3\*Log[1 + (b\*F^(c + d\*x))/a])/(b\*d\*Log[F]) + (3\*x^2\*PolyLog[2, -((b\*F^(c + d\*x))/a)])/(b\*d^2\*Log[F]^2) - (6\*x\*PolyLog[3, -((b\*F^(c + d\*x))/a)])/(b\*d^3\*Log[F]^3) + (6\*PolyLog[4, -((b\*F^(c + d\*x))/a)])/(b\*d^4\*Log[F]^4)

**fricas [C]** time = 0.44, size = 134, normalized size = 1.17

$$\frac{3d^2x^2\text{Li}_2\left(-\frac{F^{dx+cb+a}}{a} + 1\right)\log(F)^2 - c^3\log(F^{dx+cb+a})\log(F)^3 + (d^3x^3 + c^3)\log(F)^3\log\left(\frac{F^{dx+cb+a}}{a}\right) - 6dx\log(F)}{bd^4\log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^3/(a+b\*F^(d\*x+c)), x, algorithm="fricas")

[Out] (3\*d^2\*x^2\*dilog(-F^(d\*x + c)\*b + a)/a + 1)\*log(F)^2 - c^3\*log(F^(d\*x + c)\*b + a)\*log(F)^3 + (d^3\*x^3 + c^3)\*log(F)^3\*log((F^(d\*x + c)\*b + a)/a) - 6\*d\*x\*log(F)\*polylog(3, -F^(d\*x + c)\*b/a) + 6\*polylog(4, -F^(d\*x + c)\*b/a))/(b\*d^4\*log(F)^4)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}x^3}{F^{dx+c}b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^3/(a+b\*F^(d\*x+c)), x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x^3/(F^(d\*x + c)\*b + a), x)

**maple [A]** time = 0.07, size = 225, normalized size = 1.96

$$\frac{x^3 \ln\left(\frac{bF^cF^{dx}}{a} + 1\right)}{bd \ln(F)} - \frac{c^3x}{bd^3} - \frac{3c^4}{4bd^4} + \frac{c^3 \ln(F^cF^{dx})}{bd^4 \ln(F)} + \frac{c^3 \ln\left(\frac{bF^cF^{dx}}{a} + 1\right)}{bd^4 \ln(F)} - \frac{c^3 \ln(bF^cF^{dx} + a)}{bd^4 \ln(F)} + \frac{3x^2 \text{polylog}\left(2, -\frac{bF^cF^{dx}}{a}\right)}{bd^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x)`

[Out] 
$$-1/d^3/b*c^3*x-3/4/d^4/b*c^4+1/d/\ln(F)/b*\ln(1+b*F^(d*x)*F^c/a)*x^3+1/d^4/\ln(F)/b*\ln(1+b*F^(d*x)*F^c/a)*c^3+3/d^2/\ln(F)^2/b*polylog(2,-b*F^(d*x)*F^c/a)*x^2-6/d^3/\ln(F)^3/b*polylog(3,-b*F^(d*x)*F^c/a)*x+6/d^4/\ln(F)^4/b*polylog(4,-b*F^(d*x)*F^c/a)+1/d^4/\ln(F)/b*c^3*\ln(F^(d*x)*F^c)-1/d^4/\ln(F)/b*c^3*\ln(a+F^c*F^(d*x)*b)$$

**maxima** [A] time = 0.48, size = 133, normalized size = 1.16

$$\frac{x^4}{4b} - \frac{\log(F^{dx})^4}{4bd^4 \log(F)^4} + \frac{\log\left(\frac{F^{dx}F^cb}{a} + 1\right) \log(F^{dx})^3 + 3 \operatorname{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right) \log(F^{dx})^2 - 6 \log(F^{dx}) \operatorname{Li}_3\left(-\frac{F^{dx}F^cb}{a}\right) + 6 \operatorname{Li}_4\left(-\frac{F^{dx}F^cb}{a}\right)}{bd^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x, algorithm="maxima")`

[Out] 
$$1/4*x^4/b - 1/4*\log(F^(d*x))^4/(b*d^4*\log(F)^4) + (\log(F^(d*x)*F^c*b/a + 1) * \log(F^(d*x))^3 + 3*\operatorname{dilog}(-F^(d*x)*F^c*b/a)*\log(F^(d*x))^2 - 6*\log(F^(d*x)) * \operatorname{polylog}(3, -F^(d*x)*F^c*b/a) + 6*\operatorname{polylog}(4, -F^(d*x)*F^c*b/a))/b*d^4*\log(F)^4$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c+dx} x^3}{a + F^{c+dx} b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b),x)`

[Out] `int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx} x^3}{F^c F^{dx} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c)),x)`

[Out] `Integral(F**(c + d*x)*x**3/(F**c*F**(d*x)*b + a), x)`

$$3.77 \quad \int \frac{F^{c+dx} x^2}{a+bF^{c+dx}} dx$$

**Optimal.** Leaf size=85

$$-\frac{2\text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{2x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} + \frac{x^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

[Out]  $x^2 \ln(1+bF^{(d*x+c)}/a)/b/d/\ln(F)+2*x*\text{polylog}(2,-bF^{(d*x+c)}/a)/b/d^2/\ln(F)^2-2*\text{polylog}(3,-bF^{(d*x+c)}/a)/b/d^3/\ln(F)^3$

**Rubi [A]** time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2190, 2531, 2282, 6589}

$$\frac{2x\text{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{2\text{PolyLog}\left(3, -\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{x^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] `Int[(F^(c + d*x)*x^2)/(a + bF^(c + d*x)),x]`

[Out]  $(x^2*\text{Log}[1 + (bF^{(c + d*x)}/a)]/(b*d*\text{Log}[F]) + (2*x*\text{PolyLog}[2, -((bF^{(c + d*x)}/a)])/(b*d^2*\text{Log}[F]^2) - (2*\text{PolyLog}[3, -((bF^{(c + d*x)}/a)])/(b*d^3*\text{Log}[F]^3)$

Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*(f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int \frac{F^{c+dx} x^2}{a + bF^{c+dx}} dx &= \frac{x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} - \frac{2 \int x \log\left(1 + \frac{bF^{c+dx}}{a}\right) dx}{bd \log(F)} \\ &= \frac{x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} + \frac{2x \operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{2 \int \operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right) dx}{bd^2 \log^2(F)} \\ &= \frac{x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} + \frac{2x \operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{2 \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{bx}{a}\right)}{x} dx, x, F^{c+dx}\right)}{bd^3 \log^3(F)} \\ &= \frac{x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} + \frac{2x \operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} - \frac{2 \operatorname{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 85, normalized size = 1.00

$$-\frac{2 \operatorname{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3 \log^3(F)} + \frac{2x \operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} + \frac{x^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x)),x]
```

```
[Out] (x^2*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + (2*x*PolyLog[2, -((b*F^(c +
d*x))/a)])/(b*d^2*Log[F]^2) - (2*PolyLog[3, -((b*F^(c + d*x))/a)])/(b*d^3*
Log[F]^3)
```

**fricas** [C] time = 0.43, size = 108, normalized size = 1.27

$$\frac{c^2 \log(F^{dx+c}b+a) \log(F)^2 + 2 dx \operatorname{Li}_2\left(-\frac{F^{dx+c}b+a}{a}\right) \log(F) + (d^2 x^2 - c^2) \log(F)^2 \log\left(\frac{F^{dx+c}b+a}{a}\right) - 2 \operatorname{polylog}\left(3, -\frac{F^{dx+c}b+a}{a}\right)}{bd^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c)),x, algorithm="fricas")

[Out] (c^2\*log(F^(d\*x + c)\*b + a)\*log(F)^2 + 2\*d\*x\*dilog(-(F^(d\*x + c)\*b + a)/a + 1)\*log(F) + (d^2\*x^2 - c^2)\*log(F)^2\*log((F^(d\*x + c)\*b + a)/a) - 2\*polylog(3, -F^(d\*x + c)\*b/a))/(b\*d^3\*log(F)^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c} x^2}{F^{dx+c} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c)),x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x^2/(F^(d\*x + c)\*b + a), x)

**maple** [B] time = 0.06, size = 194, normalized size = 2.28

$$\frac{c^2 x}{b d^2} + \frac{x^2 \ln\left(\frac{b F^c F^{dx}}{a} + 1\right)}{b d \ln(F)} + \frac{2 c^3}{3 b d^3} - \frac{c^2 \ln(F^c F^{dx})}{b d^3 \ln(F)} - \frac{c^2 \ln\left(\frac{b F^c F^{dx}}{a} + 1\right)}{b d^3 \ln(F)} + \frac{c^2 \ln(b F^c F^{dx} + a)}{b d^3 \ln(F)} + \frac{2 x \operatorname{polylog}\left(2, -\frac{b F^c F^{dx}}{a}\right)}{b d^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c)),x)

[Out] 1/d^2/b\*c^2\*x+2/3/d^3/b\*c^3+1/d/ln(F)/b\*ln(1/a\*b\*F^c\*F^(d\*x)+1)\*x^2-1/d^3/ln(F)/b\*ln(1/a\*b\*F^c\*F^(d\*x)+1)\*c^2+2/d^2/ln(F)^2/b\*polylog(2,-1/a\*b\*F^c\*F^(d\*x))\*x-2/d^3/ln(F)^3/b\*polylog(3,-1/a\*b\*F^c\*F^(d\*x))-1/d^3/ln(F)/b\*c^2\*ln(F^c\*F^(d\*x))+1/d^3/ln(F)/b\*c^2\*ln(b\*F^c\*F^(d\*x)+a)

**maxima** [A] time = 0.49, size = 107, normalized size = 1.26

$$\frac{x^3}{3b} - \frac{\log(F^{dx})^3}{3bd^3 \log(F)^3} + \frac{\log\left(\frac{F^{dx} F^c b}{a} + 1\right) \log(F^{dx})^2 + 2 \operatorname{Li}_2\left(-\frac{F^{dx} F^c b}{a}\right) \log(F^{dx}) - 2 \operatorname{Li}_3\left(-\frac{F^{dx} F^c b}{a}\right)}{bd^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{3}x^3/b - \frac{1}{3}\log(F^{(d*x)})^3/(b*d^3*\log(F)^3) + (\log(F^{(d*x)})*F^{c*b/a} + 1) * \log(F^{(d*x)})^2 + 2*\operatorname{dilog}(-F^{(d*x)}*F^{c*b/a})*\log(F^{(d*x)}) - 2*\operatorname{polylog}(3, -F^{(d*x)}*F^{c*b/a})/(b*d^3*\log(F)^3)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c+dx} x^2}{a + F^{c+dx} b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(c + d\*x)\*x^2)/(a + F^(c + d\*x)\*b),x)

[Out] int((F^(c + d\*x)\*x^2)/(a + F^(c + d\*x)\*b), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx} x^2}{F^c F^{dx} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)\*x\*\*2/(a+b\*F\*\*(d\*x+c)),x)

[Out] Integral(F\*\*(c + d\*x)\*x\*\*2/(F\*\*c\*F\*\*(d\*x)\*b + a), x)

$$3.78 \quad \int \frac{F^{c+dx} x}{a+bF^{c+dx}} dx$$

**Optimal.** Leaf size=54

$$\frac{\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} + \frac{x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

[Out]  $x \ln(1+bF^{(d*x+c)/a})/b/d/\ln(F)+\text{polylog}(2,-bF^{(d*x+c)/a})/b/d^2/\ln(F)^2$

**Rubi [A]** time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} + \frac{x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(F^(c + d\*x)\*x)/(a + b\*F^(c + d\*x)), x]

[Out] (x\*Log[1 + (b\*F^(c + d\*x))/a])/(b\*d\*Log[F]) + PolyLog[2, -((b\*F^(c + d\*x))/a)]/(b\*d^2\*Log[F]^2)

Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{F^{c+dx} x}{a + bF^{c+dx}} dx &= \frac{x \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} - \frac{\int \log\left(1 + \frac{bF^{c+dx}}{a}\right) dx}{bd \log(F)} \\
&= \frac{x \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} - \frac{\text{Subst}\left(\int \frac{\log\left(1 + \frac{bx}{a}\right)}{x} dx, x, F^{c+dx}\right)}{bd^2 \log^2(F)} \\
&= \frac{x \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd \log(F)} + \frac{\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2 \log^2(F)} + \frac{x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c + d\*x)\*x)/(a + b\*F^(c + d\*x)),x]

[Out] (x\*Log[1 + (b\*F^(c + d\*x))/a])/(b\*d\*Log[F]) + PolyLog[2, -((b\*F^(c + d\*x))/a)]/(b\*d^2\*Log[F]^2)

**fricas** [A] time = 0.43, size = 75, normalized size = 1.39

$$\frac{c \log(F^{dx+c} b + a) \log(F) - (dx + c) \log(F) \log\left(\frac{F^{dx+c} b + a}{a}\right) - \text{Li}_2\left(-\frac{F^{dx+c} b + a}{a}\right)}{bd^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c)),x, algorithm="fricas")

[Out] -(c\*log(F^(d\*x + c)\*b + a)\*log(F) - (d\*x + c)\*log(F)\*log((F^(d\*x + c)\*b + a)/a) - dilog(-(F^(d\*x + c)\*b + a)/a + 1))/(b\*d^2\*log(F)^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c} x}{F^{dx+c} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c)),x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x/(F^(d\*x + c)\*b + a), x)

**maple** [B] time = 0.06, size = 154, normalized size = 2.85

$$-\frac{cx}{bd} - \frac{c^2}{2bd^2} + \frac{x \ln\left(\frac{bF^c F^{dx}}{a} + 1\right)}{bd \ln(F)} + \frac{c \ln(F^c F^{dx})}{bd^2 \ln(F)} + \frac{c \ln\left(\frac{bF^c F^{dx}}{a} + 1\right)}{bd^2 \ln(F)} - \frac{c \ln(bF^c F^{dx} + a)}{bd^2 \ln(F)} + \frac{\text{polylog}\left(2, -\frac{bF^c F^{dx}}{a}\right)}{bd^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c)),x)

[Out] -1/d/b\*c\*x-1/2\*c^2/d^2/b+1/d/ln(F)/b\*ln(1/a\*b\*F^c\*F^(d\*x)+1)\*x+1/d^2/ln(F)/b\*ln(1/a\*b\*F^c\*F^(d\*x)+1)\*c+1/d^2/ln(F)^2/b\*polylog(2,-1/a\*b\*F^c\*F^(d\*x))+1/d^2/ln(F)/b\*c\*ln(F^c\*F^(d\*x))-1/d^2/ln(F)/b\*c\*ln(b\*F^c\*F^(d\*x)+a)

**maxima** [A] time = 0.48, size = 79, normalized size = 1.46

$$\frac{x^2}{2b} - \frac{\log(F^{dx})^2}{2bd^2 \log(F)^2} + \frac{\log\left(\frac{F^{dx} F^c b}{a} + 1\right) \log(F^{dx}) + \text{Li}_2\left(-\frac{F^{dx} F^c b}{a}\right)}{bd^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*x^2/b - 1/2\*log(F^(d\*x))^2/(b\*d^2\*log(F)^2) + (log(F^(d\*x))\*F^c\*b/a + 1)\*log(F^(d\*x)) + dilog(-F^(d\*x)\*F^c\*b/a)/(b\*d^2\*log(F)^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c+dx} x}{a + F^{c+dx} b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(c + d\*x)\*x)/(a + F^(c + d\*x)\*b),x)

[Out] int((F^(c + d\*x)\*x)/(a + F^(c + d\*x)\*b), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx} x}{F^c F^{dx} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)\*x/(a+b\*F\*\*(d\*x+c)),x)

[Out] Integral(F\*\*(c + d\*x)\*x/(F\*\*c\*F\*\*(d\*x)\*b + a), x)



$$3.79 \quad \int \frac{F^{c+dx}}{a+bF^{c+dx}} dx$$

Optimal. Leaf size=23

$$\frac{\log(a + bF^{c+dx})}{bd \log(F)}$$

[Out]  $\ln(a+bF^{(d*x+c)})/b/d/\ln(F)$

**Rubi [A]** time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2246, 31}

$$\frac{\log(a + bF^{c+dx})}{bd \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c + d*x)}/(a + bF^{(c + d*x)}), x]$

[Out]  $\text{Log}[a + bF^{(c + d*x)}]/(b*d*\text{Log}[F])$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 2246

$\text{Int}[(F_)^{(e_)*((c_ + (d_)*(x_)))^{(n_)*((a_ + (b_)*(F_)^{(e_)*((c_ + (d_)*(x_)))^{(n_))^{(p_))}}), x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[(a + b*x)^p, x], x, (F^{(e*(c + d*x)))^n], x] \text{ ; FreeQ}\{F, a, b, c, d, e, n, p\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{F^{c+dx}}{a + bF^{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx} dx, x, F^{c+dx}\right)}{d \log(F)} \\ &= \frac{\log(a + bF^{c+dx})}{bd \log(F)} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log(a + bF^{c+dx})}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c + d\*x)/(a + b\*F^(c + d\*x)),x]

[Out] Log[a + b\*F^(c + d\*x)]/(b\*d\*Log[F])

**fricas** [A] time = 0.42, size = 23, normalized size = 1.00

$$\frac{\log(F^{dx+c}b + a)}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c)),x, algorithm="fricas")

[Out] log(F^(d\*x + c)\*b + a)/(b\*d\*log(F))

**giac** [A] time = 0.42, size = 24, normalized size = 1.04

$$\frac{\log(|F^{dx+c}b + a|)}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c)),x, algorithm="giac")

[Out] log(abs(F^(d\*x + c)\*b + a))/(b\*d\*log(F))

**maple** [A] time = 0.00, size = 24, normalized size = 1.04

$$\frac{\ln(bF^{dx+c} + a)}{bd \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(a+b\*F^(d\*x+c)),x)

[Out] ln(a+b\*F^(d\*x+c))/b/d/ln(F)

**maxima** [A] time = 0.43, size = 23, normalized size = 1.00

$$\frac{\log(F^{dx+c}b + a)}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c)),x, algorithm="maxima")

[Out] log(F^(d\*x + c)\*b + a)/(b\*d\*log(F))

mupad [B] time = 3.43, size = 23, normalized size = 1.00

$$\frac{\ln(a + F^{c+dx} b)}{b d \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(a + F^(c + d\*x)\*b),x)

[Out] log(a + F^(c + d\*x)\*b)/(b\*d\*log(F))

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\frac{\log\left(F^{c+dx} + \frac{a}{b}\right)}{b d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)/(a+b\*F\*\*(d\*x+c)),x)

[Out] log(F\*\*(c + d\*x) + a/b)/(b\*d\*log(F))

$$3.80 \quad \int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx$$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{F^{c+dx}}{x(a+bF^{c+dx})}, x\right)$$

[Out] Unintegrable(F^(d\*x+c)/(a+b\*F^(d\*x+c))/x,x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx$$

Verification is Not applicable to the result.

[In] Int[F^(c + d\*x)/((a + b\*F^(c + d\*x))\*x), x]

[Out] Defer[Int][F^(c + d\*x)/((a + b\*F^(c + d\*x))\*x), x]

Rubi steps

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx$$

Mathematica [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(c + d\*x)/((a + b\*F^(c + d\*x))\*x), x]

[Out] Integrate[F^(c + d\*x)/((a + b\*F^(c + d\*x))\*x), x]

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{F^{dx+c}}{F^{dx+c}bx + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))/x,x, algorithm="fricas")

[Out] integral(F^(d\*x + c)/(F^(d\*x + c)\*b\*x + a\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(F^{dx+c}b + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))/x,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)/((F^(d\*x + c)\*b + a)\*x), x)

**maple** [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(bF^{dx+c} + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)/x,x)

[Out] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-a \int \frac{1}{F^{dx}F^c b^2 x + abx} dx + \frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))/x,x, algorithm="maxima")

[Out] -a\*integrate(1/(F^(d\*x)\*F^c\*b^2\*x + a\*b\*x), x) + log(x)/b

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F^{c+dx}}{x(a + F^{c+dx}b)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(x\*(a + F^(c + d\*x)\*b)),x)

[Out] `int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx}}{x(F^c F^{dx} b + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(d*x+c)/(a+b*F**(d*x+c))/x,x)`

[Out] `Integral(F**(c + d*x)/(x*(F**c*F**(d*x)*b + a)), x)`

$$3.81 \quad \int \frac{F^{c+dx}}{(a+bF^{c+dx})x^2} dx$$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{F^{c+dx}}{x^2(a+bF^{c+dx})}, x\right)$$

[Out] Unintegrable(F^(d\*x+c)/(a+b\*F^(d\*x+c))/x^2, x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})x^2} dx$$

Verification is Not applicable to the result.

[In] Int[F^(c + d\*x)/((a + b\*F^(c + d\*x))\*x^2), x]

[Out] Defer[Int][F^(c + d\*x)/((a + b\*F^(c + d\*x))\*x^2), x]

Rubi steps

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})x^2} dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})x^2} dx$$

Mathematica [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(c + d\*x)/((a + b\*F^(c + d\*x))\*x^2), x]

[Out] Integrate[F^(c + d\*x)/((a + b\*F^(c + d\*x))\*x^2), x]

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{F^{dx+c}}{F^{dx+c}bx^2 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))/x^2,x, algorithm="fricas")

[Out] integral(F^(d\*x + c)/(F^(d\*x + c)\*b\*x^2 + a\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(F^{dx+c}b + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))/x^2,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)/((F^(d\*x + c)\*b + a)\*x^2), x)

**maple** [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(bF^{dx+c} + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)/x^2,x)

[Out] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-a \int \frac{1}{F^{dx}F^c b^2 x^2 + abx^2} dx - \frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))/x^2,x, algorithm="maxima")

[Out] -a\*integrate(1/(F^(d\*x)\*F^c\*b^2\*x^2 + a\*b\*x^2), x) - 1/(b\*x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F^{c+dx}}{x^2 (a + F^{c+dx} b)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(x^2\*(a + F^(c + d\*x)\*b)),x)



[Out] `int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx}}{x^2 (F^c F^{dx} b + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(d*x+c)/(a+b*F**(d*x+c))/x**2,x)`

[Out] `Integral(F**(c + d*x)/(x**2*(F**c*F**(d*x)*b + a)), x)`

$$3.82 \quad \int \frac{F^{c+dx} x^3}{(a+bF^{c+dx})^2} dx$$

**Optimal.** Leaf size=140

$$\frac{6\text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{abd^4 \log^4(F)} - \frac{6x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F)} - \frac{3x^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{abd^2 \log^2(F)} - \frac{x^3}{bd \log(F)(a + bF^{c+dx})} + \frac{x^3}{abd \log(F)}$$

[Out]  $x^3/a/b/d/\ln(F) - x^3/b/d/(a+bF^{(d*x+c)})/\ln(F) - 3*x^2*\ln(1+bF^{(d*x+c)}/a)/a/b/d^2/\ln(F)^2 - 6*x*\text{polylog}(2, -bF^{(d*x+c)}/a)/a/b/d^3/\ln(F)^3 + 6*\text{polylog}(3, -bF^{(d*x+c)}/a)/a/b/d^4/\ln(F)^4$

**Rubi [A]** time = 0.25, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2191, 2184, 2190, 2531, 2282, 6589}

$$-\frac{6x\text{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F)} + \frac{6\text{PolyLog}\left(3, -\frac{bF^{c+dx}}{a}\right)}{abd^4 \log^4(F)} - \frac{3x^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{abd^2 \log^2(F)} - \frac{x^3}{bd \log(F)(a + bF^{c+dx})} + \frac{x^3}{abd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(F^(c + d\*x)\*x^3)/(a + bF^(c + d\*x))^2, x]

[Out]  $x^3/(a*b*d*\text{Log}[F]) - x^3/(b*d*(a + bF^{(c + d*x)})*\text{Log}[F]) - (3*x^2*\text{Log}[1 + (bF^{(c + d*x)})/a])/(a*b*d^2*\text{Log}[F]^2) - (6*x*\text{PolyLog}[2, -((bF^{(c + d*x)})/a)])/(a*b*d^3*\text{Log}[F]^3) + (6*\text{PolyLog}[3, -((bF^{(c + d*x)})/a)])/(a*b*d^4*\text{Log}[F]^4)$

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2191

```
Int[((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))^(p_)*((c_) + (d_)*(x_))^(m_), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Log[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a + b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{c+dx} x^3}{(a + bF^{c+dx})^2} dx &= -\frac{x^3}{bd(a + bF^{c+dx}) \log(F)} + \frac{3 \int \frac{x^2}{a+bF^{c+dx}} dx}{bd \log(F)} \\
&= \frac{x^3}{abd \log(F)} - \frac{x^3}{bd(a + bF^{c+dx}) \log(F)} - \frac{3 \int \frac{F^{c+dx} x^2}{a+bF^{c+dx}} dx}{ad \log(F)} \\
&= \frac{x^3}{abd \log(F)} - \frac{x^3}{bd(a + bF^{c+dx}) \log(F)} - \frac{3x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{abd^2 \log^2(F)} + \frac{6 \int x \log\left(1 + \frac{bF^{c+dx}}{a}\right) dx}{abd^2 \log^2(F)} \\
&= \frac{x^3}{abd \log(F)} - \frac{x^3}{bd(a + bF^{c+dx}) \log(F)} - \frac{3x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{abd^2 \log^2(F)} - \frac{6x \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F)} + \frac{6 \int \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right) dx}{abd^3 \log^3(F)} \\
&= \frac{x^3}{abd \log(F)} - \frac{x^3}{bd(a + bF^{c+dx}) \log(F)} - \frac{3x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{abd^2 \log^2(F)} - \frac{6x \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F)} + \frac{6 \text{Subst}\left(\int \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right) dx\right)}{abd^3 \log^3(F)} \\
&= \frac{x^3}{abd \log(F)} - \frac{x^3}{bd(a + bF^{c+dx}) \log(F)} - \frac{3x^2 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{abd^2 \log^2(F)} - \frac{6x \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F)} + \frac{6 \text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{abd^4 \log^4(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 137, normalized size = 0.98

$$\frac{3 \left( \frac{2 \text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{ad^3 \log^3(F)} - \frac{2x \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{ad^2 \log^2(F)} - \frac{x^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{ad \log(F)} + \frac{x^3}{3a} \right)}{bd \log(F)} - \frac{x^3}{bd \log(F) (a + bF^{c+dx})}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c + d\*x)\*x^3)/(a + b\*F^(c + d\*x))^2,x]

[Out] -(x^3/(b\*d\*(a + b\*F^(c + d\*x))\*Log[F])) + (3\*(x^3/(3\*a) - (x^2\*Log[1 + (b\*F^(c + d\*x))/a])/(a\*d\*Log[F]) - (2\*x\*PolyLog[2, -((b\*F^(c + d\*x))/a)])/(a\*d^2\*Log[F]^2) + (2\*PolyLog[3, -((b\*F^(c + d\*x))/a)])/(a\*d^3\*Log[F]^3)))/(b\*d\*Log[F])

**fricas [C]** time = 0.42, size = 246, normalized size = 1.76

$$\frac{ac^3 \log(F)^3 + (bd^3 x^3 + bc^3) F^{dx+c} \log(F)^3 - 6(F^{dx+c} bdx \log(F) + adx \log(F)) \text{Li}_2\left(-\frac{F^{dx+c} b+a}{a} + 1\right) - 3(F^{dx+c} bc^2 \log(F) + \dots)}{F}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^3/(a+b\*F^(d\*x+c))^2,x, algorithm="fricas")

[Out] (a\*c^3\*log(F)^3 + (b\*d^3\*x^3 + b\*c^3)\*F^(d\*x + c)\*log(F)^3 - 6\*(F^(d\*x + c) \*b\*d\*x\*log(F) + a\*d\*x\*log(F))\*dilog(-(F^(d\*x + c)\*b + a)/a + 1) - 3\*(F^(d\*x + c)\*b\*c^2\*log(F)^2 + a\*c^2\*log(F)^2)\*log(F^(d\*x + c)\*b + a) - 3\*((b\*d^2\*x^2 - b\*c^2)\*F^(d\*x + c)\*log(F)^2 + (a\*d^2\*x^2 - a\*c^2)\*log(F)^2)\*log((F^(d\*x + c)\*b + a)/a) + 6\*(F^(d\*x + c)\*b + a)\*polylog(3, -F^(d\*x + c)\*b/a)/(F^(d\*x + c)\*a\*b^2\*d^4\*log(F)^4 + a^2\*b\*d^4\*log(F)^4)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c} x^3}{(F^{dx+c} b + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^3/(a+b\*F^(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x^3/(F^(d\*x + c)\*b + a)^2, x)

**maple** [A] time = 0.05, size = 274, normalized size = 1.96

$$-\frac{x^3}{(b F^{dx+c} + a) b d \ln(F)} + \frac{x^3}{a b d \ln(F)} - \frac{3c^2 x}{a b d^3 \ln(F)} - \frac{3x^2 \ln\left(\frac{b F^c F^{dx}}{a} + 1\right)}{a b d^2 \ln(F)^2} - \frac{2c^3}{a b d^4 \ln(F)} + \frac{3c^2 \ln(F^c F^{dx})}{a b d^4 \ln(F)^2} + \frac{3c^2 \ln\left(\frac{b F^c F^{dx}}{a}\right)}{a b d^4 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)\*x^3/(b\*F^(d\*x+c)+a)^2,x)

[Out] -x^3/b/d/(b\*F^(d\*x+c)+a)/ln(F)+x^3/a/b/d/ln(F)-3/d^3/ln(F)/b/a\*c^2\*x-2/d^4/ln(F)/b/a\*c^3-3/d^2/ln(F)^2/b/a\*ln(1/a\*b\*F^c\*F^(d\*x)+1)\*x^2+3/d^4/ln(F)^2/b/a\*ln(1/a\*b\*F^c\*F^(d\*x)+1)\*c^2-6/d^3/ln(F)^3/b/a\*polylog(2,-1/a\*b\*F^c\*F^(d\*x))\*x+6/d^4/ln(F)^4/b/a\*polylog(3,-1/a\*b\*F^c\*F^(d\*x))+3/d^4/ln(F)^2/b\*c^2/a\*ln(F^c\*F^(d\*x))-3/d^4/ln(F)^2/b\*c^2/a\*ln(b\*F^c\*F^(d\*x)+a)

**maxima** [A] time = 0.52, size = 134, normalized size = 0.96

$$-\frac{x^3}{F^{dx} F^c b^2 d \log(F) + a b d \log(F)} + \frac{\log(F^{dx})^3}{a b d^4 \log(F)^4} - \frac{3 \left( \log\left(\frac{F^{dx} F^c b}{a} + 1\right) \log(F^{dx})^2 + 2 \operatorname{Li}_2\left(-\frac{F^{dx} F^c b}{a}\right) \log(F^{dx}) - 2 \operatorname{Li}_3\left(-\frac{F^{dx} F^c b}{a}\right) \right)}{a b d^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^3/(a+b\*F^(d\*x+c))^2,x, algorithm="maxima")

[Out]  $-x^3/(F^{(d*x)}*F^c*b^2*d*\log(F) + a*b*d*\log(F)) + \log(F^{(d*x)})^3/(a*b*d^4*\log(F)^4) - 3*(\log(F^{(d*x)})*F^c*b/a + 1)*\log(F^{(d*x)})^2 + 2*\operatorname{dilog}(-F^{(d*x)}*F^c*b/a)*\log(F^{(d*x)}) - 2*\operatorname{polylog}(3, -F^{(d*x)}*F^c*b/a))/(a*b*d^4*\log(F)^4)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c+dx} x^3}{(a + F^{c+dx} b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^2, x)`

[Out] `int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^3}{F^{c+dx} b^2 d \log(F) + a b d \log(F)} + \frac{3 \int \frac{x^2}{a + b e^{c \log(F)} e^{dx \log(F)}} dx}{b d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c))**2, x)`

[Out]  $-x^3/(F^{(c + d*x)}*b^2*d*\log(F) + a*b*d*\log(F)) + 3*\operatorname{Integral}(x^2/(a + b*\exp(c*\log(F))*\exp(d*x*\log(F))), x)/(b*d*\log(F))$

$$3.83 \quad \int \frac{F^{c+dx} x^2}{(a+bF^{c+dx})^2} dx$$

**Optimal.** Leaf size=107

$$-\frac{2\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F)} - \frac{2x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{abd^2 \log^2(F)} - \frac{x^2}{bd \log(F)(a + bF^{c+dx})} + \frac{x^2}{abd \log(F)}$$

[Out]  $x^2/a/b/d/\ln(F) - x^2/b/d/(a+bF^{(d*x+c)})/\ln(F) - 2*x*\ln(1+bF^{(d*x+c)}/a)/a/b/d^2/\ln(F)^2 - 2*polylog(2, -bF^{(d*x+c)}/a)/a/b/d^3/\ln(F)^3$

**Rubi [A]** time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {2191, 2184, 2190, 2279, 2391}

$$-\frac{2\text{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F)} - \frac{2x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{abd^2 \log^2(F)} - \frac{x^2}{bd \log(F)(a + bF^{c+dx})} + \frac{x^2}{abd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(F^(c + d\*x)\*x^2)/(a + bF^(c + d\*x))^2, x]

[Out]  $x^2/(a*b*d*\text{Log}[F]) - x^2/(b*d*(a + bF^{(c + d*x)})*\text{Log}[F]) - (2*x*\text{Log}[1 + (bF^{(c + d*x)})/a])/(a*b*d^2*\text{Log}[F]^2) - (2*\text{PolyLog}[2, -((bF^{(c + d*x)})/a)])/(a*b*d^3*\text{Log}[F]^3)$

Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2191

```

Int[((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))*((a_) + (b_)*((F_)^((g_)*
(e_) + (f_)*(x_)))^(n_))^(p_)*((c_) + (d_)*(x_))^(m_), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Log
[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]

```

### Rule 2279

```

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{F^{c+dx} x^2}{(a + bF^{c+dx})^2} dx &= -\frac{x^2}{bd(a + bF^{c+dx}) \log(F)} + \frac{2 \int \frac{x}{a + bF^{c+dx}} dx}{bd \log(F)} \\
&= \frac{x^2}{abd \log(F)} - \frac{x^2}{bd(a + bF^{c+dx}) \log(F)} - \frac{2 \int \frac{F^{c+dx} x}{a + bF^{c+dx}} dx}{ad \log(F)} \\
&= \frac{x^2}{abd \log(F)} - \frac{x^2}{bd(a + bF^{c+dx}) \log(F)} - \frac{2x \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{abd^2 \log^2(F)} + \frac{2 \int \log\left(1 + \frac{bF^{c+dx}}{a}\right) dx}{abd^2 \log^2(F)} \\
&= \frac{x^2}{abd \log(F)} - \frac{x^2}{bd(a + bF^{c+dx}) \log(F)} - \frac{2x \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{abd^2 \log^2(F)} + \frac{2 \operatorname{Subst}\left(\int \frac{\log\left(1 + \frac{bx}{a}\right)}{x} dx, x, F^{c+dx}\right)}{abd^3 \log^3(F)} \\
&= \frac{x^2}{abd \log(F)} - \frac{x^2}{bd(a + bF^{c+dx}) \log(F)} - \frac{2x \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{abd^2 \log^2(F)} - \frac{2 \operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F)}
\end{aligned}$$



**Mathematica [A]** time = 0.09, size = 103, normalized size = 0.96

$$\frac{dx \log(F) \left( bdx \log(F) F^{c+dx} - 2(a + bF^{c+dx}) \log\left(\frac{bF^{c+dx}}{a} + 1\right) \right) - 2(a + bF^{c+dx}) \operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3 \log^3(F) (a + bF^{c+dx})}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c + d\*x)\*x^2)/(a + b\*F^(c + d\*x))^2,x]

[Out] (d\*x\*Log[F]\*(b\*d\*F^(c + d\*x)\*x\*Log[F] - 2\*(a + b\*F^(c + d\*x))\*Log[1 + (b\*F^(c + d\*x))/a]) - 2\*(a + b\*F^(c + d\*x))\*PolyLog[2, -((b\*F^(c + d\*x))/a)]/(a\*b\*d^3\*(a + b\*F^(c + d\*x))\*Log[F]^3)

**fricas [A]** time = 0.42, size = 186, normalized size = 1.74

$$\frac{ac^2 \log(F)^2 - (bd^2x^2 - bc^2)F^{dx+c} \log(F)^2 + 2(F^{dx+c}b + a) \operatorname{Li}_2\left(-\frac{F^{dx+c}b+a}{a} + 1\right) - 2(F^{dx+c}bc \log(F) + ac \log(F))}{F^{dx+c}ab^2d^3 \log(F)^3 + a^2bd^3 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c))^2,x, algorithm="fricas")

[Out] -(a\*c^2\*log(F)^2 - (b\*d^2\*x^2 - b\*c^2)\*F^(d\*x + c)\*log(F)^2 + 2\*(F^(d\*x + c)\*b + a)\*dilog(-(F^(d\*x + c)\*b + a)/a + 1) - 2\*(F^(d\*x + c)\*b\*c\*log(F) + a\*c\*log(F))\*log(F^(d\*x + c)\*b + a) + 2\*((b\*d\*x + b\*c)\*F^(d\*x + c)\*log(F) + (a\*d\*x + a\*c)\*log(F))\*log((F^(d\*x + c)\*b + a)/a)/(F^(d\*x + c)\*a\*b^2\*d^3\*log(F)^3 + a^2\*b\*d^3\*log(F)^3)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}x^2}{(F^{dx+c}b + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x^2/(F^(d\*x + c)\*b + a)^2, x)

**maple [B]** time = 0.05, size = 231, normalized size = 2.16

$$-\frac{x^2}{(bF^{dx+c} + a)bd \ln(F)} + \frac{x^2}{abd \ln(F)} + \frac{2cx}{abd^2 \ln(F)} + \frac{c^2}{abd^3 \ln(F)} - \frac{2x \ln\left(\frac{bF^c F^{dx}}{a} + 1\right)}{abd^2 \ln(F)^2} - \frac{2c \ln(F^c F^{dx})}{abd^3 \ln(F)^2} - \frac{2c \ln\left(\frac{bF^c F^{dx}}{a} + 1\right)}{abd^3 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(d*x+c)*x^2/(b*F^(d*x+c)+a)^2,x)`

[Out] 
$$-x^2/b/d/(b*F^(d*x+c)+a)/\ln(F)+x^2/a/b/d/\ln(F)+2/d^2/\ln(F)/b/a*c*x+1/d^3/\ln(F)/b/a*c^2-2/d^2/\ln(F)^2/b/a*\ln(1/a*b*F^c*F^(d*x)+1)*x-2/d^3/\ln(F)^2/b/a*\ln(1/a*b*F^c*F^(d*x)+1)*c-2/d^3/\ln(F)^3/b/a*\text{polylog}(2,-1/a*b*F^c*F^(d*x))-2/d^3/\ln(F)^2/b*c/a*\ln(F^c*F^(d*x))+2/d^3/\ln(F)^2/b*c/a*\ln(b*F^c*F^(d*x)+a)$$

**maxima** [A] time = 0.50, size = 106, normalized size = 0.99

$$-\frac{x^2}{F^{dx}F^c b^2 d \log(F) + abd \log(F)} + \frac{\log(F^{dx})^2}{abd^3 \log(F)^3} - \frac{2 \left( \log\left(\frac{F^{dx}F^c b}{a} + 1\right) \log(F^{dx}) + \text{Li}_2\left(-\frac{F^{dx}F^c b}{a}\right) \right)}{abd^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x, algorithm="maxima")`

[Out] 
$$-x^2/(F^(d*x)*F^c*b^2*d*\log(F) + a*b*d*\log(F)) + \log(F^(d*x))^2/(a*b*d^3*\log(F)^3) - 2*(\log(F^(d*x)*F^c*b/a + 1)*\log(F^(d*x)) + \text{dilog}(-F^(d*x)*F^c*b/a))/ (a*b*d^3*\log(F)^3)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c+dx} x^2}{(a + F^{c+dx} b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((F^(c+d*x)*x^2)/(a+F^(c+d*x)*b)^2,x)`

[Out] `int((F^(c+d*x)*x^2)/(a+F^(c+d*x)*b)^2,x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^2}{F^{c+dx} b^2 d \log(F) + abd \log(F)} + \frac{2 \int \frac{x}{a + b e^{c \log(F)} e^{dx \log(F)}} dx}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(d*x+c)*x**2/(a+b*F**(d*x+c))**2,x)`

[Out] 
$$-x^{**2}/(F^{**}(c+d*x)*b^{**2}*d*\log(F) + a*b*d*\log(F)) + 2*\text{Integral}(x/(a + b*\exp(c*\log(F))*\exp(d*x*\log(F))), x)/(b*d*\log(F))$$

$$3.84 \quad \int \frac{F^{c+dx} x}{(a+bF^{c+dx})^2} dx$$

Optimal. Leaf size=69

$$-\frac{\log(a+bF^{c+dx})}{abd^2 \log^2(F)} - \frac{x}{bd \log(F)(a+bF^{c+dx})} + \frac{x}{abd \log(F)}$$

[Out]  $x/a/b/d/\ln(F)-x/b/d/(a+b*F^{(d*x+c)})/\ln(F)-\ln(a+b*F^{(d*x+c)})/a/b/d^2/\ln(F)^2$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {2191, 2282, 36, 29, 31}

$$-\frac{\log(a+bF^{c+dx})}{abd^2 \log^2(F)} - \frac{x}{bd \log(F)(a+bF^{c+dx})} + \frac{x}{abd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(F^(c + d\*x)\*x)/(a + b\*F^(c + d\*x))^2,x]

[Out]  $x/(a*b*d*\text{Log}[F]) - x/(b*d*(a + b*F^{(c + d*x)})*\text{Log}[F]) - \text{Log}[a + b*F^{(c + d*x)}]/(a*b*d^2*\text{Log}[F]^2)$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_)\*(x\_))^-1, x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 2191

Int[((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^((p\_)\*((c\_) + (d\_)\*(x\_))^(m\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x))))^n)^(p + 1)/(b\*f\*g\*n\*(p + 1)\*Lo

$g[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*(p + 1)*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*(a + b*(F^{(g*(e + f*x)))^n})^{(p + 1)}, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, m, n, p\}, x] \&\& \text{NeQ}[p, -1]$

### Rule 2282

$\text{Int}[u_, x\_Symbol] := \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_}))^{(m\_)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{((c\_)*((a\_)+(b\_)*x))} (F\_)[v_] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

### Rubi steps

$$\begin{aligned} \int \frac{F^{c+dx} x}{(a + bF^{c+dx})^2} dx &= -\frac{x}{bd(a + bF^{c+dx}) \log(F)} + \frac{\int \frac{1}{a+bF^{c+dx}} dx}{bd \log(F)} \\ &= -\frac{x}{bd(a + bF^{c+dx}) \log(F)} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, F^{c+dx}\right)}{bd^2 \log^2(F)} \\ &= -\frac{x}{bd(a + bF^{c+dx}) \log(F)} - \frac{\text{Subst}\left(\int \frac{1}{a+bx} dx, x, F^{c+dx}\right)}{ad^2 \log^2(F)} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, F^{c+dx}\right)}{abd^2 \log^2(F)} \\ &= \frac{x}{abd \log(F)} - \frac{x}{bd(a + bF^{c+dx}) \log(F)} - \frac{\log(a + bF^{c+dx})}{abd^2 \log^2(F)} \end{aligned}$$

**Mathematica** [A] time = 0.07, size = 54, normalized size = 0.78

$$\frac{\frac{dx \log(F)^{F^{c+dx}}}{a+bF^{c+dx}} - \frac{\log(a+bF^{c+dx})}{b}}{ad^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(F^{(c + d*x)*x})/(a + b*F^{(c + d*x)})^2, x]$

[Out]  $((d*F^{(c + d*x)*x}*\text{Log}[F])/(a + b*F^{(c + d*x)}) - \text{Log}[a + b*F^{(c + d*x)}])/b)/(a*d^2*\text{Log}[F]^2)$

**fricas** [A] time = 0.42, size = 74, normalized size = 1.07

$$\frac{F^{dx+c} b dx \log(F) - (F^{dx+c} b + a) \log(F^{dx+c} b + a)}{F^{dx+c} a b^2 d^2 \log(F)^2 + a^2 b d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c))^2,x, algorithm="fricas")

[Out] (F^(d\*x + c)\*b\*d\*x\*log(F) - (F^(d\*x + c)\*b + a)\*log(F^(d\*x + c)\*b + a))/(F^(d\*x + c)\*a\*b^2\*d^2\*log(F)^2 + a^2\*b\*d^2\*log(F)^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c} x}{(F^{dx+c} b + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x/(F^(d\*x + c)\*b + a)^2, x)

**maple** [A] time = 0.02, size = 67, normalized size = 0.97

$$\frac{x e^{(dx+c) \ln(F)}}{(b e^{(dx+c) \ln(F)} + a) a d \ln(F)} - \frac{\ln(b e^{(dx+c) \ln(F)} + a)}{a b d^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)\*x/(b\*F^(d\*x+c)+a)^2,x)

[Out] 1/ln(F)/a/d\*x\*exp((d\*x+c)\*ln(F))/(b\*exp((d\*x+c)\*ln(F))+a)-1/ln(F)^2/b/d^2/a\*ln(b\*exp((d\*x+c)\*ln(F))+a)

**maxima** [A] time = 0.47, size = 72, normalized size = 1.04

$$\frac{F^{dx} F^c x}{F^{dx} F^c a b d \log(F) + a^2 d \log(F)} - \frac{\log\left(\frac{F^{dx} F^c b + a}{F^c b}\right)}{a b d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c))^2,x, algorithm="maxima")

[Out] F^(d\*x)\*F^c\*x/(F^(d\*x)\*F^c\*a\*b\*d\*log(F) + a^2\*d\*log(F)) - log((F^(d\*x)\*F^c\*b + a)/(F^c\*b))/(a\*b\*d^2\*log(F)^2)

**mupad** [B] time = 3.62, size = 63, normalized size = 0.91

$$\frac{F^c F^{dx} x}{a d \ln(F) (a + F^c F^{dx} b)} - \frac{\ln(a + F^c F^{dx} b)}{a b d^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((F^(c + d*x)*x)/(a + F^(c + d*x)*b)^2,x)`

[Out]  $(F^c F^{(d*x)*x}) / (a*d*\log(F)*(a + F^c F^{(d*x)*b})) - \log(a + F^c F^{(d*x)*b}) / (a*b*d^2*\log(F)^2)$

sympy [A] time = 0.18, size = 58, normalized size = 0.84

$$-\frac{x}{F^{c+dx} b^2 d \log(F) + a b d \log(F)} + \frac{x}{a b d \log(F)} - \frac{\log\left(F^{c+dx} + \frac{a}{b}\right)}{a b d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(d*x+c)*x/(a+b*F**(d*x+c))**2,x)`

[Out]  $-x / (F^{(c + d*x)*b**2*d*\log(F) + a*b*d*\log(F)} + x / (a*b*d*\log(F)) - \log(F^{(c + d*x) + a/b}) / (a*b*d**2*\log(F)**2)$

$$3.85 \quad \int \frac{F^{c+dx}}{(a+bF^{c+dx})^2} dx$$

Optimal. Leaf size=25

$$-\frac{1}{bd \log(F) (a + bF^{c+dx})}$$

[Out] -1/b/d/(a+b\*F^(d\*x+c))/ln(F)

**Rubi [A]** time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2246, 32}

$$-\frac{1}{bd \log(F) (a + bF^{c+dx})}$$

Antiderivative was successfully verified.

[In] Int[F^(c + d\*x)/(a + b\*F^(c + d\*x))^2,x]

[Out] -(1/(b\*d\*(a + b\*F^(c + d\*x))\*Log[F]))

Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*(F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{F^{c+dx}}{(a + bF^{c+dx})^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx)^2} dx, x, F^{c+dx}\right)}{d \log(F)} \\ &= -\frac{1}{bd (a + bF^{c+dx}) \log(F)} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 25, normalized size = 1.00

$$-\frac{1}{bd \log(F) (a + bF^{c+dx})}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c + d\*x)/(a + b\*F^(c + d\*x))^2,x]

[Out] -(1/(b\*d\*(a + b\*F^(c + d\*x))\*Log[F]))

**fricas** [A] time = 0.43, size = 25, normalized size = 1.00

$$-\frac{1}{F^{dx+c}b^2d \log(F) + abd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2,x, algorithm="fricas")

[Out] -1/(F^(d\*x + c)\*b^2\*d\*log(F) + a\*b\*d\*log(F))

**giac** [A] time = 0.32, size = 26, normalized size = 1.04

$$-\frac{1}{(F^{dx}F^c b + a)bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2,x, algorithm="giac")

[Out] -1/((F^(d\*x)\*F^c\*b + a)\*b\*d\*log(F))

**maple** [A] time = 0.00, size = 26, normalized size = 1.04

$$-\frac{1}{(b F^{dx+c} + a)bd \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^2,x)

[Out] -1/b/d/(b\*F^(d\*x+c)+a)/ln(F)

**maxima** [A] time = 0.43, size = 25, normalized size = 1.00

$$-\frac{1}{(F^{dx+c}b + a)bd \log(F)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2,x, algorithm="maxima")

[Out] -1/((F^(d\*x + c)\*b + a)\*b\*d\*log(F))

mupad [B] time = 3.46, size = 31, normalized size = 1.24

$$\frac{F^{c+dx}}{a^2 d \ln(F) + F^{c+dx} a b d \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(a + F^(c + d\*x)\*b)^2,x)

[Out] F^(c + d\*x)/(a^2\*d\*log(F) + F^(c + d\*x)\*a\*b\*d\*log(F))

sympy [A] time = 0.12, size = 26, normalized size = 1.04

$$\frac{1}{F^{c+dx} b^2 d \log(F) + a b d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)/(a+b\*F\*\*(d\*x+c))\*\*2,x)

[Out] -1/(F\*\*(c + d\*x)\*b\*\*2\*d\*log(F) + a\*b\*d\*log(F))

$$3.86 \quad \int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x} dx$$

Optimal. Leaf size=61

$$-\frac{\text{Int}\left(\frac{1}{x^2(a+bF^{c+dx})}, x\right)}{bd \log(F)} - \frac{1}{bdx \log(F)(a+bF^{c+dx})}$$

[Out] -1/b/d/(a+b\*F^(d\*x+c))/x/ln(F)-Unintegrable(1/(a+b\*F^(d\*x+c))/x^2,x)/b/d/ln(F)

**Rubi [A]** time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x} dx$$

Verification is Not applicable to the result.

[In] Int[F^(c + d\*x)/((a + b\*F^(c + d\*x))^2\*x), x]

[Out] -(1/(b\*d\*(a + b\*F^(c + d\*x))\*x\*Log[F])) - Defer[Int][1/((a + b\*F^(c + d\*x))\*x^2), x]/(b\*d\*Log[F])

Rubi steps

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x} dx = -\frac{1}{bd(a+bF^{c+dx})x \log(F)} - \frac{\int \frac{1}{(a+bF^{c+dx})x^2} dx}{bd \log(F)}$$

**Mathematica [A]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(c + d\*x)/((a + b\*F^(c + d\*x))^2\*x), x]

[Out] Integrate[F^(c + d\*x)/((a + b\*F^(c + d\*x))^2\*x), x]

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{F^{dx+c}}{2F^{dx+c}abx + F^{2dx+2c}b^2x + a^2x', x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2/x,x, algorithm="fricas")

[Out] integral(F^(d\*x + c)/(2\*F^(d\*x + c)\*a\*b\*x + F^(2\*d\*x + 2\*c)\*b^2\*x + a^2\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(F^{dx+c}b + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2/x,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)/((F^(d\*x + c)\*b + a)^2\*x), x)

**maple** [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(bF^{dx+c} + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^2/x,x)

[Out] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^2/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{F^{dx}F^c b^2 dx \log(F) + ab dx \log(F)} - \int \frac{1}{F^{dx}F^c b^2 dx^2 \log(F) + ab dx^2 \log(F)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2/x,x, algorithm="maxima")

[Out] -1/(F^(d\*x)\*F^c\*b^2\*d\*x\*log(F) + a\*b\*d\*x\*log(F)) - integrate(1/(F^(d\*x)\*F^c\*b^2\*d\*x^2\*log(F) + a\*b\*d\*x^2\*log(F)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c+dx}}{x(a + F^{c+dx}b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(x\*(a + F^(c + d\*x)\*b)^2), x)

[Out] int(F^(c + d\*x)/(x\*(a + F^(c + d\*x)\*b)^2), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{F^{c+dx}b^2dx \log(F) + abdx \log(F)} - \frac{\int \frac{1}{ax^2+bx^2e^{c \log(F)}e^{dx \log(F)}} dx}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)/(a+b\*F\*\*(d\*x+c))\*\*2/x,x)

[Out] -1/(F\*\*(c + d\*x)\*b\*\*2\*d\*x\*log(F) + a\*b\*d\*x\*log(F)) - Integral(1/(a\*x\*\*2 + b\*x\*\*2\*exp(c\*log(F))\*exp(d\*x\*log(F))), x)/(b\*d\*log(F))

$$3.87 \quad \int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x^2} dx$$

Optimal. Leaf size=61

$$-\frac{2 \operatorname{Int}\left(\frac{1}{x^3(a+bF^{c+dx})}, x\right)}{bd \log(F)} - \frac{1}{bdx^2 \log(F)(a+bF^{c+dx})}$$

[Out]  $-1/b/d/(a+bF^{(d*x+c)})/x^2/\ln(F)-2*\operatorname{Unintegrable}(1/(a+bF^{(d*x+c)})/x^3,x)/b/d/\ln(F)$

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[F^{(c+d*x)} / ((a+bF^{(c+d*x)})^2 * x^2), x]$

[Out]  $-(1/(b*d*(a+bF^{(c+d*x)}) * x^2 * \operatorname{Log}[F])) - (2*\operatorname{Defer}[\operatorname{Int}[1/((a+bF^{(c+d*x)}) * x^3), x]) / (b*d * \operatorname{Log}[F])$

Rubi steps

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x^2} dx = -\frac{1}{bd(a+bF^{c+dx})x^2 \log(F)} - \frac{2 \int \frac{1}{(a+bF^{c+dx})x^3} dx}{bd \log(F)}$$

Mathematica [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[F^{(c+d*x)} / ((a+bF^{(c+d*x)})^2 * x^2), x]$

[Out]  $\operatorname{Integrate}[F^{(c+d*x)} / ((a+bF^{(c+d*x)})^2 * x^2), x]$

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{F^{dx+c}}{2F^{dx+c}abx^2 + F^{2dx+2c}b^2x^2 + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2/x^2,x, algorithm="fricas")

[Out] integral(F^(d\*x + c)/(2\*F^(d\*x + c)\*a\*b\*x^2 + F^(2\*d\*x + 2\*c)\*b^2\*x^2 + a^2\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(F^{dx+c}b + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2/x^2,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)/((F^(d\*x + c)\*b + a)^2\*x^2), x)

**maple** [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(bF^{dx+c} + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^2/x^2,x)

[Out] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^2/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{F^{dx}F^c b^2 dx^2 \log(F) + ab dx^2 \log(F)} - 2 \int \frac{1}{F^{dx}F^c b^2 dx^3 \log(F) + ab dx^3 \log(F)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^2/x^2,x, algorithm="maxima")

[Out] -1/(F^(d\*x)\*F^c\*b^2\*d\*x^2\*log(F) + a\*b\*d\*x^2\*log(F)) - 2\*integrate(1/(F^(d\*x)\*F^c\*b^2\*d\*x^3\*log(F) + a\*b\*d\*x^3\*log(F)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c+dx}}{x^2 (a + F^{c+dx} b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(x^2\*(a + F^(c + d\*x)\*b)^2), x)

[Out] int(F^(c + d\*x)/(x^2\*(a + F^(c + d\*x)\*b)^2), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{F^{c+dx} b^2 dx^2 \log(F) + ab dx^2 \log(F)} - \frac{2 \int \frac{1}{ax^3 + bx^3 e^{c \log(F)} e^{dx \log(F)}} dx}{bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)/(a+b\*F\*\*(d\*x+c))\*\*2/x\*\*2, x)

[Out] -1/(F\*\*(c + d\*x)\*b\*\*2\*d\*x\*\*2\*log(F) + a\*b\*d\*x\*\*2\*log(F)) - 2\*Integral(1/(a\*x\*\*3 + b\*x\*\*3\*exp(c\*log(F))\*exp(d\*x\*log(F))), x)/(b\*d\*log(F))

$$3.88 \quad \int \frac{F^{c+dx} x^3}{(a+bF^{c+dx})^3} dx$$

**Optimal.** Leaf size=261

$$\frac{3\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4 \log^4(F)} + \frac{3\text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4 \log^4(F)} - \frac{3x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3 \log^3(F)} + \frac{3x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{a^2bd^3 \log^3(F)} - \frac{3x^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{2a^2bd^2 \log^2(F)} - \frac{3x^2}{2a^2bd^2 \log^2(F)} + \frac{3x^2}{2a^2bd^2 \log^2(F)} + \frac{3x^2}{2a^2bd^2 \log^2(F)}$$

[Out]  $-3/2*x^2/a^2/b/d^2/\ln(F)^2+3/2*x^2/a/b/d^2/(a+bF^{(d*x+c)})/\ln(F)^2+1/2*x^3/a^2/b/d/\ln(F)-1/2*x^3/b/d/(a+bF^{(d*x+c)})^2/\ln(F)+3*x*\ln(1+bF^{(d*x+c)})/a/a^2/b/d^3/\ln(F)^3-3/2*x^2*\ln(1+bF^{(d*x+c)})/a/a^2/b/d^2/\ln(F)^2+3*\text{polylog}(2,-bF^{(d*x+c)})/a/a^2/b/d^4/\ln(F)^4-3*x*\text{polylog}(2,-bF^{(d*x+c)})/a/a^2/b/d^3/\ln(F)^3+3*\text{polylog}(3,-bF^{(d*x+c)})/a/a^2/b/d^4/\ln(F)^4$

**Rubi [A]** time = 0.50, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2191, 2185, 2184, 2190, 2531, 2282, 6589, 2279, 2391}

$$-\frac{3x\text{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{a^2bd^3 \log^3(F)} + \frac{3\text{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{a^2bd^4 \log^4(F)} + \frac{3\text{PolyLog}\left(3, -\frac{bF^{c+dx}}{a}\right)}{a^2bd^4 \log^4(F)} - \frac{3x^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{2a^2bd^2 \log^2(F)} + \frac{3x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{a^2bd^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{(c+d*x)}*x^3)/(a+bF^{(c+d*x)})^3, x]$

[Out]  $(-3*x^2)/(2*a^2*b*d^2*\text{Log}[F]^2) + (3*x^2)/(2*a*b*d^2*(a+bF^{(c+d*x)})*\text{Log}[F]^2) + x^3/(2*a^2*b*d*\text{Log}[F]) - x^3/(2*b*d*(a+bF^{(c+d*x)})^2*\text{Log}[F]) + (3*x*\text{Log}[1+(bF^{(c+d*x)})/a])/(a^2*b*d^3*\text{Log}[F]^3) - (3*x^2*\text{Log}[1+(bF^{(c+d*x)})/a])/(2*a^2*b*d^2*\text{Log}[F]^2) + (3*\text{PolyLog}[2, -((bF^{(c+d*x)})/a)])/(a^2*b*d^4*\text{Log}[F]^4) - (3*x*\text{PolyLog}[2, -((bF^{(c+d*x)})/a)])/(a^2*b*d^3*\text{Log}[F]^3) + (3*\text{PolyLog}[3, -((bF^{(c+d*x)})/a)])/(a^2*b*d^4*\text{Log}[F]^4)$

**Rule 2184**

$\text{Int}[(c_1 + d_1*x)^{m_1}/(a_1 + (b_1*(F^{(g_1*(e_1 + f_1*x))}) + (f_1*x)))^{n_1}], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{m+1}/(a*d*(m+1)), x] - \text{Dist}[b/a, \text{Int}[(c + d*x)^m*(F^{(g*(e + f*x))})^n]/(a + b*(F^{(g*(e + f*x))})^n), x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2185**

$\text{Int}[(a_1 + (b_1*(F^{(g_1*(e_1 + f_1*x))}) + (f_1*x)))^{n_1}]^{p_1}*(c_1 + (d_1*x)^{m_1}), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(c + d*x)^m*(a + b*(F^{(g*(e + f*x))})^n), x]]$



```

f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]

```

### Rule 2190

```

Int[(((F_)^((g_)*(e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2191

```

Int[((F_)^((g_)*(e_) + (f_)*(x_))))^(n_)*((a_) + (b_)*((F_)^((g_)*(
e_) + (f_)*(x_))))^(n_))^(p_)*((c_) + (d_)*(x_))^(m_), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]

```

### Rule 2279

```

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 2391

```

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rule 2531

```

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)])*((f_) + (g_)
*(x_)^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -

```

1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{c+dx} x^3}{(a + bF^{c+dx})^3} dx &= -\frac{x^3}{2bd(a + bF^{c+dx})^2 \log(F)} + \frac{3 \int \frac{x^2}{(a+bF^{c+dx})^2} dx}{2bd \log(F)} \\
 &= -\frac{x^3}{2bd(a + bF^{c+dx})^2 \log(F)} - \frac{3 \int \frac{F^{c+dx} x^2}{(a+bF^{c+dx})^2} dx}{2ad \log(F)} + \frac{3 \int \frac{x^2}{a+bF^{c+dx}} dx}{2abd \log(F)} \\
 &= \frac{3x^2}{2abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^3}{2a^2bd \log(F)} - \frac{x^3}{2bd(a + bF^{c+dx})^2 \log(F)} - \frac{3 \int \frac{x}{a+bF^{c+dx}} dx}{abd^2 \log^2(F)} \\
 &= -\frac{3x^2}{2a^2bd^2 \log^2(F)} + \frac{3x^2}{2abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^3}{2a^2bd \log(F)} - \frac{x^3}{2bd(a + bF^{c+dx})^2 \log(F)} \\
 &= -\frac{3x^2}{2a^2bd^2 \log^2(F)} + \frac{3x^2}{2abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^3}{2a^2bd \log(F)} - \frac{x^3}{2bd(a + bF^{c+dx})^2 \log(F)} \\
 &= -\frac{3x^2}{2a^2bd^2 \log^2(F)} + \frac{3x^2}{2abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^3}{2a^2bd \log(F)} - \frac{x^3}{2bd(a + bF^{c+dx})^2 \log(F)} \\
 &= -\frac{3x^2}{2a^2bd^2 \log^2(F)} + \frac{3x^2}{2abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^3}{2a^2bd \log(F)} - \frac{x^3}{2bd(a + bF^{c+dx})^2 \log(F)}
 \end{aligned}$$

**Mathematica [A]** time = 0.32, size = 220, normalized size = 0.84

$$\frac{dx \log(F) \left( b d^2 x^2 \log^2(F) F^{c+dx} (2a + b F^{c+dx}) + 6 (a + b F^{c+dx})^2 \log\left(\frac{b F^{c+dx}}{a} + 1\right) - 3 dx \log(F) (a + b F^{c+dx}) \left( (a + b F^{c+dx}) \log(F) + \frac{2 a^2 b d^4 \log^4(F)}{a^2} \right) \right)}{2 a^2 b d^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c + d\*x)\*x^3)/(a + b\*F^(c + d\*x))^3,x]

[Out] (d\*x\*Log[F]\*(b\*d^2\*F^(c + d\*x)\*(2\*a + b\*F^(c + d\*x))\*x^2\*Log[F]^2 + 6\*(a + b\*F^(c + d\*x))^2\*Log[1 + (b\*F^(c + d\*x))/a] - 3\*d\*(a + b\*F^(c + d\*x))\*x\*Log[F]\*(b\*F^(c + d\*x) + (a + b\*F^(c + d\*x))\*Log[1 + (b\*F^(c + d\*x))/a])) - 6\*(a + b\*F^(c + d\*x))^2\*(-1 + d\*x\*Log[F])\*PolyLog[2, -((b\*F^(c + d\*x))/a)] + 6\*(a + b\*F^(c + d\*x))^2\*PolyLog[3, -((b\*F^(c + d\*x))/a)]/(2\*a^2\*b\*d^4\*(a + b\*F^(c + d\*x))^2\*Log[F]^4)

**fricas [C]** time = 0.46, size = 577, normalized size = 2.21

$$\frac{a^2 c^3 \log(F)^3 + 3 a^2 c^2 \log(F)^2 + \left( (b^2 d^3 x^3 + b^2 c^3) \log(F)^3 - 3 (b^2 d^2 x^2 - b^2 c^2) \log(F)^2 \right) F^{2dx+2c} + \left( 2 (abd^3 x^3 + ab^2 c^3) \log(F)^3 - 3 (b^2 d^2 x^2 - b^2 c^2) \log(F)^2 \right) F^{2dx+2c} + \left( 2 (abd^3 x^3 + ab^2 c^3) \log(F)^3 - 3 (b^2 d^2 x^2 - b^2 c^2) \log(F)^2 \right) F^{2dx+2c} + \left( 2 (abd^3 x^3 + ab^2 c^3) \log(F)^3 - 3 (b^2 d^2 x^2 - b^2 c^2) \log(F)^2 \right) F^{2dx+2c}}{2 a^2 b d^4 \log^4(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^3/(a+b\*F^(d\*x+c))^3,x, algorithm="fricas")

[Out] 1/2\*(a^2\*c^3\*log(F)^3 + 3\*a^2\*c^2\*log(F)^2 + ((b^2\*d^3\*x^3 + b^2\*c^3)\*log(F)^3 - 3\*(b^2\*d^2\*x^2 - b^2\*c^2)\*log(F)^2)\*F^(2\*d\*x + 2\*c) + (2\*(a\*b\*d^3\*x^3 + a\*b\*c^3)\*log(F)^3 - 3\*(a\*b\*d^2\*x^2 - 2\*a\*b\*c^2)\*log(F)^2)\*F^(d\*x + c) - 6\*(a^2\*d\*x\*log(F) + (b^2\*d\*x\*log(F) - b^2)\*F^(2\*d\*x + 2\*c) + 2\*(a\*b\*d\*x\*log(F) - a\*b)\*F^(d\*x + c) - a^2)\*dilog(-(F^(d\*x + c)\*b + a)/a + 1) - 3\*(a^2\*c^2\*log(F)^2 + 2\*a^2\*c\*log(F) + (b^2\*c^2\*log(F)^2 + 2\*b^2\*c\*log(F))\*F^(2\*d\*x + 2\*c) + 2\*(a\*b\*c^2\*log(F)^2 + 2\*a\*b\*c\*log(F))\*F^(d\*x + c))\*log(F^(d\*x + c)\*b + a) - 3\*((a^2\*d^2\*x^2 - a^2\*c^2)\*log(F)^2 + ((b^2\*d^2\*x^2 - b^2\*c^2)\*log(F)^2 - 2\*(b^2\*d\*x + b^2\*c)\*log(F))\*F^(2\*d\*x + 2\*c) + 2\*((a\*b\*d^2\*x^2 - a\*b\*c^2)\*log(F)^2 - 2\*(a\*b\*d\*x + a\*b\*c)\*log(F))\*F^(d\*x + c) - 2\*(a^2\*d\*x + a^2\*c)\*log(F))\*log((F^(d\*x + c)\*b + a)/a) + 6\*(2\*F^(d\*x + c)\*a\*b + F^(2\*d\*x + 2\*c)\*b^2 + a^2)\*polylog(3, -F^(d\*x + c)\*b/a)/(2\*F^(d\*x + c)\*a^3\*b^2\*d^4\*log(F)^4 + F^(2\*d\*x + 2\*c)\*a^2\*b^3\*d^4\*log(F)^4 + a^4\*b\*d^4\*log(F)^4)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c} x^3}{(F^{dx+c} b + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^3/(a+bF^(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x^3/(F^(d\*x + c)\*b + a)^3, x)

**maple** [A] time = 0.08, size = 501, normalized size = 1.92

$$\frac{x^3}{2a^2bd \ln(F)} - \frac{(adx \ln(F) - 3bF^{dx+c} - 3a)x^2}{2(bF^{dx+c} + a)^2 ab d^2 \ln(F)^2} - \frac{3c^2x}{2a^2b d^3 \ln(F)} - \frac{3x^2 \ln\left(\frac{bF^{cF^{dx}}}{a} + 1\right)}{2a^2b d^2 \ln(F)^2} - \frac{c^3}{a^2b d^4 \ln(F)} - \frac{3x^2}{2a^2b d^2 \ln(F)^2} + \frac{3c}{2a^2b d^4 \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)\*x^3/(bF^(d\*x+c)+a)^3,x)

[Out] 
$$-1/2*x^2*(\ln(F)*a*d*x-3*b*F^{d*x+c}-3*a)/\ln(F)^2/d^2/a/b/(b*F^{d*x+c}+a)^2+1/2*x^3/a^2/b/d/\ln(F)-3/2/b/a^2/d^3/\ln(F)*c^2*x-1/b/a^2/d^4/\ln(F)*c^3-3/2/b/a^2/d^2/\ln(F)^2*\ln(1/a*b*F^c*F^{d*x}+1)*x^2+3/2/b/a^2/d^4/\ln(F)^2*\ln(1/a*b*F^c*F^{d*x}+1)*c^2-3/b/a^2/d^3/\ln(F)^3*\text{polylog}(2,-1/a*b*F^c*F^{d*x})*x+3/b/a^2/d^4/\ln(F)^4*\text{polylog}(3,-1/a*b*F^c*F^{d*x}))+3/2/b/a^2/d^4/\ln(F)^2*c^2*\ln(F^c*F^{d*x})-3/2/b/a^2/d^4/\ln(F)^2*c^2*\ln(b*F^c*F^{d*x}+a)-3/2*x^2/a^2/b/d^2/\ln(F)^2-3/b/a^2/d^3/\ln(F)^2*c*x-3/2/b/a^2/d^4/\ln(F)^2*c^2+3/b/a^2/d^3/\ln(F)^3*\ln(1/a*b*F^c*F^{d*x}+1)*x+3/b/a^2/d^4/\ln(F)^3*\ln(1/a*b*F^c*F^{d*x}+1)*c+3/b/a^2/d^4/\ln(F)^4*\text{polylog}(2,-1/a*b*F^c*F^{d*x}))+3/b/a^2/d^4/\ln(F)^3*c*\ln(F^c*F^{d*x})-3/b/a^2/d^4/\ln(F)^3*c*\ln(b*F^c*F^{d*x}+a)$$

**maxima** [A] time = 0.53, size = 263, normalized size = 1.01

$$\frac{adx^3 \log(F) - 3F^{dx}F^c b x^2 - 3ax^2}{2(2F^{dx}F^c a^2 b^2 d^2 \log(F)^2 + F^{2dx}F^{2c} ab^3 d^2 \log(F)^2 + a^3 b d^2 \log(F)^2)} - \frac{3\left(\log\left(\frac{F^{dx}F^c b}{a} + 1\right)\log(F^{dx})^2 + 2\text{Li}_2\left(-\frac{F^{dx}}{a}\right)\right)}{2a^2 b d^4 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^3/(a+bF^(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$-1/2*(a*d*x^3*\log(F) - 3*F^{d*x}*F^c*b*x^2 - 3*a*x^2)/(2*F^{d*x}*F^c*a^2*b^2*d^2*\log(F)^2 + F^{(2*d*x)}*F^{(2*c)}*a*b^3*d^2*\log(F)^2 + a^3*b*d^2*\log(F)^2) - 3/2*(\log(F^{d*x}*F^c*b/a + 1)*\log(F^{d*x})^2 + 2*\text{dilog}(-F^{d*x}*F^c*b/a)*\log(F^{d*x}) - 2*\text{polylog}(3, -F^{d*x}*F^c*b/a))/(a^2*b*d^4*\log(F)^4) + 1/2*(\log(F^{d*x})^3 - 3*\log(F^{d*x})^2)/(a^2*b*d^4*\log(F)^4) + 3*(\log(F^{d*x}*F^c*b/a + 1)*\log(F^{d*x}) + \text{dilog}(-F^{d*x}*F^c*b/a))/(a^2*b*d^4*\log(F)^4)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{F^{c+dx} x^3}{(a + F^{c+dx} b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^3,x)`

[Out] `int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3F^{c+dx}bx^2 - adx^3 \log(F) + 3ax^2}{4F^{c+dx}a^2b^2d^2 \log(F)^2 + 2F^{2c+2dx}ab^3d^2 \log(F)^2 + 2a^3bd^2 \log(F)^2} + \frac{3 \left( \int \left( -\frac{2x}{a+be^{c \log(F)}e^{dx \log(F)}} \right) dx + \int \frac{dx^2 \log(F)}{a+be^{c \log(F)}e^{dx}} \right)}{2abd^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c))**3,x)`

[Out] `(3*F**(c + d*x)*b*x**2 - a*d*x**3*log(F) + 3*a*x**2)/(4*F**(c + d*x)*a**2*b**2*d**2*log(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*log(F)**2 + 2*a**3*b*d**2*log(F)**2) + 3*(Integral(-2*x/(a + b*exp(c*log(F))*exp(d*x*log(F))), x) + Integral(d*x**2*log(F)/(a + b*exp(c*log(F))*exp(d*x*log(F))), x))/(2*a*b*d**2*log(F)**2)`

$$3.89 \quad \int \frac{F^{c+dx} x^2}{(a+bF^{c+dx})^3} dx$$

**Optimal.** Leaf size=182

$$\frac{\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3 \log^3(F)} + \frac{\log(a+bF^{c+dx})}{a^2bd^3 \log^3(F)} - \frac{x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{a^2bd^2 \log^2(F)} - \frac{x}{a^2bd^2 \log^2(F)} + \frac{x^2}{2a^2bd \log(F)} + \frac{x}{abd^2 \log^2(F)(a+bF^{c+dx})}$$

[Out]  $-x/a^2/b/d^2/\ln(F)^2+x/a/b/d^2/(a+bF^{(d*x+c)})/\ln(F)^2+1/2*x^2/a^2/b/d/\ln(F)-1/2*x^2/b/d/(a+bF^{(d*x+c)})^2/\ln(F)+\ln(a+bF^{(d*x+c)})/a^2/b/d^3/\ln(F)^3-x*\ln(1+bF^{(d*x+c)}/a)/a^2/b/d^2/\ln(F)^2-\text{polylog}(2,-bF^{(d*x+c)}/a)/a^2/b/d^3/\ln(F)^3$

**Rubi [A]** time = 0.29, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {2191, 2185, 2184, 2190, 2279, 2391, 2282, 36, 29, 31}

$$\frac{\text{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{a^2bd^3 \log^3(F)} - \frac{x \log\left(\frac{bF^{c+dx}}{a} + 1\right)}{a^2bd^2 \log^2(F)} + \frac{\log(a+bF^{c+dx})}{a^2bd^3 \log^3(F)} - \frac{x}{a^2bd^2 \log^2(F)} + \frac{x^2}{2a^2bd \log(F)} + \frac{x}{abd^2 \log^2(F)(a+bF^{c+dx})}$$

Antiderivative was successfully verified.

[In] Int[(F^(c + d\*x)\*x^2)/(a + b\*F^(c + d\*x))^3,x]

[Out]  $-(x/(a^2*b*d^2*\text{Log}[F]^2)) + x/(a*b*d^2*(a + b*F^{(c + d*x)})*\text{Log}[F]^2) + x^2/(2*a^2*b*d*\text{Log}[F]) - x^2/(2*b*d*(a + b*F^{(c + d*x)})^2*\text{Log}[F]) + \text{Log}[a + b*F^{(c + d*x)}]/(a^2*b*d^3*\text{Log}[F]^3) - (x*\text{Log}[1 + (b*F^{(c + d*x)})/a])/ (a^2*b*d^2*\text{Log}[F]^2) - \text{PolyLog}[2, -(b*F^{(c + d*x)})/a]/(a^2*b*d^3*\text{Log}[F]^3)$

**Rule 29**

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 36**

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x],

x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2185

Int[((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/a, Int[(c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x)))^n\*(a + b\*(F^(g\*(e + f\*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && ILtQ[p, 0] && IGtQ[m, 0]

### Rule 2190

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2191

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1)/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2,  
, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{F^{c+dx} x^2}{(a + bF^{c+dx})^3} dx &= -\frac{x^2}{2bd(a + bF^{c+dx})^2 \log(F)} + \frac{\int \frac{x}{(a + bF^{c+dx})^2} dx}{bd \log(F)} \\
 &= -\frac{x^2}{2bd(a + bF^{c+dx})^2 \log(F)} - \frac{\int \frac{F^{c+dx} x}{(a + bF^{c+dx})^2} dx}{ad \log(F)} + \frac{\int \frac{x}{a + bF^{c+dx}} dx}{abd \log(F)} \\
 &= \frac{x}{abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^2}{2a^2bd \log(F)} - \frac{x^2}{2bd(a + bF^{c+dx})^2 \log(F)} - \frac{\int \frac{1}{a + bF^{c+dx}} dx}{abd^2 \log^2(F)} - \frac{\int \frac{x}{a + bF^{c+dx}} dx}{abd^2 \log^2(F)} \\
 &= \frac{x}{abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^2}{2a^2bd \log(F)} - \frac{x^2}{2bd(a + bF^{c+dx})^2 \log(F)} - \frac{x \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{a^2bd^2 \log^2(F)} \\
 &= \frac{x}{abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^2}{2a^2bd \log(F)} - \frac{x^2}{2bd(a + bF^{c+dx})^2 \log(F)} - \frac{x \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{a^2bd^2 \log^2(F)} \\
 &= -\frac{x}{a^2bd^2 \log^2(F)} + \frac{x}{abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x^2}{2a^2bd \log(F)} - \frac{x^2}{2bd(a + bF^{c+dx})^2 \log(F)} +
 \end{aligned}$$

**Mathematica** [A] time = 0.17, size = 177, normalized size = 0.97

$$\frac{bd^2 x^2 \log^2(F) F^{c+dx} (2a + bF^{c+dx}) - 2(a + bF^{c+dx})^2 \operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right) + 2(a + bF^{c+dx})^2 \log\left(\frac{bF^{c+dx}}{a} + 1\right) - 2dx \log(F)}{2a^2bd^3 \log^3(F) (a + bF^{c+dx})^2}$$

Antiderivative was successfully verified.



[In] Integrate[(F^(c + d\*x)\*x^2)/(a + b\*F^(c + d\*x))^3,x]

[Out] (b\*d^2\*F^(c + d\*x)\*(2\*a + b\*F^(c + d\*x))\*x^2\*Log[F]^2 + 2\*(a + b\*F^(c + d\*x))^2\*Log[1 + (b\*F^(c + d\*x))/a] - 2\*d\*(a + b\*F^(c + d\*x))\*x\*Log[F]\*(b\*F^(c + d\*x) + (a + b\*F^(c + d\*x))\*Log[1 + (b\*F^(c + d\*x))/a]) - 2\*(a + b\*F^(c + d\*x))^2\*PolyLog[2, -((b\*F^(c + d\*x))/a)])/(2\*a^2\*b\*d^3\*(a + b\*F^(c + d\*x))^2\*Log[F]^3)

**fricas** [B] time = 0.42, size = 379, normalized size = 2.08

$$a^2c^2 \log(F)^2 + 2a^2c \log(F) - \left( (b^2d^2x^2 - b^2c^2) \log(F)^2 - 2(b^2dx + b^2c) \log(F) \right) F^{2dx+2c} - 2 \left( (abd^2x^2 - abc^2) \log(F) \right) F^{2dx+2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c))^3,x, algorithm="fricas")

[Out] -1/2\*(a^2\*c^2\*log(F)^2 + 2\*a^2\*c\*log(F) - ((b^2\*d^2\*x^2 - b^2\*c^2)\*log(F)^2 - 2\*(b^2\*d\*x + b^2\*c)\*log(F))\*F^(2\*d\*x + 2\*c) - 2\*((a\*b\*d^2\*x^2 - a\*b\*c^2)\*log(F)^2 - (a\*b\*d\*x + 2\*a\*b\*c)\*log(F))\*F^(d\*x + c) + 2\*(2\*F^(d\*x + c)\*a\*b + F^(2\*d\*x + 2\*c)\*b^2 + a^2)\*dilog(-(F^(d\*x + c)\*b + a)/a + 1) - 2\*(a^2\*c\*log(F) + (b^2\*c\*log(F) + b^2)\*F^(2\*d\*x + 2\*c) + 2\*(a\*b\*c\*log(F) + a\*b)\*F^(d\*x + c) + a^2)\*log(F^(d\*x + c)\*b + a) + 2\*((b^2\*d\*x + b^2\*c)\*F^(2\*d\*x + 2\*c)\*log(F) + 2\*(a\*b\*d\*x + a\*b\*c)\*F^(d\*x + c)\*log(F) + (a^2\*d\*x + a^2\*c)\*log(F))\*log((F^(d\*x + c)\*b + a)/a)/(2\*F^(d\*x + c)\*a^3\*b^2\*d^3\*log(F)^3 + F^(2\*d\*x + 2\*c)\*a^2\*b^3\*d^3\*log(F)^3 + a^4\*b\*d^3\*log(F)^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c} x^2}{(F^{dx+c} b + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x^2/(F^(d\*x + c)\*b + a)^3, x)

**maple** [A] time = 0.08, size = 304, normalized size = 1.67

$$\frac{x^2}{2a^2bd \ln(F)} + \frac{cx}{a^2b d^2 \ln(F)} - \frac{(adx \ln(F) - 2b F^{dx+c} - 2a)x}{2(b F^{dx+c} + a)^2 ab d^2 \ln(F)^2} + \frac{c^2}{2a^2b d^3 \ln(F)} - \frac{x \ln\left(\frac{b F^c F^{dx}}{a} + 1\right)}{a^2b d^2 \ln(F)^2} - \frac{c \ln(F^c F^{dx})}{a^2b d^3 \ln(F)^2} - \frac{c \ln(F^c F^{dx})}{a^2b d^3 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)\*x^2/(b\*F^(d\*x+c)+a)^3,x)

[Out] 
$$-1/2*x*(a*d*x*\ln(F)-2*b*F^(d*x+c)-2*a)/\ln(F)^2/d^2/a/b/(b*F^(d*x+c)+a)^2+1/2*x^2/a^2/b/d/\ln(F)+1/b/a^2/d^2/\ln(F)*c*x+1/2/b/a^2/d^3/\ln(F)*c^2-1/b/a^2/d^2/\ln(F)^2*\ln(1/a*b*F^c*F^(d*x)+1)*x-1/b/a^2/d^3/\ln(F)^2*\ln(1/a*b*F^c*F^(d*x)+1)*c-1/b/a^2/d^3/\ln(F)^3*\text{polylog}(2,-1/a*b*F^c*F^(d*x))+1/b/a^2/d^3/\ln(F)^3*\ln(b*F^c*F^(d*x)+a)-1/b/a^2/d^3/\ln(F)^3*\ln(F^c*F^(d*x))-1/b/a^2/d^3/\ln(F)^2*c*\ln(F^c*F^(d*x))+1/b/a^2/d^3/\ln(F)^2*c*\ln(b*F^c*F^(d*x)+a)$$

**maxima** [A] time = 0.51, size = 214, normalized size = 1.18

$$\frac{adx^2 \log(F) - 2F^{dx}F^c bx - 2ax}{2(2F^{dx}F^c a^2 b^2 d^2 \log(F)^2 + F^{2dx}F^{2c} ab^3 d^2 \log(F)^2 + a^3 b d^2 \log(F)^2)} + \frac{\log(F^{dx})^2}{2a^2 b d^3 \log(F)^3} - \frac{\log\left(\frac{F^{dx}F^c b}{a} + 1\right) \log(F^{dx})}{a^2 b d^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x^2/(a+b\*F^(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$-1/2*(a*d*x^2*\log(F) - 2*F^(d*x)*F^c*b*x - 2*a*x)/(2*F^(d*x)*F^c*a^2*b^2*d^2*\log(F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*\log(F)^2 + a^3*b*d^2*\log(F)^2) + 1/2*\log(F^(d*x))^2/(a^2*b*d^3*\log(F)^3) - (\log(F^(d*x)*F^c*b/a + 1)*\log(F^(d*x)) + \text{dilog}(-F^(d*x)*F^c*b/a))/(a^2*b*d^3*\log(F)^3) + \log(F^(d*x)*F^c*b + a)/(a^2*b*d^3*\log(F)^3) - \log(F^(d*x))/(a^2*b*d^3*\log(F)^3)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c+dx} x^2}{(a + F^{c+dx} b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(c + d\*x)\*x^2)/(a + F^(c + d\*x)\*b)^3,x)

[Out] int((F^(c + d\*x)\*x^2)/(a + F^(c + d\*x)\*b)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2F^{c+dx}bx - adx^2 \log(F) + 2ax}{4F^{c+dx}a^2b^2d^2 \log(F)^2 + 2F^{2c+2dx}ab^3d^2 \log(F)^2 + 2a^3bd^2 \log(F)^2} + \frac{\int \frac{dx \log(F)}{a+be^c \log(F)e^{dx \log(F)}} dx + \int \left(-\frac{1}{a+be^c \log(F)e^{dx \log(F)}}\right)}{abd^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)\*x\*\*2/(a+b\*F\*\*(d\*x+c))\*\*3,x)

[Out] 
$$(2*F**(c + d*x)*b*x - a*d*x**2*\log(F) + 2*a*x)/(4*F**(c + d*x)*a**2*b**2*d**2*\log(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*\log(F)**2 + 2*a**3*b*d**2*\log(F)**2)$$

$$(F)^{**2} + (\text{Integral}(d*x*\log(F)/(a + b*\exp(c*\log(F))*\exp(d*x*\log(F))), x) + \text{Integral}(-1/(a + b*\exp(c*\log(F))*\exp(d*x*\log(F))), x))/(a*b*d^{**2}*\log(F)^{**2})$$

$$3.90 \quad \int \frac{F^{c+dx} x}{(a+bF^{c+dx})^3} dx$$

**Optimal.** Leaf size=106

$$-\frac{\log(a+bF^{c+dx})}{2a^2bd^2\log^2(F)} + \frac{x}{2a^2bd\log(F)} + \frac{1}{2abd^2\log^2(F)(a+bF^{c+dx})} - \frac{x}{2bd\log(F)(a+bF^{c+dx})^2}$$

[Out] 1/2/a/b/d^2/(a+bF^(d\*x+c))/ln(F)^2+1/2\*x/a^2/b/d/ln(F)-1/2\*x/b/d/(a+bF^(d\*x+c))^2/ln(F)-1/2\*ln(a+bF^(d\*x+c))/a^2/b/d^2/ln(F)^2

**Rubi [A]** time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2191, 2282, 44}

$$-\frac{\log(a+bF^{c+dx})}{2a^2bd^2\log^2(F)} + \frac{x}{2a^2bd\log(F)} + \frac{1}{2abd^2\log^2(F)(a+bF^{c+dx})} - \frac{x}{2bd\log(F)(a+bF^{c+dx})^2}$$

Antiderivative was successfully verified.

[In] Int[(F^(c + d\*x)\*x)/(a + bF^(c + d\*x))^3,x]

[Out] 1/(2\*a\*b\*d^2\*(a + bF^(c + d\*x))\*Log[F]^2) + x/(2\*a^2\*b\*d\*Log[F]) - x/(2\*b\*d\*(a + bF^(c + d\*x))^2\*Log[F]) - Log[a + bF^(c + d\*x)]/(2\*a^2\*b\*d^2\*Log[F]^2)

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2191

Int[((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((c + d\*x)^m\*(a + b\*(F^(g\*(e + f\*x))))^n)^(p + 1)/(b\*f\*g\*n\*(p + 1)\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*(p + 1)\*Log[F]), Int[(c + d\*x)^(m - 1)\*(a + b\*(F^(g\*(e + f\*x))))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rubi steps

$$\begin{aligned} \int \frac{F^{c+dx} x}{(a + bF^{c+dx})^3} dx &= -\frac{x}{2bd(a + bF^{c+dx})^2 \log(F)} + \frac{\int \frac{1}{(a+bF^{c+dx})^2} dx}{2bd \log(F)} \\ &= -\frac{x}{2bd(a + bF^{c+dx})^2 \log(F)} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, F^{c+dx}\right)}{2bd^2 \log^2(F)} \\ &= -\frac{x}{2bd(a + bF^{c+dx})^2 \log(F)} + \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right) dx, x, F^{c+dx}\right)}{2bd^2 \log^2(F)} \\ &= \frac{1}{2abd^2(a + bF^{c+dx}) \log^2(F)} + \frac{x}{2a^2bd \log(F)} - \frac{x}{2bd(a + bF^{c+dx})^2 \log(F)} - \frac{\log(a + bF^{c+dx})}{2a^2bd^2 \log^2(F)} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 98, normalized size = 0.92

$$\frac{bdx \log(F) F^{c+dx} (2a + bF^{c+dx}) - (a + bF^{c+dx}) ((a + bF^{c+dx}) \log(a + bF^{c+dx}) - a)}{2a^2bd^2 \log^2(F) (a + bF^{c+dx})^2}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c + d\*x)\*x)/(a + b\*F^(c + d\*x))^3, x]

[Out] (b\*d\*F^(c + d\*x)\*(2\*a + b\*F^(c + d\*x))\*x\*Log[F] - (a + b\*F^(c + d\*x))\*(-a + (a + b\*F^(c + d\*x))\*Log[a + b\*F^(c + d\*x)]))/(2\*a^2\*b\*d^2\*(a + b\*F^(c + d\*x))^2\*Log[F]^2)

**fricas [A]** time = 0.43, size = 148, normalized size = 1.40

$$\frac{F^{2dx+2c} b^2 dx \log(F) + (2abdx \log(F) + ab) F^{dx+c} + a^2 - (2F^{dx+c} ab + F^{2dx+2c} b^2 + a^2) \log(F^{dx+c} b + a)}{2(2F^{dx+c} a^3 b^2 d^2 \log(F)^2 + F^{2dx+2c} a^2 b^3 d^2 \log(F)^2 + a^4 b d^2 \log(F)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{2}*(F^{(2*d*x + 2*c)}*b^2*d*x*\log(F) + (2*a*b*d*x*\log(F) + a*b)*F^{(d*x + c)} + a^2 - (2*F^{(d*x + c)}*a*b + F^{(2*d*x + 2*c)}*b^2 + a^2)*\log(F^{(d*x + c)}*b + a))/(2*F^{(d*x + c)}*a^3*b^2*d^2*\log(F)^2 + F^{(2*d*x + 2*c)}*a^2*b^3*d^2*\log(F)^2 + a^4*b*d^2*\log(F)^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}x}{(F^{dx+c}b + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)\*x/(F^(d\*x + c)\*b + a)^3, x)

**maple** [A] time = 0.03, size = 127, normalized size = 1.20

$$-\frac{\ln(b e^{(dx+c)\ln(F)} + a)}{2a^2 b d^2 \ln(F)^2} + \frac{\frac{x e^{(dx+c)\ln(F)}}{ad \ln(F)} + \frac{bx e^{(2dx+2c)\ln(F)}}{2a^2 d \ln(F)} + \frac{e^{(dx+c)\ln(F)}}{2a d^2 \ln(F)^2} + \frac{1}{2b d^2 \ln(F)^2}}{(b e^{(dx+c)\ln(F)} + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)\*x/(b\*F^(d\*x+c)+a)^3,x)

[Out]  $(1/2/\ln(F)^2/a/d^2*\exp((d*x+c)*\ln(F))+1/\ln(F)/a/d*x*\exp((d*x+c)*\ln(F))+1/2/\ln(F)/a^2/d*b*x*\exp((d*x+c)*\ln(F))^2+1/2/\ln(F)^2/b/d^2)/(b*\exp((d*x+c)*\ln(F))+a)^2-1/2/\ln(F)^2/b/d^2/a^2*\ln(b*\exp((d*x+c)*\ln(F))+a)$

**maxima** [A] time = 0.48, size = 150, normalized size = 1.42

$$\frac{F^{2dx}F^{2c}b^2dx \log(F) + (2F^c ab dx \log(F) + F^c ab)F^{dx} + a^2}{2(2F^{dx}F^c a^3 b^2 d^2 \log(F)^2 + F^{2dx}F^{2c} a^2 b^3 d^2 \log(F)^2 + a^4 b d^2 \log(F)^2)} - \frac{\log\left(\frac{F^{dx}F^c b + a}{F^c b}\right)}{2a^2 b d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)\*x/(a+b\*F^(d\*x+c))^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}*(F^{(2*d*x)}*F^{(2*c)}*b^2*d*x*\log(F) + (2*F^c*a*b*d*x*\log(F) + F^c*a*b)*F^{(d*x)} + a^2)/(2*F^{(d*x)}*F^c*a^3*b^2*d^2*\log(F)^2 + F^{(2*d*x)}*F^{(2*c)}*a^2*b^3*d^2*\log(F)^2 + a^4*b*d^2*\log(F)^2) - 1/2*\log((F^{(d*x)}*F^c*b + a)/(F^c*b))/(a^2*b*d^2*\log(F)^2)$

**mupad [B]** time = 3.75, size = 155, normalized size = 1.46

$$\frac{\frac{F^c F^{dx}}{2 a d^2 \ln(F)^2} - \frac{F^c F^{dx} x}{a d \ln(F)} + \frac{F^{2c} F^{2dx} b}{2 a^2 d^2 \ln(F)^2} - \frac{F^{2c} F^{2dx} b x}{2 a^2 d \ln(F)}}{a^2 + F^{2c} F^{2dx} b^2 + 2 F^c F^{dx} a b} - \frac{\ln(a + F^c F^{dx} b)}{2 a^2 b d^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(c + d\*x)\*x)/(a + F^(c + d\*x)\*b)^3,x)

[Out] - ((F^c\*F^(d\*x))/(2\*a\*d^2\*log(F)^2) - (F^c\*F^(d\*x)\*x)/(a\*d\*log(F)) + (F^(2\*c)\*F^(2\*d\*x)\*b)/(2\*a^2\*d^2\*log(F)^2) - (F^(2\*c)\*F^(2\*d\*x)\*b\*x)/(2\*a^2\*d\*log(F)))/(a^2 + F^(2\*c)\*F^(2\*d\*x)\*b^2 + 2\*F^c\*F^(d\*x)\*a\*b) - log(a + F^c\*F^(d\*x)\*b)/(2\*a^2\*b\*d^2\*log(F)^2)

**sympy [A]** time = 0.23, size = 122, normalized size = 1.15

$$\frac{F^{c+dx}b - adx \log(F) + a}{4F^{c+dx}a^2b^2d^2 \log(F)^2 + 2F^{2c+2dx}ab^3d^2 \log(F)^2 + 2a^3bd^2 \log(F)^2} + \frac{x}{2a^2bd \log(F)} - \frac{\log\left(F^{c+dx} + \frac{a}{b}\right)}{2a^2bd^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)\*x/(a+b\*F\*\*(d\*x+c))\*\*3,x)

[Out] (F\*\*(c + d\*x)\*b - a\*d\*x\*log(F) + a)/(4\*F\*\*(c + d\*x)\*a\*\*2\*b\*\*2\*d\*\*2\*log(F)\*\*2 + 2\*F\*\*(2\*c + 2\*d\*x)\*a\*b\*\*3\*d\*\*2\*log(F)\*\*2 + 2\*a\*\*3\*b\*d\*\*2\*log(F)\*\*2) + x/(2\*a\*\*2\*b\*d\*log(F)) - log(F\*\*(c + d\*x) + a/b)/(2\*a\*\*2\*b\*d\*\*2\*log(F)\*\*2)

$$3.91 \quad \int \frac{F^{c+dx}}{(a+bF^{c+dx})^3} dx$$

Optimal. Leaf size=27

$$-\frac{1}{2bd \log(F) (a + bF^{c+dx})^2}$$

[Out]  $-1/2/b/d/(a+b*F^{(d*x+c)})^2/\ln(F)$

**Rubi [A]** time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2246, 32}

$$-\frac{1}{2bd \log(F) (a + bF^{c+dx})^2}$$

Antiderivative was successfully verified.

[In] Int[F^(c + d\*x)/(a + b\*F^(c + d\*x))^3,x]

[Out]  $-1/(2*b*d*(a + b*F^{(c + d*x)})^2*\text{Log}[F])$

Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2246

Int[((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)\*((a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(p\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[(a + b\*x)^p, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{F^{c+dx}}{(a+bF^{c+dx})^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx)^3} dx, x, F^{c+dx}\right)}{d \log(F)} \\ &= -\frac{1}{2bd (a + bF^{c+dx})^2 \log(F)} \end{aligned}$$



**Mathematica** [A] time = 0.03, size = 27, normalized size = 1.00

$$\frac{1}{2bd \log(F) (a + bF^{c+dx})^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c + d\*x)/(a + b\*F^(c + d\*x))^3,x]

[Out] -1/2\*1/(b\*d\*(a + b\*F^(c + d\*x))^2\*Log[F])

**fricas** [A] time = 0.41, size = 46, normalized size = 1.70

$$\frac{1}{2 \left( 2 F^{dx+c} ab^2 d \log(F) + F^{2dx+2c} b^3 d \log(F) + a^2 bd \log(F) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3,x, algorithm="fricas")

[Out] -1/2/(2\*F^(d\*x + c)\*a\*b^2\*d\*log(F) + F^(2\*d\*x + 2\*c)\*b^3\*d\*log(F) + a^2\*b\*d\*log(F))

**giac** [A] time = 0.33, size = 26, normalized size = 0.96

$$\frac{1}{2 \left( F^{dx} F^c b + a \right)^2 bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3,x, algorithm="giac")

[Out] -1/2/((F^(d\*x)\*F^c\*b + a)^2\*b\*d\*log(F))

**maple** [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{1}{2 \left( b F^{dx+c} + a \right)^2 bd \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^3,x)

[Out] -1/2/b/d/(b\*F^(d\*x+c)+a)^2/ln(F)

**maxima** [A] time = 0.43, size = 25, normalized size = 0.93

$$-\frac{1}{2(F^{dx+cb} + a)^2 bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3,x, algorithm="maxima")

[Out] -1/2/((F^(d\*x + c)\*b + a)^2\*b\*d\*log(F))

**mupad** [B] time = 3.63, size = 25, normalized size = 0.93

$$-\frac{1}{2bd \ln(F) (a + F^{c+dx} b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(a + F^(c + d\*x)\*b)^3,x)

[Out] -1/(2\*b\*d\*log(F)\*(a + F^(c + d\*x)\*b)^2)

**sympy** [B] time = 0.15, size = 53, normalized size = 1.96

$$-\frac{1}{4F^{c+dx} ab^2 d \log(F) + 2F^{2c+2dx} b^3 d \log(F) + 2a^2 bd \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)/(a+b\*F\*\*(d\*x+c))\*\*3,x)

[Out] -1/(4\*F\*\*(c + d\*x)\*a\*b\*\*2\*d\*log(F) + 2\*F\*\*(2\*c + 2\*d\*x)\*b\*\*3\*d\*log(F) + 2\*a\*\*2\*b\*d\*log(F))

$$3.92 \quad \int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x} dx$$

Optimal. Leaf size=65

$$-\frac{\text{Int}\left(\frac{1}{x^2(a+bF^{c+dx})^2}, x\right)}{2bd \log(F)} - \frac{1}{2bdx \log(F) (a+bF^{c+dx})^2}$$

[Out] -1/2/b/d/(a+bF^(d\*x+c))^2/x/ln(F)-1/2\*Unintegrable(1/(a+bF^(d\*x+c))^2/x^2, x)/b/d/ln(F)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x} dx$$

Verification is Not applicable to the result.

[In] Int[F^(c + d\*x)/((a + bF^(c + d\*x))^3\*x), x]

[Out] -1/(2\*b\*d\*(a + bF^(c + d\*x))^2\*x\*Log[F]) - Defer[Int][1/((a + bF^(c + d\*x))^2\*x^2), x]/(2\*b\*d\*Log[F])

Rubi steps

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x} dx = -\frac{1}{2bd (a+bF^{c+dx})^2 x \log(F)} - \frac{\int \frac{1}{(a+bF^{c+dx})^2 x^2} dx}{2bd \log(F)}$$

Mathematica [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x} dx$$

Verification is Not applicable to the result.

[In] Integrate[F^(c + d\*x)/((a + bF^(c + d\*x))^3\*x), x]

[Out] Integrate[F^(c + d\*x)/((a + b\*F^(c + d\*x))^3\*x), x]

**fricas** [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{F^{dx+c}}{3F^{dx+c}a^2bx + 3F^{2dx+2c}ab^2x + F^{3dx+3c}b^3x + a^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3/x,x, algorithm="fricas")

[Out] integral(F^(d\*x + c)/(3\*F^(d\*x + c)\*a^2\*b\*x + 3\*F^(2\*d\*x + 2\*c)\*a\*b^2\*x + F^(3\*d\*x + 3\*c)\*b^3\*x + a^3\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(F^{dx+c}b + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3/x,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)/((F^(d\*x + c)\*b + a)^3\*x), x)

**maple** [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(bF^{dx+c} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^3/x,x)

[Out] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^3/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{adx \log(F) + F^{dx}F^c b + a}{2(2F^{dx}F^c a^2 b^2 d^2 x^2 \log(F)^2 + F^{2dx}F^{2c} ab^3 d^2 x^2 \log(F)^2 + a^3 b d^2 x^2 \log(F)^2)} - \int \frac{dx \log(F) + 2}{2(F^{dx}F^c ab^2 d^2 x^3 \log(F)^2 + a^2 b d^2 x^2 \log(F)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3/x,x, algorithm="maxima")

[Out] -1/2\*(a\*d\*x\*log(F) + F^(d\*x)\*F^c\*b + a)/(2\*F^(d\*x)\*F^c\*a^2\*b^2\*d^2\*x^2\*log(F)^2 + F^(2\*d\*x)\*F^(2\*c)\*a\*b^3\*d^2\*x^2\*log(F)^2 + a^3\*b\*d^2\*x^2\*log(F)^2) -

integrate(1/2\*(d\*x\*log(F) + 2)/(F^(d\*x)\*F^c\*a\*b^2\*d^2\*x^3\*log(F)^2 + a^2\*b\*d^2\*x^3\*log(F)^2), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c+dx}}{x(a + F^{c+dx}b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(x\*(a + F^(c + d\*x)\*b)^3), x)

[Out] int(F^(c + d\*x)/(x\*(a + F^(c + d\*x)\*b)^3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-F^{c+dx}b - adx \log(F) - a}{4F^{c+dx}a^2b^2d^2x^2 \log(F)^2 + 2F^{2c+2dx}ab^3d^2x^2 \log(F)^2 + 2a^3bd^2x^2 \log(F)^2} - \frac{\int \frac{dx \log(F)}{ax^3+bx^3e^{c \log(F)}e^{dx \log(F)}} dx + \int \frac{1}{ax^3+bx^3e^c}}{2abd^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)/(a+b\*F\*\*(d\*x+c))\*\*3/x, x)

[Out] (-F\*\*(c + d\*x)\*b - a\*d\*x\*log(F) - a)/(4\*F\*\*(c + d\*x)\*a\*\*2\*b\*\*2\*d\*\*2\*x\*\*2\*log(F)\*\*2 + 2\*F\*\*(2\*c + 2\*d\*x)\*a\*b\*\*3\*d\*\*2\*x\*\*2\*log(F)\*\*2 + 2\*a\*\*3\*b\*d\*\*2\*x\*\*2\*log(F)\*\*2) - (Integral(d\*x\*log(F)/(a\*x\*\*3 + b\*x\*\*3\*exp(c\*log(F))\*exp(d\*x\*log(F))), x) + Integral(2/(a\*x\*\*3 + b\*x\*\*3\*exp(c\*log(F))\*exp(d\*x\*log(F))), x))/(2\*a\*b\*d\*\*2\*log(F)\*\*2)

$$3.93 \quad \int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x^2} dx$$

Optimal. Leaf size=63

$$-\frac{\text{Int}\left(\frac{1}{x^3(a+bF^{c+dx})^2}, x\right)}{bd \log(F)} - \frac{1}{2bdx^2 \log(F) (a + bF^{c+dx})^2}$$

[Out]  $-1/2/b/d/(a+bF^{(d*x+c)})^2/x^2/\ln(F)-\text{Unintegrable}(1/(a+bF^{(d*x+c)})^2/x^3,x)/b/d/\ln(F)$

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{F^{c+dx}}{(a + bF^{c+dx})^3 x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[F^{(c + d*x)/((a + bF^{(c + d*x)})^3*x^2)}, x]$

[Out]  $-1/(2*b*d*(a + bF^{(c + d*x)})^2*x^2*\text{Log}[F]) - \text{Defer}[\text{Int}][1/((a + bF^{(c + d*x)})^2*x^3), x]/(b*d*\text{Log}[F])$

Rubi steps

$$\int \frac{F^{c+dx}}{(a + bF^{c+dx})^3 x^2} dx = -\frac{1}{2bd (a + bF^{c+dx})^2 x^2 \log(F)} - \frac{\int \frac{1}{(a+bF^{c+dx})^2 x^3} dx}{bd \log(F)}$$

**Mathematica [A]** time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{F^{c+dx}}{(a + bF^{c+dx})^3 x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[F^{(c + d*x)/((a + bF^{(c + d*x)})^3*x^2)}, x]$

[Out] Integrate[F^(c + d\*x)/((a + b\*F^(c + d\*x))^3\*x^2), x]

**fricas** [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{F^{dx+c}}{3 F^{dx+c} a^2 b x^2 + 3 F^{2 dx+2 c} a b^2 x^2 + F^{3 dx+3 c} b^3 x^2 + a^3 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3/x^2,x, algorithm="fricas")

[Out] integral(F^(d\*x + c)/(3\*F^(d\*x + c)\*a^2\*b\*x^2 + 3\*F^(2\*d\*x + 2\*c)\*a\*b^2\*x^2 + F^(3\*d\*x + 3\*c)\*b^3\*x^2 + a^3\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(F^{dx+c}b + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3/x^2,x, algorithm="giac")

[Out] integrate(F^(d\*x + c)/((F^(d\*x + c)\*b + a)^3\*x^2), x)

**maple** [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{F^{dx+c}}{(b F^{dx+c} + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^3/x^2,x)

[Out] int(F^(d\*x+c)/(b\*F^(d\*x+c)+a)^3/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{adx \log(F) + 2 F^{dx} F^c b + 2 a}{2 (2 F^{dx} F^c a^2 b^2 d^2 x^3 \log(F)^2 + F^{2 dx} F^{2 c} a b^3 d^2 x^3 \log(F)^2 + a^3 b d^2 x^3 \log(F)^2)} - \int \frac{dx \log(F) + 3}{F^{dx} F^c a b^2 d^2 x^4 \log(F)^2 + a^2 b d^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(d\*x+c)/(a+b\*F^(d\*x+c))^3/x^2,x, algorithm="maxima")

[Out] -1/2\*(a\*d\*x\*log(F) + 2\*F^(d\*x)\*F^c\*b + 2\*a)/(2\*F^(d\*x)\*F^c\*a^2\*b^2\*d^2\*x^3\*log(F)^2 + F^(2\*d\*x)\*F^(2\*c)\*a\*b^3\*d^2\*x^3\*log(F)^2 + a^3\*b\*d^2\*x^3\*log(F)^2)

2) - integrate((d\*x\*log(F) + 3)/(F^(d\*x)\*F^c\*a\*b^2\*d^2\*x^4\*log(F)^2 + a^2\*b\*d^2\*x^4\*log(F)^2), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c+dx}}{x^2 (a + F^{c+dx} b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c + d\*x)/(x^2\*(a + F^(c + d\*x)\*b)^3), x)

[Out] int(F^(c + d\*x)/(x^2\*(a + F^(c + d\*x)\*b)^3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-2F^{c+dx}b - adx \log(F) - 2a}{4F^{c+dx}a^2b^2d^2x^3 \log(F)^2 + 2F^{2c+2dx}ab^3d^2x^3 \log(F)^2 + 2a^3bd^2x^3 \log(F)^2} \int \frac{dx \log(F)}{ax^4+bx^4e^{c \log(F)}e^{dx \log(F)}} dx + \int \frac{3}{ax^4+bx^4e^{c \log(F)}e^{dx \log(F)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(d\*x+c)/(a+b\*F\*\*(d\*x+c))\*\*3/x\*\*2, x)

[Out] (-2\*F\*\*(c + d\*x)\*b - a\*d\*x\*log(F) - 2\*a)/(4\*F\*\*(c + d\*x)\*a\*\*2\*b\*\*2\*d\*\*2\*x\*\*3\*log(F)\*\*2 + 2\*F\*\*(2\*c + 2\*d\*x)\*a\*b\*\*3\*d\*\*2\*x\*\*3\*log(F)\*\*2 + 2\*a\*\*3\*b\*d\*\*2\*x\*\*3\*log(F)\*\*2) - (Integral(d\*x\*log(F)/(a\*x\*\*4 + b\*x\*\*4\*exp(c\*log(F))\*exp(d\*x\*log(F))), x) + Integral(3/(a\*x\*\*4 + b\*x\*\*4\*exp(c\*log(F))\*exp(d\*x\*log(F))), x))/(a\*b\*d\*\*2\*log(F)\*\*2)



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],

```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
            ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```



```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(6,m1) #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```